SHIVAJI UNIVERSITY, KOLHAPUR

****

Accredited By NAAC with 'A' Grade

CHOICE BASED CREDIT SYSTEM

Syllabus For

B.Sc. Mathematics Part -II

SEMESTER III AND IV

(Syllabus to be implemented from June, 2019 onwards.)

B.Sc. (Mathematics) (Part-II) (Semester–III)

(Choice Based Credit System)

(Introduced from June 2019)

Course Code: DSC – 5C

Title of Course: Real Analysis–I

Theory: 32Hrs. (40 Lectures of 48 minutes) Marks – 50 (Credits: 02)

Course Objectives: Upon successful completion of this course, the student will be able to:

(1) understand types of functions and how to identify them.

(2) use mathematical induction to prove various properties.

(3) understand the basic ideas of Real Analysis.

(4) prove order properties of real numbers, completeness property and the Archimedean property.

Unit1:Functions and Countable sets **(16hrs)**

1.1. **Sets.**

1.1.1. Revision of basic notions in sets.

1.1.2. Operations on sets:-Union, Intersection, Complement, Relative complement, Cartesian product of sets, Relation.

1.2. **Functions**

1.2.1. Definitions: Function, Domain, Co-domain, Range, Graph of a function, Direct image and Inverse image of a subset under a function. Examples of direct image and inverse image of a subset.

1.2.2. **Theorem:** If $f: A \to B$ and if $X \subseteq B, Y \subseteq B$, then

$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$

1.2.3. **Theorem:** If $f: A \to B$ and if $X \subseteq B, Y \subseteq B$, then

$f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$

1.2.4. **Theorem:** If $f: A \to B$ and if $X \subseteq A, Y \subseteq A$, then $f(X \cup Y) = f(X) \cup f(Y)$

1.2.5. **Theorem:** If $f: A \to B$ and if $X \subseteq A, Y \subseteq A$, then $f(X \cap Y) \subseteq f(X) \cap f(Y)$

1.2.6. **Definitions:** Injective, Surjective and Bijective functions (1-1 correspondance) Inverse function.

1.2.7. **Proposition:** If $f: A \to B$is injective and $E \subseteq A$, then $f^{-1}(f(E)) = E$.

1.2.8. **Proposition:** If $f: A \to B$is surjective and $H \subseteq B$, then $f(f^{-1}(H)) = H$.

1.2.9. **Definition:** Composite function, Restriction and Extension of a function.

1.2.10. **Theorem:** Let $f: A \to B$ and $g: B \to C$ be functions and let H be a subset of C. Then $(g \circ f)^{-1}(H) = f^{-1}(g^{-1}(H))$.

1.2.11. **Theorem:** Composition of two bijective functions is a bijective function.

1.2.12. **Examples**

1.3. **Mathematical Induction**

1.3.1. **Principle of Mathematical Induction** (without proof), Well ordering property of natural numbers

1.3.2. **Principle of Mathematical Induction** (second version: Statement only), Principle of strong induction (Statement only).

1.3.3. Examples based on 1.3.1 and 1.3.2

1.4. **Countable Sets**

1.4.1. **Definitions:** Denumerable sets, Countable sets, uncountable sets.

1.4.2. **Examples of denumerable sets:** Set of Natural numbers, Set of Integers, Set of even natural numbers and odd natural numbers.

1.4.3. **Proposition:** Union of two disjoint denumerable sets is denumerable.

1.4.4. **Theorem:** If A_m is a countable set for each $m \in \mathbb{N}$, then the union $A = \bigcup_{m=1}^{\infty} A_m$ is countable. (Countable union of countable sets is countable)

1.4.5. **Theorem:**The set of Rational numbers is denumerable.

1.4.6. **Theorem:** Any subset of countable set is countable.

1.4.7. **Theorem:** The closed interval $[0,1]$ is uncountable.

1.4.8. **Corollary:** The set of all real numbers is uncountable.

1.4.9. **Examples**

Unit2:The Real numbers **(16hrs)**

2.1. Algebraic and Order Properties of $\mathbb{R}$.

2.1.1. Algebraic properties of real numbers.

2.1.2. **Theorem:**Let $a, b, c \in \mathbb{R}$.

(a) If $a > b$ and $b > c$, then $a > c$

(b) If $a > b$, then $a + c > b + c$

(c) If $a > b$and $c > 0$, then $ac > bc$. If $a > b$ and $c < 0$, then $ac < bc$

2.1.3. **Theorem:**

(a) If $a \in \mathbb{R}$and $a \neq 0$, then $a^2 > 0$.

(b) $1 > 0$

(c) If $n \in \mathbb{N}$, then $n > 0$.

2.1.4. **Theorem:** If $a \in \mathbb{R}$ is such that $0 \leq a < \epsilon$ for every $\epsilon > 0$then $a = 0$.

2.1.5. **Theorem:** If $ab > 0$, then either (i) $a > 0$ and $b > 0$ or (ii) $a < 0$ and $b < 0$

2.1.6. **Corollary:** If $ab < 0$, then either (i) $a < 0$ and $b > 0$ or (ii) $a > 0$ and $b < 0$

2.2. Inequalities

2.2.1. If$a \geq 0$, $b \geq 0$, then prove that

$a < b \Leftrightarrow a^2 < b^2 \Leftrightarrow \sqrt{a} < \sqrt{b}$.

2.2.2. **Arithmetic-Geometric mean inequality** (with proof).

2.2.3. **Bernoulli's inequality** (with proof).

2.3. Absolute Value and neighbourhood

2.3.1. **Definition:** Absolute value of a real number

2.3.2. **Theorem:**

(a) $|ab| - |a|.|b|$ for all $a, b \in \mathbb{R}$

(b) $|a|^2 = a^2$ for all $a \in \mathbb{R}$

(c) If $c \geq 0$, then $|a| \leq c$ if and only if $-c \leq a \leq c$

(d) $-|a| \leq a \leq |a|$ for all $a \in \mathbb{R}$

2.3.3. **Theorem (Triangle inequality):**If $a, b \in \mathbb{R}$, then $|a + b| \leq |a| + |b|$.

2.3.4. **Corollary:**If $a, b \in \mathbb{R}$, then (i) $\big||a| - |b|\big| \leq |a - b|$ (ii) $|a - b| \leq |a| + |b|$

2.3.5. **Corollary:** If $a_1, a_2, \dots, a_n$ are any real numbers then

$$|a_1 + a_2 + \dots + a_n| \leq |a_1| + |a_2| + \dots + |a_n|$$

2.3.6. **Examples on inequalities**

2.3.7. **Definition:**ε - Neighbourhood.

2.3.8. **Theorem:**Let $a \in \mathbb{R}$. If x belongs to the neighbourhood $V_\epsilon(a)$ for every $\epsilon > 0$then $x = a$.

2.4. Completeness property of $\mathbb{R}$

2.4.1. **Definitions:** Lower bound, Upper bound of a subset of $\mathbb{R}$, Bounded set, Supremum (least upper bound), Infimum (greatest lower bound).

2.4.2. **The completeness property of $\mathbb{R}$ (The supremum property)**

2.4.3. Applications of the supremum property.

2.4.4. **Theorem: (Archimedean Property)** If $x \in \mathbb{R}$, then there exists $n_x \in \mathbb{N}$ such that $x \leq n_x$.

2.4.5. **Corollary:** If $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, then inf$S = 0$.

2.4.6. **Corollary:** If $t > 0$, then there exists $n_t \in \mathbb{N}$ such that $0 < \frac{1}{n_t} < t$.

2.4.7. **Corollary:** If $y > 0$, then there exists $n_y \in \mathbb{N}$ such that $n_y - 1 < y < n_y$.

2.4.8. **Theorem:** There exists a positive real number x such that $x^2 = 2$.

2.4.9. **Theorem: (The Density theorem)**If x and y are any real numbers with $x < y$, then there exists a rational number $r \in \mathbb{Q}$ such that $x < r < y$.

2.4.10. **Corollary:** If x and y are real numbers with $x < y$, then there existsan irrational number z such that $x < z < y$.

2.5. Intervals

2.5.1. **Characterization theorem:** If S is a subset of $\mathbb{R}$ that contains at least two points and has the property

if $x, y \in S$ and $x < y$, thenthe closed interval$[x, y] \subseteq S$,

then$S$ is an interval.

Recommended Book

1) **Introduction to Real Analysis**, Robert G. Bartle and Donald R. Sherbert, Wiley Student Edition, 2010.

Reference Books:

1) **Methods of Real Analysis**, R. R. Goldberg, Oxford and IBH Publishing House, New Delhi, 1970.
2) **A Basic Course in Real Analysis**, Ajit Kumar and S. Kumaresan, CRC Press, Taylor & Francis Group, 2014.
3) **Real Analysis**, HariKishan, Pragati Prakashan, fourth revised edition 2012
4) **An Introduction to Real Analysis**, P. K. Jain and S. K. Kaushik, S. Chand& Co., New Delhi, 2000.

B.Sc. Part II (Mathematics) (Semester III)
(Choice Based Credit System)
(Introduced from June 2019 onwards)

Course Code: DSC – 6C

Title of Course: Algebra–I

Theory: 32 hrs. (40 lectures of 48 minutes)   Marks – 50 (Credits: 02)

Course Objectives : Upon successful completion of this course, the student will be able to:

1. understand properties of matrices
2. solve System of linear homogeneous equations and linear non-homogeneous equations.
3. find Eigen values and Eigen vectors.
4. construct permutation group and relate it to other groups.
5. classify the various types of groups and subgroups.

Unit1: Matrices and Relations

1.5. **Definitions:** Hermitian and Skew Hermitian matrices.
1.6. **Properties** of Hermitian and Skew Hermitian matrices.
1.7. Rank of a matrix, Row-echelon form and reduced row echelon form.
1.8. System of linear homogeneous equations and linear non-homogeneous equations.
  1.8.1. Condition for consistency
  1.8.2. Nature of the general solution
  1.8.3. Gaussian elimination and Gauss Jordon method (Using row-echelon form and reduced row echelon form).
  1.8.4. Examples on 1.4.1 and 1.4.3
1.9. The characteristic equation of a matrix, Eigen values, Eigen vectors of a matrix.
1.10. **Cayley Hamilton theorem**
1.11. Applications of Cayley Hamilton theorem (Examples).
1.12. **Relations**: Definition, Types of relations, Equivalence relation, Partial ordering relation
1.13. **Examples** of equivalence relations and Partial ordering relations.
1.14. Digraphs of relations, matrix representation.
1.15. Composition of relations
1.16. Transitive closure, Warshall's algorithm
1.17. Equivalence classes, Partition of a set
  1.17.1. **Theorem:** Let $\sim$ be an equivalence relation on a set X. Then

(a) For every $x \in X$, $x \in \overline{X}$

(b) For every $x, y \in X$, $x \in \overline{Y}$ if and only if $\overline{X} - \overline{Y}$.

(c) For every $x, y \in X$, either $\overline{x} = \overline{y}$ or $\overline{x} \cap \overline{y} = \emptyset$.

1.17.2. **Equivalence class Theorem**

Unit2: Groups **(16 hours)**

2.1. **Definition** of Binary Operations and examples

2.2. **Groups and its Properties**

2.2.1. **Definition** of Group, Semigroup, Abelian group, finite and infinite group, Quaternion group and Order of the group and examples

2.2.2. **Theorem:** In a group G

(i) Identity element is unique

(ii) Inverses of each elements in G is unique

(iii) $(a^{-1})^{-1} = a$ for all a∈ G

(iv) $(ab)^{-1} = b^{-1}a^{-1}$ for all a, b ∈ G.

2.2.3.**Theorem:** If G is a group with binary operation $*$, then the left and right cancellation laws hold in G, that is $a * b = a * c$ implies b = c, and $b * a = c * a$ implies $b = c$ for $a, b, c \in G$.

2.2.4.**Theorem:** If G is a group with binary operation *, and if a and b are any elements of G, then linear equations a * x = b and y * a = b have unique solutions in G.

2.3. **Subgroups**

2.3.1. **Definition** of Subgroup, Improper and Proper subgroups, Trivial subgroup and examples

2.3.2.**Theorem:** A subset H of a group G is a subgroup of G if and only if

(i) H is closed under the binary operation of G.

(ii) The identity e of G is in H,

(iii) For all a∈ H it is true that a^{-1} ∈H also.

2.3.3. **Theorem:** A non empty subset H of a group G is a subgroup of G if and only if for all a, b ∈ H, $a * b^{-1}$ ∈ H.

2.3.4. **Theorem:** Intersection of any two subgroups of a group is again a subgroup.

2.3.5. **Definition** of Normalizer of an element in group G, Center of group G.

2.3.6. **Theorem:** If G is a group and a ∈ G, then the set N(a) = { x ∈ G | xa = ax } is a subgroup of G.

2.3.7. **Theorem:** If G is a group, then the set C = { x∈ G | xa = ax, for all a ∈ G } is the set of all the elements of G which commutes with every elements of G.

2.4. **Cyclic Groups and its Properties**

2.4.1. **Definition** of Cyclic group generated by an element, Cyclic subgroup of a group and examples

2.4.2.**Theorem:** If G is a group and a ∈ G is a fixed element of G, then the set H = { a^n | n ∈ Z } is a subgroup of G.

2.4.3. **Definition** of Order of an element of a group and its properties.

2.4.4. **Theorem:** Every cyclic group is abelian.

2.4.5. **Theorem:** If a is a generator of a cyclic group G, so is a^{-1}.

2.4.6. **Theorem:** If a is a generator of a cyclic group G, then O(a) = O(G).

2.4.7. **Theorem:** If G is a finite group of order n containing an element of order n, then G is cyclic.

2.4.8. **Theorem:** If in a cyclic group <a> of order k, $a^m = a^n$ ($m \neq n$), then $m \equiv n (\text{mod } k)$.

2.4.9. **Theorem:** Every subgroup of a cyclic group is cyclic.

2.4.10. **Theorem:** A cyclic group of order d has $\emptyset(d)$ generators.

2.5. **Cosets**

2.5.1. **Definition** of Left and Right Cosets in group G and examples

2.5.2. **Theorem:** If H is a subgroup of G, then

(i) Ha = H if and only if $a \in H$

(ii) Ha = Hb if and only if $ab^{-1} \in H$

(iii) Ha is a subgroup of G if and only if $a \in H$

2.5.3. **Theorem:** If H is a subgroup of G, then for all $a \in G$ Ha = $\{x \in G \mid x \equiv a \text{ mod } H\}$.

2.5.4.**Theorem:** If H is a subgroup of G then there exists a one to one correspondence between any two right (left) cosets of H in G.

Recommended Books

1. Howard Anton—Elementary Linear Algebra, Fifth Edition John Wiley & Sons.
2. **J. B. Fraleigh**, A First Course in Abstract Algebra, Narosa Publishing House New Delhi.

Reference Books---

1. Kenneth Hoffman,Raykunze---Linear Algebra, Second Edition, PHI Learning Private LTD.New Delhi-110001-2010.
2. Vivek Sahai, Vikas Bist—Linear Algebra, Alpha Science International LTD. Pangboume.
3. I. N. Herstein-- Topics in Algebra, Wiley India Pvt. Ltd.
4. S. kumaresan—Linear Algebra,A Geometric Approach

B.Sc. Part II (Mathematics) (Semester IV)
(Choice Based Credit System)
(Introduced from June 2019 onwards)

Course Code: DSC – 5D

Title of Course: Real Analysis – II

Theory: 32Hrs. (40 Lectures of 48 minutes) Marks – 50 (Credits: 02)**Course Objectives :** Upon successful completion of this course, the student will be able to:

1. understand sequence and subsequence.
2. prove The Bolzano-Weierstrass Theorem.
3. derive Cauchy Convergence Criterion.
4. find convergence of series.
5. apply Leibnitz Test.

UNIT 1: Sequence of real numbers (20 Lectures)

1.1 Sequence and subsequence

1.1.1 Definition and examples.

1.1.2 Limit of sequence and examples using definition.

1.1.3 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is sequence of non-negative real numbers and if $\lim S_n = L$ then L $\geq$ 0.

1.1.4 Convergent sequences and examples.

1.1.5 Theorem: If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L, then $\{S_n\}_{n=1}^{\infty}$ can not converge to limit distinct from L.

1.1.6 Theorem (without proof) : If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L, then any subsequence of $\{S_n\}_{n=1}^{\infty}$ is also convergent to L.

1.1.7 Theorem (without proof): All subsequences of a convergent sequence of real numbers converge to the same limit.

1.1.8 Bounded sequences and examples.

1.1.9 Theorem: If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent, then it is bounded.

1.2 Monotone Sequences

1.2.1 Definition and examples.

1.2.2 Theorem: A non-decreasing sequence which is bounded above is convergent.

1.2.3 Theorem: A non-increasing sequence which is bounded below is convergent.

1.2.4 Corollary**:** The sequence **Error! Objects cannot be created from editing field codes.**$\{(1 + 1/n)^n\}$ is convergent.

1.2.5 Theorem (without proof): A non-decreasing sequence which is not bounded above diverges to infinity.

1.2.6 Theorem (without proof)**:** A non-increasing sequence which is not bounded below diverges to minus infinity.

1.2.7 Theorem : Abounded sequence of real numbers has convergent subsequence.

1.3 Operations on convergent sequences

1.3.1 Theorem: If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if lim S_n = L and lim t_n = M then lim (S_n + t_n) = L + M.

1.3.2 Theorem: If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if lim S_n = L and lim t_n = M then lim (S_n - t_n) = L – M.

1.3.3 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is sequence of real numbers, if c ∈ R , and if lim S_n = L. then lim c$S_n = cL$.

1.3.4 Theorem: If 0< x < 1 , then the sequence { x^n } converges to 0.

1.3.5 Lemma: If $\{S_n\}_{n=1}^{\infty}$ is sequence of real numbers which converges to L then $\{S_n 2\}_{n=1}^{\infty}$ converges to L^2.

1.3.6 Theorem: If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if lim S_n = L and lim t_n = M then lim (S_n . t_n) = LM.

1.3.7 Theorem: If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if lim S_n = L and lim t_n = M then lim (S_n / t_n) = L / M.

1.4 Limit Superior and Limit Inferior of Sequences

1.4.1 Definition and examples.

1.4.2 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers, then

$\lim_{n\to\infty} sup\ S_n = \lim_{n\to\infty} S_n$.

1.4.3 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers, then

$\lim_{n\to\infty} inf\ S_n = \lim_{n\to\infty} S_n$.

1.4.4 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is a sequences of real numbers, and

if $\lim_{n\to\infty} sup\ S_n = \lim_{n\to\infty} inf\ S_n$ = L where L ∈ R, then $\{S_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} S_n$ = L.

1.4.5 Theorem: If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers

and if $S_n \le t_n$ then i) $\lim_{n\to\infty} \sup\ S_n \le \lim_{n\to\infty} \sup\ t_n$.

ii) $\lim_{n\to\infty} \inf\ S_n \le \lim_{n\to\infty} \inf\ t_n$.

1.4.6 Theorem: If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers

then i) $\lim_{n\to\infty} \sup\ (S_n+t_n) \le \lim_{n\to\infty} \sup\ S_n + \lim_{n\to\infty} \sup\ t_n$.

ii) $\lim_{n\to\infty} \inf\ (S_n+t_n) \ge \lim_{n\to\infty} \inf\ S_n + \lim_{n\to\infty} \inf\ t_n$.

1.5 The Cauchy Sequence

1.5.1 Definition and examples

1.5.2 Theorem: If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges, then $\{S_n\}_{n=1}^{\infty}$ is Cauchy sequence.

1.5.3 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is the Cauchy sequence of real numbers then $\{S_n\}_{n=1}^{\infty}$ is bounded.

1.5.4 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is the Cauchy sequence of real numbers then $\{S_n\}_{n=1}^{\infty}$ is convergent.

1.5.5 Definition and examples of (C , 1) summability of sequence.

UNIT 2 Infinite Series (20 Lectures)

2.1 Convergent and Divergent Series

2.1.1 Definition**:** Infinite series, convergent and divergent series, sequence of partial sum of series and examples.

2.1.2 A necessary condition for convergence**:** A necessary condition for convergence of an infinite series $\sum u_n$ is that lim $u_n = 0$.

2.1.3 Cauchy's General Principal of Convergence (statement only).

2.1.3 Theorem**:** A series $\sum u_n$ converges iff for every $\epsilon > 0$ there exists a positive number m such that $|u_{n+1} + u_{n+2} + \cdots + u_{n+p}| < \epsilon$, for every all n $\geq$ m and p $\geq$ 1.

2.2 Positive Term Series

2.2.1 Definition and examples.

2.2.2 Theorem**:** A positive term series converges iff its sequence of partial sums is bounded above.

2.2.3 Geometric Series**:** The positive term geometric series **Error! Objects cannot be created from editing field codes.**^{n}, converges for r <1, and diverges to infinity for $\geq$ 1.

2.2.4 Theorem**:** A positive term series **Error! Objects cannot be created from editing field codes.**^{p} is convergent if and only if p > 1.

2.3 Comparison Tests For Positive Term Series

2.3.1 Comparison Test (First Type)**):** If $\sum u_n$ and $\sum v_n$ are two positive term series, and k $\neq$ 0, a fixed positive real number (independent of n) and there exists a positive integer m such that $u_n \leq kv_n$, for every n $\geq$ m,then

(a) $\sum u_n$ is convergent , if $\sum v_n$ is convergent, and

(b) $\sum v_n$ is divergent, if $\sum u_n$ is divergent.

2.3.2 Examples.

2.3.3 Limit Form**:** If $\sum u_n$ and $\sum v_n$ are two positive term series such that lim (u_n / v_n) = L, where L is a non zero finite number, then the two series converge or diverge together.

2.3.4 Comparison Test (Second Type): If $\sum u_n$and $\sum v_n$ are two positive term series, and there exists a positive number m such that $(u_n / u_{n+1}) \geq (v_n / v_{n+1})$, for every n $\geq$ m, then (a) $\sum u_n$ is convergent, if $\sum v_n$ is convergent, and (b) $\sum v_n$ is divergent , if $\sum u_n$ is divergent.

2.3.5 Examples.

2.3.6 Cauchy's Root Test: If $\sum u_n$ is a positive term series such that lim $(u_n)^{1/n}$ = L, then the series (i) converges, if L < 1, (ii) diverges, if L > 1, and (iii) the test fails to give any definite information, if L= 1.

23.7 Examples.

2.3.8 D'Alembert's Ratio Test**:** If $\sum u_n$ is a positive term series, such that lim (u_{n+1} / u_n) = L, then the Series (i) converges, if L < 1. (ii) diverges, if L > 1, and (iii) the test fails, if L = 1.

2.3.9 Examples.

2.3.10 Raabe's Test**:** If $\sum u_n$ is a positive term series such that Lim n{ (u_n / u_{n+1}) – 1 } = L, then the series (i) converges, if L >1. (ii) diverges, if L < 1, and (iii) the test fails, if L = 1.

2.3.11 Examples**.**

2.4 Alternating Series

2.4.1 Definition and examples.

2.4.2 Leibnitz Test**:** If the alternating series $u_1 - u_2 + u_3 - u_4 + \cdots$, ($u_n > 0$, for every n) is such that (i) $u_{n+1} \leq u_n$, for every n and (ii) lim u_n = 0, then the series converges.

2.4.3 Examples.

2.5 Absolute and Conditional Convergence

2.5.1 Definition and examples .

2.5.2 Theorem**:** Every absolutely convergent series is convergent.

2.5.3 Examples.

Recommended Books:

1. **R.R.Goldberg**, Methods of Real Analysis, Oxford & IBH Publishing Co. Pvt. Ltd., New Delhi. **For Unit 1**

2. **S.C.Malik and SavitaArora**, Mathematical Analysis (Fifth Edition), New Age International (P) Limited, 2017. **For Unit 2**

Reference Books:

1. **R.G.Bartle and D.R.Sherbert**, Introduction to Real Analysis, Wiley India Pvt. Ltd., Fourth Edition 2016.

2. **D Somasundaram and B Choudhary**, First Course in Mathematical Analysis, Narosa Publishing House New Delhi, Eighth Reprint 2013.

3. **P.K.Jain and S.K.Kaushik,** An Introduction to Real Analysis, S.Chand& Company Ltd. New Delhi, First Edition 2000.

4. **Shanti Narayan and M.D.Raisinghania,** Elements of Real Analysis, S.Chand& Company Ltd. New Delhi, Fifteenth Revised Edition 2014

5. **Shanti Narayan and P.K.Mittal,** A Course of Mathematical Analysis, S.Chand& Company Ltd. New Delhi, Reprint 2016

B.Sc. Part II (Mathematics) (Semester IV)
(Choice Based Credit System)
(Introduced from June 2019 onwards)

Course Code: DSC – 6D

Title of Course: Algebra-II

Theory: 32 hrs. (40 lectures of 48 minutes) Marks – 50 (Credits: 02)

Course Objectives : Upon successful completion of this course, the student will be able to:

1. prove Lagrange's theorem.
2. derive Fermat's theorem.
3. understand properties of normal subgroups, factor group.
4. define homomorphism and isomorphism's in group and rings.
5. derive basic properties of rings and subrings.

Unit - 1 **Groups** **(16 hours)**

1.1 **Lagrange's theorem and its Consequences**

1.1.1 **Definition**of Index of a subgroup

1.1.2 **Theorem(Lagrange):** If G is any finite group and H is any subgroup of G, then O(H) divides O(G).

1.1.3 **Corollary:** The index of any subgroup of a finite group is a divisor of the order of the group.

1.1.4 **Corollary:** If G is a finite group and a $\in$ G, then O(a) divides O(G).

1.1.5 **Corollary:** If G is a finite group of order n then for all a $\in$ G, $a^n = e$, where e is the identity element of G.

1.1.6 **Theorem(Euler's theorem):** If n is any positive integer and a is relatively prime to n, then $a^{\emptyset(n)} \equiv 1 (\mathrm{mod}\ n)$

1.1.7 **Theorem(Fermat's theorem):** If a is any integer and p is any positive prime, then $a^p \equiv a (\mathrm{mod}\ p)$.

1.2 **Normal subgroups and its Properties**

1.2.1 **Definition** of Normal subgroup and examples

1.2.2 **Theorem:** A subgroup H of a group G is normal if and only if $gHg^{-1} = H$ for all g $\in$ G.

1.2.3 **Theorem:** A subgroup H of a group G is normal if and only if every right coset of H in G is a left coset of H in G.

1.2.4 **Corollary:** Every subgroup of an abelian group is a normal subgroup.

1.2.5 **Theorem:** A subgroup H of a group G is normal in G if and only if the product of any two right (or left) cosets H in G is again a right (or left) coset of H in G.

1.2.6 **Results related to Normal subgroups**

(i) The intersection of any two normal subgroups of a group is also a normal subgroup.
(ii) The product of any two normal subgroups of a group is a subgroup of the group.
(iii) Let H be a subgroup and K be normal subgroup of the group G. Then H $\cap$ K is normal in H.
(iv) If N is a normal subgroup of G and H is any subgroup of G, then NH is a subgroup of G.
(v) The center Z of a group G is a normal subgroup of G.
(vi) The center Z of a group is a normal subgroup of a normalizer of an element.

1.3 **Factor Group (Quotient Group)**

1.3.1 **Definition** of Factor Group or Quotient Group and examples

1.3.2 **Theorem:** The set G/H = {Ha | a $\in$ G} of all cosets of a normal subgroup H, of the group G, is a group G, is a group under the binary operation defined by Ha .Hb = Hab, for all Ha, Hb$\in$ G/H.

1.3.3 **Theorem:** If H is a normal subgroup of finite order, then O(G/H) = O(G)/O(H).

1.3.4 **Theorem:** Every Quotient group of an abelian group is abelian.

1.3.5 **Theorem:** Every factor group of a cyclic group is cyclic

1.4 **Homomorphism of Groups**

1.4.1 **Definition** of Homomorphism, Isomorphism, Automorphism and Endomorphism of Groups and examples.

1.4.2 **Theorem:** Let f : G $\rightarrow$ G' be a homomorphism from the group (G, .) into the group (G', *). Then

(i) f(e) = e', where e and e' are the identity elements of the groups G and G' respectively.
(ii) $f(a^{-1}) = [f(a)]^{-1}$, for all a $\in$ G.

1.4.3 **Theorem:** If f is a homomorphism of a group G into a group G', then the range f(G) = {f(g) | for all g $\in$ G} is a subgroup of G'.

1.4.4 **Theorem:** The homomorphic image of the group G in the group G' is a subgroup of G'.

1.4.5 **Theorem:** Let f : G $\rightarrow$ G' be a homomorphism from the group G into the group G' and H is a subgroup of G, then f(H) is also a subgroup of G'.

1.4.6 **Theorem:**Let f : G $\rightarrow$ G be a homomorphism of the group G into itself and H is a cyclic subgroup of G, then f(H) is again a cyclic subgroup of G.

Unit – 2 Normal subgroups (16 hours)

2.1. **Kernel of a Homomorphism**

2.1.1. **Definition**of Kernel of a Homomorphism and examples.

2.1.2. **Theorem:** Let f : G $\rightarrow$ G' be a homomorphism of a group G into G' with Kernel K. Then K is a normal subgroup of G.

2.1.3. **Theorem:** Let f : G $\rightarrow$ G' be a homomorphism of a group G into G' with Kernel K. Then f is one – one if and only if K = {e}, where e is the identity element of G.

2.1.4. **Corollary:** A homomorphism f from the group G onto the group G' is an isomorphism if and only if Ker f = {e}.

2.1.5. **Theorem:** Let G be a group and H be a normal subgroup of G. Then G/H is a homomorphic image of G with H as its Kernel.

2.1.6. **Theorem(Fundamental Homomorphism Theorem):** Let f be a homomorphism of a group G into a group G', with kernel K. Then f(G) is isomorphic to factor group G/K.

2.1.7. **Results related to Isomorphism**

(i) If f : G → G' be an isomorphism of a group G onto a group G' and a is any element of G then the order of f(a) equals the order of a.

(ii) If f : G → G' be an isomorphism and G is an abelian group then G' is also abelian.

(iii) Any infinite cyclic group is isomorphic to the group Z of integers, under addition.

(iv) Any finite cyclic group of order n is isomorphic to additive group of integers modulo n.

2.2. **Permutation Group**

2.2.1. **Definition** of Permutation, Degree of permutation, Equality of two permutations, Identity permutations, Inverse and Composition of permutation and Symmetric group and examples.

2.2.2. **Theorem:** Let S be a non empty finite set of n elements. The set S_n of all permutations of degree n defined on S, is a finite group of order n!, under the permutation multiplication.

2.2.3. **Theorem(Cayley's Theorem):** Every finite group is isomorphic to a group of permutation.

2.3. Rings

2.3.1. Definition and examples.

2.3.2. Basic Properties.

2.3.3. Homomorphism and isomorphism in a ring.

2.3.4. Multiplicative questions: Fields

2.3.5. Examples of Commutative and non-commutative rings.

2.3.6. Rings from number system, Z_n the ring of integers modulo n.

2.4. Subrings

2.4.1. Definition and examples.

2.4.2. Basic properties

2.4.3. Ideals: Definition and examples.

2.4.4. Examples of subring which are not ideals.

Recommended Books:

1 **J. B. Fraleigh**, A First Course in Abstract Algebra, Narosa Publishing House New Delhi, Tenth Reprint 2003.

2 **V. K. Khanna and S. K. Bhambri**, A Course in Abstract Algebra, Vikas Publishing House Pvt Ltd., New Delhi, Fifth Edition 2016.

Reference Books:

1 **I.N. Herstein**, Topics in Algebra, Wiley indiaPvt. Ltd,

2 **M. Artin**, Algebra, Prentice Hall of India, New Delhi, 1994

3 **N. S. Gopalkrishnan,** University Algebra, New Age International New Delhi, Second Edition 1986

4 **A. R. Vasishtha,** Modern Algebra, Krishna Prakashan, Meerut 1994.

B. A./B. Sc. (Mathematics) (Part II)

(Choice Based Credit System)

(Introduced from June 2019 onwards)

Core Course Practical in Mathematics (CCPM – II)

Marks 50 (Credit 04)

(Analysis I & II and Algebra I & II)

Sr. No.	Topic	No. of Practicals
	SEMESTER-III	
1	Eigen values and Eigen vectors	**1**
2	Cayley Hamilton theorem (Verification and finding inverse of matrix)	**1**
3	Range of function, Image and inverse image of a subset	**1**
4	Types of Function (Injective, Surjective, Bijective , Inverse function, Composition of two functions)	**1**
5	Mathematical induction	**1**
	SEMESTER-IV	
6	Limit of a sequence (using definition)	**1**
7	Convergence of sequence	**1**
8	Comparison test and Cauchy's root test	**1**
9	D' Alembert's ratio test and Rabbi's test	**1**
10	Examples on Group and order of an element	**1**
11	Cyclic subgroup	**1**
12	Permutation group	**1**
13	Homomorphism and Kernel	**1**

Core Course Practical in Mathematics (CCPM – III)

Marks 50 (Credit 04)

(Numerical Recipes in Scilab)

SEMESTER-III		
Sr. No	**Content**	**No. of Practical's**
1	**Introduction:** Application, feature, scilabs environment workspace, working directory, Scilab as a calculate, operators, mathematical predefined functions, constants, variables and their types.	**1**
2	**Matrix:** Rows matrix,column matrix, general matrix, operation on matrix addition, subtraction, product.	**1**
3	**Accessing element of matrix:** Size of Matrix, Length of matrix, accessing element using one index, two indices	**1**
4	**Sub Matrix:** Accessing sub matrix of given matrix using ':' operator &'$' operator	**1**
5	**Advanced matrix operations:** Matrix functions: eye(), zero (), ones (), empty matrix, element-wise operation, determinant, inverse, trace of matrix & diagonal element of matrix.	**1**
6	**Polynomial:** Creating a polynomial 1) using roots 2) using coefficients, roots of polynomial, derivative companion matrix, numerator & denominator of rational, simplifying rational.	**1**
7	**Plotting graph:** Creating graphs of simple functions.	**1**
8	**Introduction Scilab programming:** disp(), Boolean operators, conditional statement (if select), find() and () or (), looping statement.	**1**
SEMESTER-IV		
9	**Advanced Scilab programming using function:** Creating Scilab function and its execution.	**1**
10	**Numerical Methods to find roots of a given of a given function:**	**1**

	(a) Bisection Method (b) Newton-Raphson Method.	
11	**Interpolation** (a) Lagrange's interpolation formula (b) Newton Gregory forward interpolation formula. (c) Newton Gregory backward interpolation formula.	**1**
12	**Graph theory:** Havel-Hakimi Theorem, Transitive closure.	**1**
13	**Numerical Integration:** (a) Trapezoidal Rule (b) Simpson's 1/3rd Rule (c) Simpson's 3/8th Rule	**1**
14	**Characteristic Polynomial:** Characteristic polynomial, its coefficients, roots(Eigen values), Derivation of Eigen Vectors using roots, Direct Derivation of Eigen Values and vectors using Spec(), Verify Cayley-Hamilton theorem using coefficients.	**1**
15	**Numerical Methods for solution of Ordinary Differential Equations:** (a) Euler Method (b) Euler's Modified Method (c) Runge-Kutta Second and Fourth order Method	**1**
16	**Numerical Methods for solution of a system of Linear Equations: (Unique solution case only)** (a) Gauss-Elimination Method. (b) Gauss-Jordan Method.	**1**

Recommended Books:

1) **R. G. Bartle and D. R. Sherbert**, Introduction to Real Analysis, Wiley India Pvt. Ltd., Fourth Edition 2016.

2) **S. C. Malik and Savita Arora**, Mathematical Analysis (Fifth Edition), New Age International (P) Limited, 2017

Reference Books:

1) **R. R. Goldberg**, Methods of Real Analysis, Oxford & IBH Publishing Co. Pvt. Ltd., New Delhi.

2) **D Somasundaram and B Choudhary**, First Course in Mathematical Analysis, Narosa Publishing House New Delhi, Eighth Reprint 2013

3) **P. K. Jain and S. K. Kaushik,** An Introduction to Real Analysis, S.Chand & Company Ltd. New Delhi, First Edition 2000

4) **Shanti Narayan and Dr. M. D. Raisinghania,** Elements of Real Analysis, S.Chand & Company Ltd. New Delhi, Fifteenth Revised Edition 2014

5) **Shanti Narayan and P. K. Mittal,** A Course of Mathematical Analysis, S.Chand & Company Ltd. New Delhi, Reprint 2016

6) **Dr. Hari Kishan**, Real Analysis, Pragati Prakashan, Meerut, Fourth Edition 2012

7) SCILAB: A Practical Introduction to Programming and Problem Solving [Print Replica] Kindle Edition by Tejas Sheth (Author)

8) **Scilab A Hands on Introduction** by Satish Annigeri .

9) **Engineering and Scientific Computing with Scilab 1999th** Edition by Claude Gomez (Editor), C. Bunks (Contributor), J.-P. Chancelier (Contributor), F. Delebecque (Contributor), M. Goursat (Contributor), R. Nikoukhah (Contributor), S. Steer (Contributor)

10) **Scilab: from Theory to Practice - I. Fundamentals Book by Philippe Roux**

11) Introduction to Scilab: For Engineers and Scientists **Book by Sandeep Nagar**