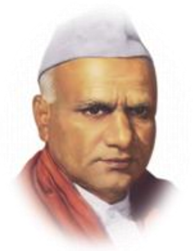

"ज्ञान, विज्ञान आणि सुसंस्कार यासाठी शिक्षण प्रसार"

शिक्षणमहर्षी- डॉ बापूजी साळुंखे .

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DEPARTMENT OF PHYSICS

QUESTION BANK

B.Sc. Part-III, Semester-V, PHYSICS Paper-X

DSE-E2 Quantum Mechanics

❖ Multiple Choice Questions

1. In case of rectangular potential barrier, the relation is…………….

   a) R-T=1 b) R+T=1 c) R+T=0 d) R-T=0

2. The quantity $|\Psi^2|$ is called…………

   a) probability density b) probability current density

   c) probability current d) current density

3. The lowest energy of harmonic oscillator is obtained by putting n equal to …………

   a) 0 b) 1 c) ½ d) -1

4. In a normal state of the atom, the number of electrons in a sub shell of the atom is given by,

   a) $l\sqrt{(l+1)}$ b) $(2l+1)$ c) $(2l-1)$ d) $2(2l+1)$

5. The energy spectrum of a particle in one dimensional rigid box has nature of

   a) exponential increasing b) exponential decreasing

   c) infinite sequence of equidistance energy levels

   d) infinite sequence of discrete energy levels

6. The parity of the wave function is odd when………….

   a) $\Psi(-x)=-\Psi(x)$ b) $\Psi(x)=x^2$ c) $\Psi(-x)=\Psi(x)$ d) $\Psi(x)=x^{-2}$

7. Which of the following has even parity?

   a) $f(x)=x$ b) $f(x)=x^2$ c) $f(x)=x^3$ d) $f(x)=\sin(x)$

8. Which of the following condition will be obeyed by Ψ.

   a) $|\Psi|\text{--->}0$ as $r\text{--->}\infty$ b) $|\Psi|\text{--->}0$ as $r\text{--->}0$

   c) $|\Psi|\text{--->}\infty$ as $r\text{--->}0$ d) $|\Psi|\text{--->}\infty$ as $r\text{--->}\infty$

9. Which of the following wave function represents a free particle moving alone negative x-axis?

a) A $\sin(kx - \omega t)$ b) $Ae^{i(kx+\omega t)}$ c) A $\cos(kx - \omega t)$ d) $Ae^{-i(kx-\omega t)}$

10. Physical significance of $|\psi^* \psi|$ is …….

a) Probability b) Probability current

c) Probability density d) Probability function

11. What is the mathematical condition for convergence of a wave function?

a) $\psi \to 0 \; as \; x \to 0$ b) $\psi \to \infty \; as \; x \to 0$

c) $\psi \to \infty \; as \; x \to \infty$ d) $\psi \to 0 \; as \; x \to \infty$

12. If ψ_m and ψ_n are orthogonal wave functions then $\int {\psi_m}^* \psi_n \, dx =$ ………

a) Indefinite b) Zero c) One d) None of the above

13. Which of the following relation or relations are corrects?

a) $\hbar = \frac{E}{k}$ b) $\hbar = \frac{p}{k}$ c) $\hbar = \frac{E}{\omega}$ d) both b and c

14. Which of the following is momentum operator$(\widehat{p_x})$?

a) $-i\hbar \frac{\partial}{\partial t}$ b) $i\hbar \frac{\partial}{\partial t}$ c) $-i\hbar \frac{\partial}{\partial x}$ d) $i\hbar \frac{\partial}{\partial x}$

15. Eigenvalue of Hamiltonian operator is….

a) Kinetic energy b) Potential energy c) both a and b d) Total energy

16. Commutation relation of αA and B is…..

a) $\alpha^2[A, B]$ b) $\alpha[A, B]$ c) $\alpha A + \alpha B$ d) $A + \alpha B$

17. $[x_i, p_j] = i\hbar \, \delta_{ij}$, $\delta_{ij} = 0$ for …..?

a) $i = j$ b) $i \neq j$ c) both a and b d) None of the above

18. Eigenvalues of the even parity is…..

a) 0 b) +1 c) -1 d) None of the above

19. $[x, p_y]$ =?

a) 0 b) $i\hbar$ c) $-i\hbar$ d) 1

20. Raising operator is given by …

a) $L_+ = L_x - iL_y$ b) $L_+ = L_x + iL_y$ c) $L_+ = L_x + iL_z$ d) $L_+ = L_y + iL_x$

21. The eigenvalue of the operator $\frac{\partial}{\partial x}$ for e^x is …..

a) +1 b) -1 c) 0 d) None of the above

22. $[L_y, L_x] =?$

a) $i\hbar L_z$ b) $-i\hbar L_z$ c) 0 d) $i\hbar$

23. Energy of the n^{th}state for the one dimensional harmonic oscillator is….

a) $\left(n + \frac{1}{2}\right)\hbar\omega$ b) $\frac{1}{2}\hbar\omega$ c) 0 d) $n\hbar$

24. Ground state energy of one dimensional symmetric harmonic oscillator is….

a) $\frac{1}{2}\hbar\omega$ b) $\hbar\omega$ c) 0 d) $\frac{3}{2}\hbar\omega$

25. Relation between reflection coefficient (R) and transmission coefficient (T) is ….

a) $R + T = 0$ b) $R = 1 - T$ c) $T = R - 1$ d) $R - T = 1$

26. Energy of 1^{st}excited state for 1D-box having length (a) is ….

a) $\frac{\pi^2\hbar^2}{2ma^2}$ b) $\frac{4\,\pi^2\hbar^2}{8\,ma^2}$ c) $\frac{2\pi^2\hbar^2}{ma^2}$ d) $\frac{\pi^2\hbar^2}{8ma^2}$

27. Ground state of 3D-box is given by ψ_{nlm}, what are the values of *n*, *l* and *m* are …….respectively.

a) 0,0,0 b) 0,0,1 c) 1,1,1 d) 1,0,0

28. Eigenvalue of L^2 is given by …

a) $(m + 1)\hbar$ b) $l(l + 1)\hbar$ c) $l(l + 1)\hbar^2$ d) $m_l\hbar$

29. $[H, P] =?$, where H is Hamiltonian operator and p= momentum operator.

a) 0 b) $i\hbar$ c) $\hbar\omega$ d) $-i\hbar$

30. Normalization constant for 1D-box having length (L) is …..

a) $\sqrt{\frac{L}{2}}$ b) $\sqrt{\frac{2}{L}}$ c) $\sqrt{2L}$ d) $\sqrt{\frac{1}{2L}}$

31. For $E < V_0$, the reflection coefficient (R) in classical regime is …

a) 1 b) 0 c) 0.5 d) -1

32. For *l=1*, possible values of m_l are …..

a) 3 b) 1 c) 2 d) 0

33. Energy of 1^{st}excited state of the hydrogen atom is ….

a) -13.6 eV b) 0 eV c) -3.4 eV d) 1.51 eV

34. In hydrogen atom, potential energy of the electron is …

a) $\frac{-e^2}{r^2}$ b) $\frac{-e}{r^2}$ c) $\frac{-e^2}{r}$ d) $\frac{-e}{r}$

35. For n=2, possible values of m_l are .....

a) +1,0,-1 b) -1,+1 c) -2,+2 d) +2,+1,0,,-1,-2

36. In hydrogen atom, potential is proportional to r^n, where n = ?

a) n =1 b) n = -1 c) n = 2 d) n = -2

37. Eigenvalue of the orbital angular momentum (L) is given as.....

a) $l(l+1)\hbar$ b) $\sqrt{l(l+1)}\hbar$ c) $\sqrt{l(l+1)\hbar}$ d) $\sqrt{l(l+1)}\hbar^2$

38. How many number of electrons could be filled inside the n = 3?, where n is the principle quantum number.

a) 9 b) 7 c) 18 d) 10

❖ Long answer questions

1. Derive Schrodinger's time dependent wave equation for matter wave in one-dimension.
2. Derive Schrodinger's time independent wave equation for matter wave in one-dimension.
3. Derive Schrodinger's time dependent wave equation and show that $\frac{dP}{dt} + \nabla \vec{J} = 0$, where P is probability density and is probability current density.
4. Give the physical interpretation of wave function and state conditions that wave function must satisfy.
5. What is normalization of wave function? How it is mathematically expressed? Find out the normalization factor of Schrödinger's wave function. Prove that normalization is independent of time.
6. If the wave function is given by $\Psi\ (x) = Ae^{-\alpha x^2}$, find out where the particle is most likely to be found and where it is least likely to be found.
7. Obtain Schrödinger's time dependent wave equation and separate it into space and time dependent parts and show that probability density P and probability current density $\vec{J}$ satisfy the continuity equation, $\frac{dP}{dt} + \nabla.\vec{J} = 0$
8. Explain the physical significance of this equation in quantum mechanics.
9. Give the formulation of time dependent Schrödinger's wave equation. Discuss the interpretation of position probability density and normalization of wave function.
10. Show that probability current density $\vec{J}$ together with probability density P satisfy the equation of continuity $\frac{dP}{dt} + \nabla.\vec{J} = 0$ and show that $\vec{J}$ has the form $\vec{J} = \frac{\hbar}{2mi}(\Psi^* \, grad\, \Psi - \Psi\, grad\, \Psi^*)$.
11. If p_x is the x component of the momentum and V is the potential at the position x, prove that

a) $\frac{d}{dt} < p_x > = < -\frac{\partial V}{\partial x} >$

b) $\frac{d}{dt} < x > = < -\frac{p_x}{m} >$

12. Define an operator. Hence obtain expressions for (i) Linear momentum operator, (ii) Total energy operator, (iii) Kinetic energy operator, (iv) Hamiltonian operator, (v) Parity operator with examples.
13. Derive the expressions for the operators of L_x, L_y, L_z and L^2 spherical polar co-ordinates.
14. Find eigen value of L^2 and L_z.
15. Explain briefly the concept of parity as applied to wave function. When will the wave function have odd or even parity? Discuss parity operator.
16. Explain the concept of parity with suitable example. Show that parity operator has Eigen values +1 and -1.
17. Show that $[L^2, L_x] = 0$.
18. Derive the expressions for raising and lowering operators.
19. Write a note on communication relation in quantum mechanics.
20. Discuss the fundamental commutation relation in quantum mechanics.
21. Show that $[x,p_x] = i\hbar$.
22. Prove the commutation relation of L_z with L_+ and L_- i.e. $[L_z, L_\pm] = \pm\hbar L_\pm$.
23. Show that $[L_+, L_-] = 2\hbar L_z$.
24. Discuss different possible orientations of total angular momentum with respect to the axis of rotation.
25. Explain in brief space quantisation of $\vec{L}$ and precession.
26. Show that $\hat{x}p_x - \widehat{p_x} = i\hbar$ and $[\hat{x}, \widehat{p_y}] = 0$.
27. What is meant by the expectation value of dynamical variable. Find the expectation value for the linear momentum, position and kinetic energy of system in terms of corresponding operators.
28. Prove that $\widehat{L_z}\Psi_{nlm} = \hbar m\, \Psi_{nlm}$.
29. Discuss the space quantisation of angular momentum explaining the meaning of quantisation of its magnitude and direction.
30. Prove that $L^2 = L_x^2 + L_y^2 + L_z^2 = -\hbar^2 = \left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\ \frac{\partial}{\partial\theta}\right) + \frac{1}{sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]$.
31. Prove that $L^2 = \frac{1}{2}\ [L_+L_- + L_-L_+] + L_z^2$.
32. Obtain the commutation relation for L_x, L_y and L_z. Show that L^2 commutes with any of the three components.

33. Deduce communication relation for the components of Lx, Ly and Lz of the total angular momentum and show that all the three components commute with $L^2 = L_x^2 + L_y^2 + L_z^2$. Find Eigen values of L^2 and L_z.

❖ Short answer type questions

1. Write short notes on:
   i) Orthogonal and normalization conditions of wave function.
   ii) Time-dependent Schrodinger's equation
   iii) Normalization of wave function
   iv) Orthogonality of wave function.
   v) Physical interpretation of the wave function.
   vi) Wave function and probability density.
2. Explain Eigen functions and Eigen values.
3. What are the conditions for the wave function to be called as well behaved wave function?
4. Explain probability current density and probability density.
5. What do you mean by normalization and probability value of wave function?
6. State physical significance of wave function. What is well behaved wave function?
7. Derive expressions for raising and lowering operators.
8. Explain ladder operators.
9. Define operator. Obtain an expression for total energy operator, kinetic energy operator.
10. What is meant expectation value of dynamical variable? Obtain expectation value for linear momentum and position in terms of corresponding operator.
11. Discuss different possible orientations of total angular momentum with respect to axis of rotation.
12. Explain briefly the concept of parity as applied to wave function. When will the wave function have odd or even parity ?
13. Obtain the commutation relations of Lz with L+ and L-.
14. Derive the commutation between momentum and Hamiltonian.
15. Obtain the expectation values of dynamical quantities.

❖ Numerical Problems

1. Determine the normalization constant of normalized wave function $\Psi\,(x) = qe^{-\alpha x^2}$. [**Ans:** one]
2. Obtain the probability current density associated with function $\Psi\,(x) = Be^{ikx}$, where $\Psi\,(x)$ is wave function associated with the free particle moving with the velocity v in positive x direction.

[**Ans.** $J_x = v\, B^2$]

3. $\left(x, \frac{\partial}{\partial x}\right) = -1$

4. $\left(x, \frac{\partial^2}{\partial x^2}\right) = -2\frac{\partial}{\partial x}$

5. $[L^2, L_+] = 0$

6. $[L_x, r^2] = [L_y, r^2] = [L_z, r^2] = 0$, where $r^2 = x^2 + y^2 + z^2$.

7. $[L_z, x] = i\hbar y$, $[L_z, y] = -i\hbar x$, $[L_z, z] = 0$.

8. $[L_z, p_x] = i\hbar p_y$, $[L_z, p_y] = -i\hbar p_x$, $[L_z, p_z] = 0$.