$$\boxed{q = Q - \frac{p^2}{2m^2 g}}$$

So New hamiltonian is -

$$H'(PQ) = \frac{p^2}{2m} + mgq$$

$$\boxed{H'(PQ) = \frac{p^2}{2m} + mg\left(Q - \frac{p^2}{2m^2 g}\right)}$$

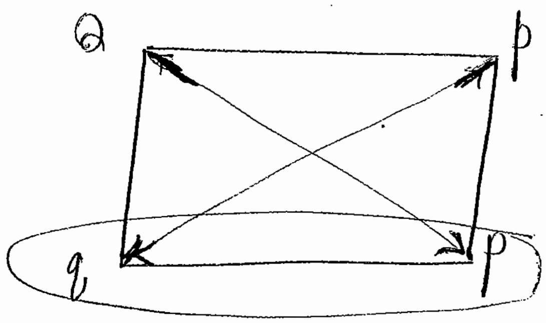
A-12
Q.11 find generating function $F(P, q)$ for transformation $p = \frac{1}{Q}$, $q = PQ^2$ is - ?

(a) $\sqrt{Pq}$ (b) $-\sqrt{Pq}$ (c) $2\sqrt{Pq}$ (d) $-2\sqrt{Pq}$.

Sol^n Given transformation relation is -

$$p = \frac{1}{Q}$$

$$q = PQ^2$$



$$p = \frac{\partial F(q,P)}{\partial q} \quad \text{—(I)}$$

$$Q = \frac{\partial F(q,P)}{\partial P} \quad \text{—(II)}$$

$$Q = \sqrt{\frac{q}{P}}$$

$$p = \frac{1}{Q}$$

$$p = \sqrt{\frac{P}{q}}$$

from (I) relation -

$$\sqrt{\frac{P}{q}} = \frac{\partial F}{\partial q}$$

$$\sqrt{P}\, 2\sqrt{q} + c_1(P) = F \quad \text{—(a)}$$

from (II)

$$\sqrt{\frac{q}{P}} = \frac{\partial F}{\partial P} \Rightarrow F = \sqrt{q} \cdot 2\sqrt{P} + c_2(q) \quad \text{—(b)}$$

So from (a) & (b) $\boxed{F = 2\sqrt{Pq}}$ Ans

Q9 Values of 'a' and 'b' for which following transformation $Q = (2q)^a \cos bp$, $P = (2q)^a \sin p$ is canonical are -

(a) $a = \frac{1}{2}, b = 1$ (b) $a = 2, b = \frac{1}{2}$ (c) $a = 1, b = 1$, (d) $a = \frac{1}{2}, b = \frac{1}{2}$

Sol^n For canonical possion bracket should be one.

So $\{Q, P\}_{q,p} = 1$

$$\Rightarrow \{(2q)^a \cos^b p, (2q)^a \sin^b p\}_{q,p} = 1$$

$$\Rightarrow 2a(2q)^{a-1} \cos^b p \cdot (2q)^a b \sin^{b-1} p \cos p + (2q)^a b \cos^{b-1} p \sin p \times 2a(2q)^a \sin^b p = 1$$

$$\Rightarrow 2ab(2q)^{2a-1} \cos^{b-1} p \sin^{b-1} p [\cos^2 p + \sin^2 p] = 1$$

$$2ab(2q)^{2a-1} (\cos p \sin p)^{b-1} = 1$$

$\because$ R.H.S is co-ordinate independent $\therefore$ L.H.S. should also be co-ordinate independent.

So $2a - 1 = 0$ and $b - 1 = 0$

$\boxed{a = \frac{1}{2}}$ $\boxed{b = 1}$

* Hamilton Jacobi Theory :-

In this theory a canonical transformation is done in such a way that new hamiltonian becomes zero. And G.F. for this C.T. is chosen to be $F[q, P]$

$$q, p \xrightarrow{C.T.} Q, P$$

$$H \xrightarrow[F(q,P)]{} H' = 0$$

Hamiltons equation in new co-ordinate -

$$\dot{Q} = \frac{\partial H'}{\partial P} = 0 \qquad \{\because H' = 0$$

$$\dot{P} = -\frac{\partial H'}{\partial Q} = 0$$

$\Rightarrow$ Q = constant

& P = constant

Importance of $F(q, P, t)$ in H. J. theory.

Total time derivative of F.

$$\frac{dF}{dt} = \frac{\partial F}{\partial q}\dot{q} + \frac{\partial F}{\partial P}\dot{P}_{\to 0} + \frac{\partial F}{\partial t}$$

$$\frac{dF}{dt} = \frac{\partial F}{\partial q}\dot{q} + \frac{\partial F}{\partial t}$$

relation for $f(q, P, t)$

$$p = \frac{\partial F}{\partial q}$$

$$Q = \frac{\partial F}{\partial P}$$

$$\underset{0}{\underset{\parallel}{H'}} = H + \frac{\partial F}{\partial t}$$

$$\frac{\partial F}{\partial t} = -H$$

$$\frac{dF}{dt} = p\dot{q} - H$$

$$\boxed{\frac{dF}{dt} = L}$$ — Lagrangian

$$\boxed{F = \int L\,dt} = S$$

Action (S)

In H-J theory F is equal to action (S).

Imp.
* H-J Equation :-

$$\boxed{H + \frac{\partial S}{\partial t} = 0}$$

In Hamilton every place of p we will write -

$$\boxed{p = \frac{\partial S}{\partial q}}$$

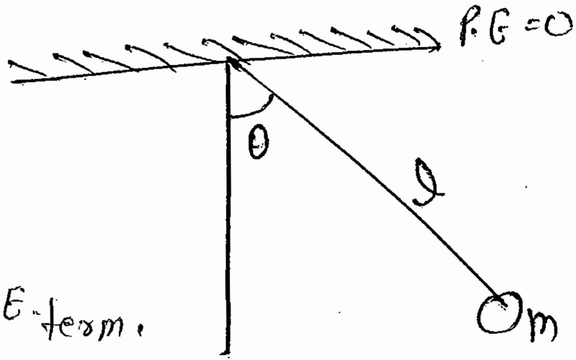
Q. Write hamilton-Jacobi eq^n for simple pendulum.

(r, θ)

$r = l$

$\dot{r} = 0$

$p_r = 0$



$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + \text{P.E. term}$$

$$= \frac{p_\theta^2}{2ml^2} - mgl\cos\theta$$

$$p_\theta = \left(\frac{\partial S}{\partial \theta}\right)$$

So H.J. eq^n for simple pendulum

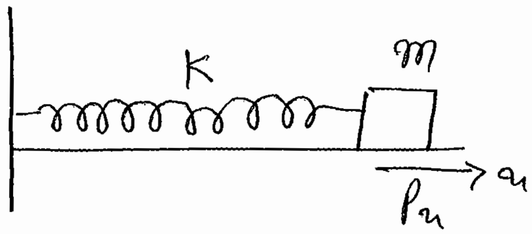
$$H + \frac{\partial S}{\partial t} = 0$$

$$\boxed{\frac{1}{2ml^2}\left(\frac{\partial S}{\partial \theta}\right)^2 - mgl\cos\theta + \frac{\partial S}{\partial t} = 0} \; *$$

$\Downarrow$

Non-linear P.D.E. (Solution of Non-linear partial diff. eq^n can be found by variable separation on sumation not multiplication)

* H-J Equation for S.H.O. :-



$$H + \frac{\partial S}{\partial t} = 0 \quad , \; p = \frac{\partial S}{\partial q}$$

$$H = \frac{p_x^2}{2m} + \frac{1}{2}Kx^2$$

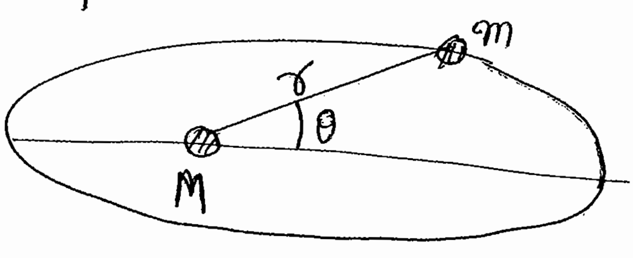
$$= \frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + \frac{1}{2}Kx^2$$

Here H-J eq^n -

$$\boxed{\frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + \frac{1}{2}Kx^2 + \frac{\partial S}{\partial t} = 0}$$

Ques: H-J equation for planatary motion.

Sol^n :-



$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} - \frac{GMm}{r}$$

$p_r = \left(\frac{\partial S}{\partial r}\right)$, $p_\theta = \frac{\partial S}{\partial \theta}$

$$\boxed{\frac{1}{2m}\left(\frac{\partial S}{\partial r}\right)^2 + \frac{1}{2mr^2}\left(\frac{\partial S}{\partial \theta}\right)^2 - \frac{GMm}{r} + \frac{\partial S}{\partial t} = 0}$$

Non-Linear P.D.E.

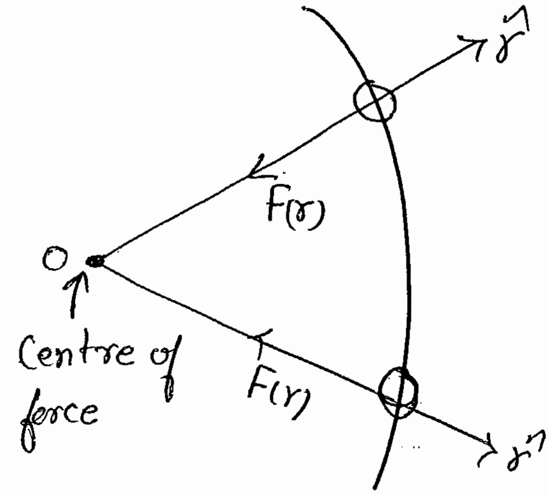
$$S = S(r, \theta, t)$$

$$S = S_1(r) + S_2(\theta) + S_3(t).$$

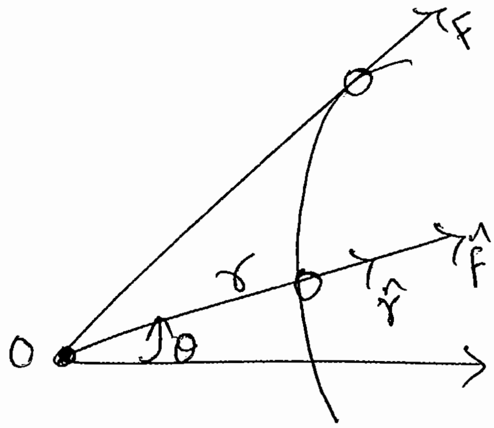
Central forces Motion :-

force is directed towards or away from a point is central force.

$$\boxed{F = \pm f(r)\,\hat{r}}$$



- When $\vec{F}$ and $\hat{r}$ is in same direction then we use +ve sign.
- When $\vec{F}$ and $\hat{r}$ is in opposite direction then we use −ve sign.



* Properties :-

$$\vec{\nabla} \times \vec{F} = 0$$

⇒ Central force is conservative.

⇒ Total energy is conserved.

$$\boxed{\vec{\tau}_0 = 0}, \boxed{L_0 \text{ is conserved.}}$$

When angular momentum is conserved then it must be two directional problem.

$\vec{F}$ is conservative. So we can define P.E. $(V(r))$

$$\boxed{V(r) = -\int f(r)\,dr}$$

, $f(r)$ is +ve for repulsive force

, $f(r)$ is −ve for attractive force

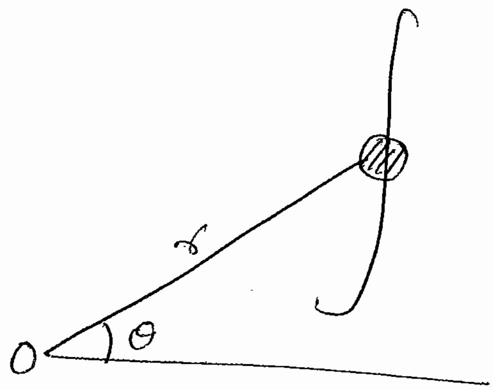
$$\vec{f} = -\vec{\nabla} V(r)$$

$$\boxed{f = -\frac{\partial}{\partial r} V(r)}$$

Equation of motion under central force :-

$$L = T - V$$

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)$$



Here L is independent of t so total energy is conserved and L is independent of θ so P_θ (angular momentum) is conserved.

Equation of motion :-

r- equation :-

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0$$

$$m\ddot{r} - mr\dot{\theta} + \frac{\partial V}{\partial r} = 0$$

$$\boxed{m\ddot{r} - mr\dot{\theta}^2 - f(r) = 0}$$ Imp. ← radial eqn of motion.

θ- equation :-

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{d}{dt}(mr^2\dot{\theta}) - 0 = 0$$

$$\frac{d}{dt}(mr^2\dot{\theta}) = 0$$

$$\boxed{mr^2\dot{\theta} = \text{Constant}}$$ Imp

$$\boxed{2\dot{r}\dot{\theta} + r\ddot{\theta} = 0}$$ Imp.

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = \boxed{mr^2\dot{\theta} = l} \text{ Angular momentum.}$$

$$\boxed{\dot{\theta} = \frac{l}{mr^2}}$$

Q. Transverse velocity of a particle under central force varies with r as -

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$ (c) r (d) $\frac{1}{r^2}$

Sol^n Transverse velocity $= r\dot{\theta}$ $\left\{ \because \dot{\theta} = \frac{l}{mr^2} \right\}$

$$= r \cdot \frac{l}{mr^2}$$

$$\left\{ K.E. = \frac{1}{2} m (\underbrace{\dot{r}^2}_{\text{Radial Velocity}} + \underbrace{r^2\dot{\theta}^2}_{\text{Trans veloc}}) \right.$$

$$= \frac{l}{m} \cdot \frac{1}{r}$$

Hence transverse velocity $\propto \frac{1}{r}$

* Equation of path in central force motion :-

↓

is independent of ~~path~~ time.

remove t from equation of motion.

Introduce $u = \frac{1}{r}$

This leads to following eq^n

$$\boxed{\frac{d^2u}{d\theta^2} + u = -\frac{mf}{l^2u^2}}$$

⇓

Diff. eq^n of path

$u = \frac{1}{r} \Rightarrow r = \frac{1}{u}$

$$\dot{r} = -\frac{1}{u^2}\frac{du}{dt}$$

$$= -\frac{1}{u^2}\frac{du}{d\theta}\cdot\frac{d\theta}{dt}$$

$$= -r^2\frac{du}{d\theta}\cdot\dot{\theta}$$

$$= -\cancel{r^2}\frac{du}{d\theta}\cdot\frac{l}{m\cancel{r^2}}$$

$$= -l\frac{du}{d\theta}$$

Use :- It is use to find $f(r)$ when relation b/w r and θ is given.

Q. If $r = a\cos\theta$ for a particle moving under central force what is force law ?

Soln

$$u = \frac{1}{r}$$

$$u = \frac{1}{a}\sec\theta$$

$$\frac{du}{d\theta} = \frac{1}{a}\sec\theta\tan\theta$$

$$\frac{d^2u}{d\theta^2} = \frac{1}{a}\left[\sec\theta\tan^2\theta + \sec^3\theta\right]$$

Differential equation of path -

$$\frac{d^2u}{d\theta^2} + u = -\frac{mf}{l^2u^2}$$

$$u\tan^2\theta + u\sec^2\theta + u = \frac{-mf}{l^2u^2}$$

$$2u\sec^2\theta = -\frac{mf}{l^2u^2}$$

$$f = -\frac{2l^2u^3\sec^2\theta}{m}$$

$$f = -\frac{2l^2a^2}{m}u^5$$

$$f = \frac{-2l^2a^2}{m}\frac{1}{r^5}$$

$$\boxed{f \propto \frac{1}{r^5}}$$ ← Force Law

Note (i) If $r = a\sin\theta$

$$\Rightarrow \boxed{f \propto \frac{1}{r^5}}$$

(ii) if $r^n = a\cos n\theta$

or $r^n = a\sin n\theta$ $\boxed{f \propto \frac{1}{r^{2n+3}}}$

Imp.

Q. A particle is moving in circle of radius R under a force which is always directed towards a point on periferi on circle (i) What is the force low. (ii) What is total energy.

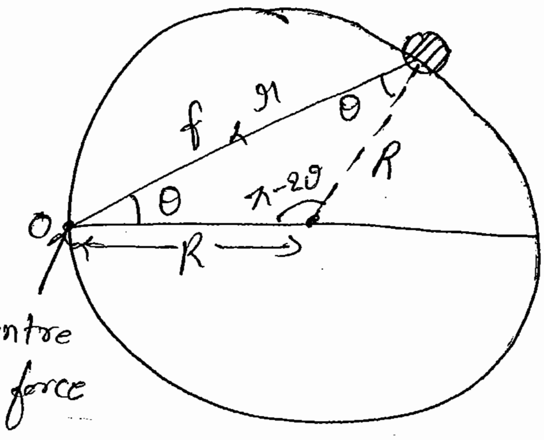
Sol^n

By sine low

$$\frac{r}{\sin(\pi-2\theta)} = \frac{R}{\sin\theta}$$

$$\frac{r}{\sin 2\theta} = \frac{R}{\sin\theta} \Rightarrow \frac{r}{2\sin\theta\cos\theta} = \frac{R}{\sin\theta}$$

$$\boxed{r = 2R\cos\theta}$$



Q. A particle is moving under a central force if eq^n of path of the particle is $r = Ae^{k\theta}$ What is Potential Energy of the particle.

Sol^n

$r = Ae^{k\theta}$ is eq^n of path becoz this relation b/w two co-ordinate and also it is time independent. when it is written in differential form then it is K/a differential eq^n of path.

Ans:- $f \propto \frac{1}{r^3}$

$$V(r) = \frac{-(k^2+1)l^2}{2mr^2}$$

T.E. = 0

For total energy:-

T.E. = K.E. + P.E.

$= \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + P.E.$

$r = Ae^{k\theta}$

$\dot{r} = kAe^{k\theta}\dot{\theta}$

$\dot{r} = kr\dot{\theta}$

$\dot{\theta} = \frac{l}{mr^2}$

* Effective Potential :- [Potential Energy] :-

It is introduce to convert 2-D problem (in central force motion) in to 1-D problem.

Total Energy -

$$E = K.E + P.E. \Rightarrow E = \frac{1}{2}mv^2 + V(r)$$

$$= \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \underbrace{V(r)}_{\text{Actual P.E.}}$$

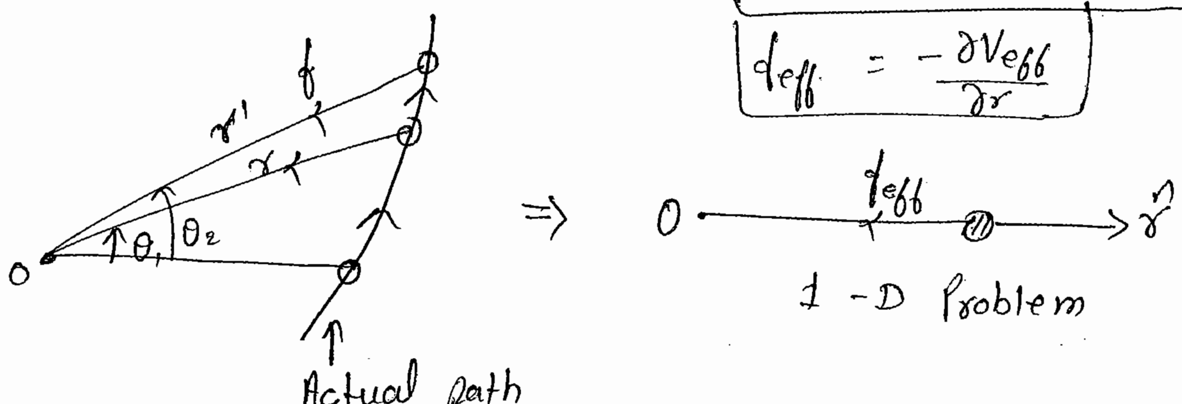
Put $\dot{\theta} = \frac{l}{mr^2}$

$$E = \underbrace{\frac{1}{2}m\dot{r}^2}_{K.E.} + \underbrace{\frac{l^2}{2mr^2} + V(r)}_{\text{P.E. or Eff. Pot.}}$$

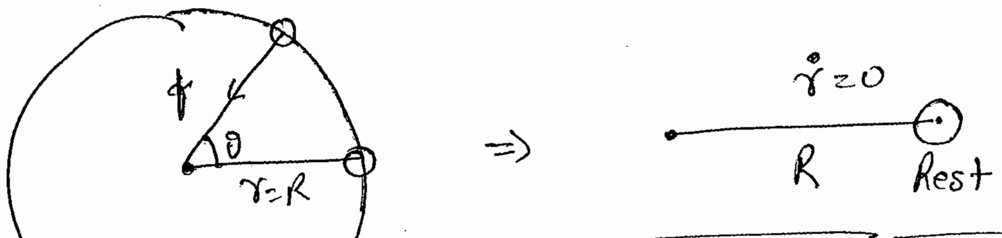
$$\boxed{E = \frac{1}{2}m\dot{r}^2 + V_{eff}(r)}$$

$$\boxed{V_{eff} = V(r) + \frac{l^2}{2mr^2}}$$

$$\boxed{f_{eff} = -\frac{\partial V_{eff}}{\partial r}}$$



2-D Problem



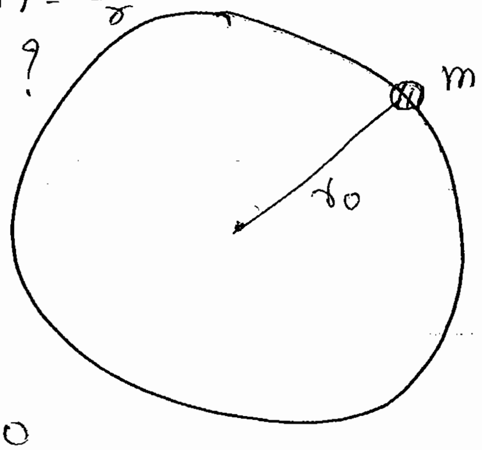
2-D circular motion

$\boxed{f_{eff} = 0}$, $\boxed{E = V_{eff}}$

Q.34

Sol^n It is circular motion $V(r) = \frac{-K}{r}$

$l = ?$



$\therefore$ $f_{eff} = 0$

$$\frac{\partial V_{eff}}{\partial r} = 0$$

$$\frac{\partial}{\partial r}\left(\frac{-K}{r} + \frac{l^2}{2mr^2}\right)_{r=r_0} = 0$$

$$\left[\frac{+K}{r^2} + \left(-\frac{l^2}{mr^3}\right)\right]_{r_0} = 0$$

$$\frac{K}{\cancel{r_0^2}} = \frac{l^2}{mr_0^{\cancel{3}}}$$

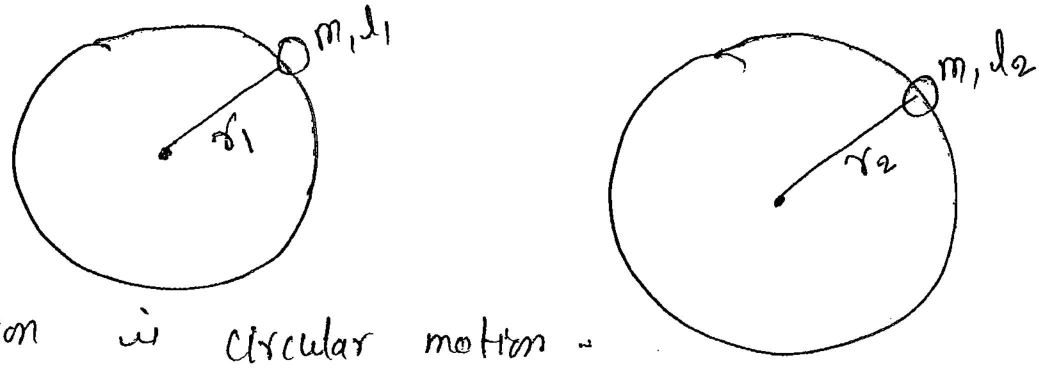
$$l^2 = Kmr_0$$

$$\boxed{l = \sqrt{Kmr_0}}$$ Ans

B.A.5
Q.20

Sol^n Given $\Rightarrow \frac{l_1}{l_2} = 2$ $V(r) = \frac{1}{2}kr^2$

$$\frac{r_1}{r_2} = ?$$



$\because$ motion is circular motion.

So $f_{eff} = 0$

$$\frac{\partial V_{eff}}{\partial r} = 0$$

$$\frac{\partial}{\partial r}\left(\frac{1}{2}Kr^2 + \frac{l^2}{2mr^2}\right) = 0$$

$$Kr - \frac{l^2}{mr^3} = 0$$

$$l = \sqrt{Kmr^4}$$

$$l = \sqrt{Km}\, r^2$$

$$l_1 = \sqrt{Km}\, r_1^2 \quad \text{—— (i)}$$

$$l_2 = \sqrt{Km}\, r_2^2 \quad \text{—— (ii)}$$

$$\frac{l_1}{l_2} = \frac{\cancel{\sqrt{Km}}\, r_1^2}{\cancel{\sqrt{Km}}\, r_2^2}$$

$$2 = \frac{r_1^2}{r_2^2}$$

So $\boxed{\frac{r_1}{r_2} = \sqrt{2}}$ Ans

Q.3

Sol^n $V(r) = kr^3$

$E = K.E. + P.E.$ — (I)

But in circular motion -

$E = V_{eff.}$ — (II)

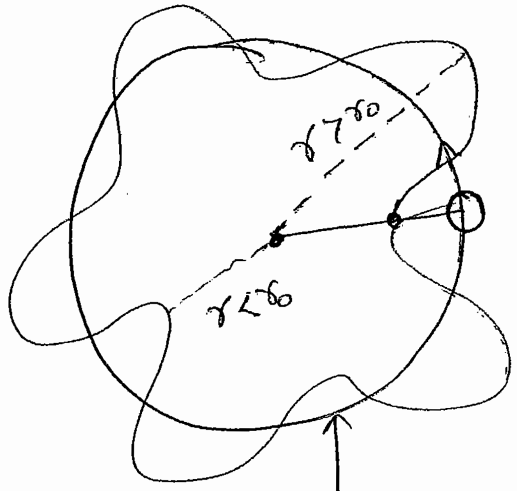
$$K.E. + P.E. = \frac{l^2}{2mr^2} + V(r)$$

$$K.E. + \cancel{V(r)} = \frac{J^2}{2mr^2} + \cancel{V(r)}$$

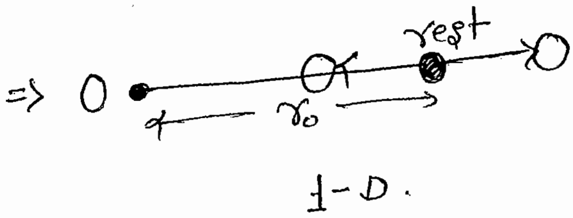
$$\boxed{K.E. = \frac{l^2}{2mr^2}} \longrightarrow$$ Always use for circular motion.

* Oscillation about Stable State :-

A stable orbit is circular orbit because ($f_{eff} = 0$)



2-D. stable circular Orbit



Frequency of Oscillation :-

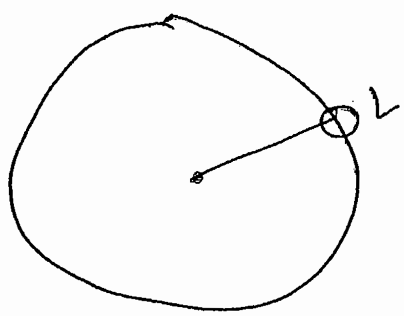
$$\boxed{\omega = \sqrt{\frac{\text{force Constant}}{m}}}$$

When ω = Radial freq. of angular oscillation.

$$\boxed{\text{Force Constant} = \frac{d^2 V_{eff}}{dr^2}\Big|_{r=r_0}}$$

where r_0 = radius of stable circular orbit.

S.A.-5
Q.32



Solⁿ

$$V(r) = \frac{-K}{r}$$

$$V_{eff} = V(r) + \frac{L^2}{2mr^2}$$

$$V_{eff} = \frac{-K}{r} + \frac{L^2}{2mr^2}$$

Let r_0 be radius of stable circular orbit -

$$\therefore \quad f_{eff} = 0$$

$$\left.\frac{\partial V_{eff}}{\partial r}\right|_{r=r_0} = 0$$

$$\frac{K}{r_0^2} - \frac{L^2}{mr_0^3} = 0$$

$$\boxed{r_0 = \frac{L^2}{mK}}$$

Hence force constant $= \left.\frac{d^2 V_{eff}}{dr^2}\right|_{r=r_0}$

$$= \frac{-2K}{r_0^3} + \frac{3L^2}{mr_0^4}$$

$$= \frac{1}{r_0^3}\left(-2K + \frac{3L^2}{mr_0}\right)$$

$$= \frac{(mk)^3}{L^6}\left(-2K + \frac{3\cancel{L^2} \times \cancel{m}k}{\cancel{m}\cancel{L^2}}\right)$$

$$= \frac{m^3k^3}{L^6}(-2K + 3K) = \frac{m^3k^4}{L^6}$$

Hence angular frequency :-

$$\omega = \sqrt{\frac{\text{force Constant}}{m}}$$

$$= \sqrt{\frac{m^{\cancel{8}} K^4}{L^6 \cancel{m}}} = \frac{m K^2}{L^3}$$

$$\boxed{\omega = \frac{mK^2}{L^3}}$$ Ans

Q. Graph of 'V_{eff} V/s 'r' :-

$$V_{eff} = V(r) + \frac{l^2}{2mr^2}$$

Consider following two limits

$r \to 0$, $r \to \infty$ and draw line in these two limits.

in most of the cases ✓

If $r \to 0$ then higher power in demomenator dominates
If $r \to \infty$ " Lower " " " "

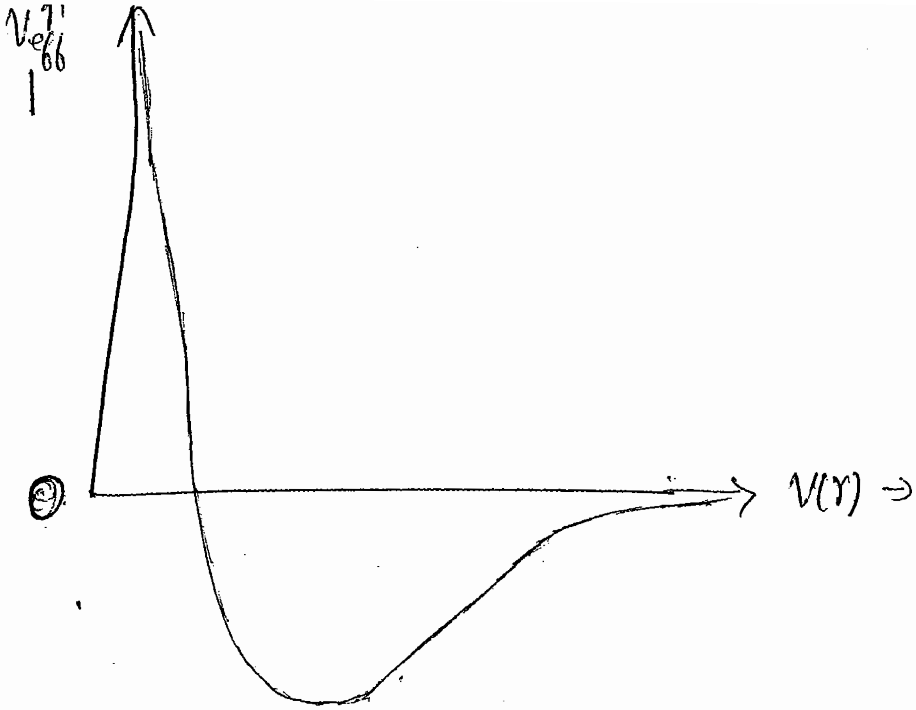
× If $r \to 0$ lowe power " " numerater "
If $r \to \infty$ higher " " " "

Ques Plot V_{eff} for $V(r) = \frac{-K}{r}$

$$V_{eff} = \frac{-K}{r} + \frac{l^2}{2mr^2}$$

$r \to 0$, $V_{eff} \to \infty$

$r \to \infty$, $V_{eff} \to 0$ (from -ve side)



$\underline{\underline{Q}}$: V_{eff} for $V(r) = \frac{-K}{r^3}$

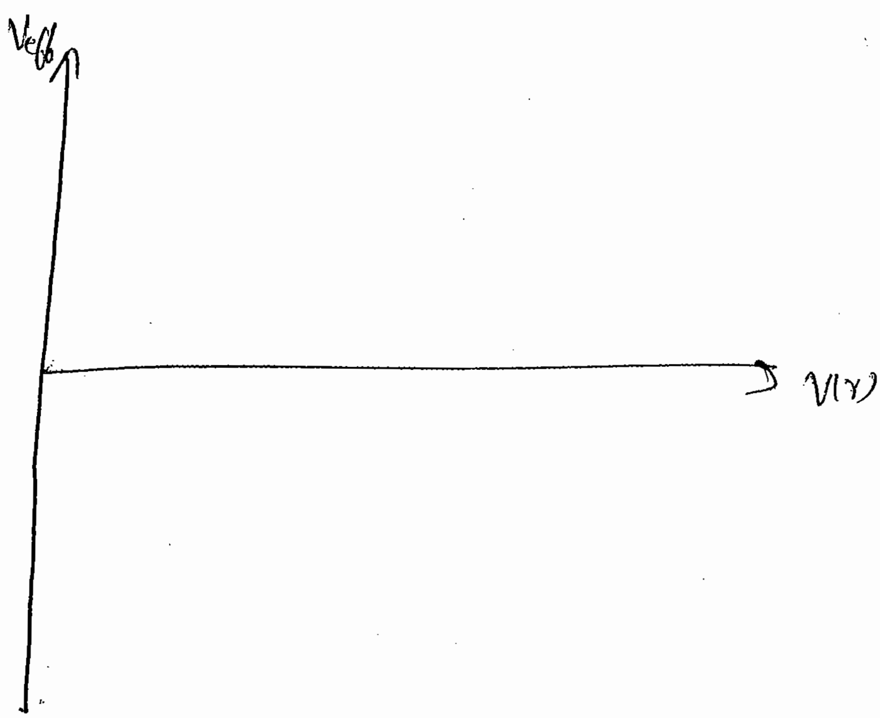
$\underline{\underline{Sol^n}}$

$$V_{eff} = V(r) + \frac{L^2}{2mr^2}$$

$$V_{eff} = -\frac{K}{r^3} + \frac{L^2}{2mr^2}$$

$r \to 0$, $V_{eff} \to -\infty$

$r \to \infty$, $V_{eff} \to 0$ (from +ve side

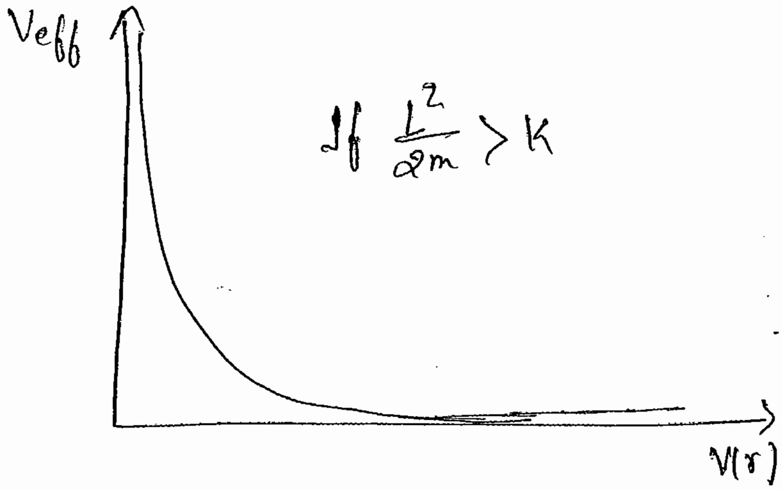


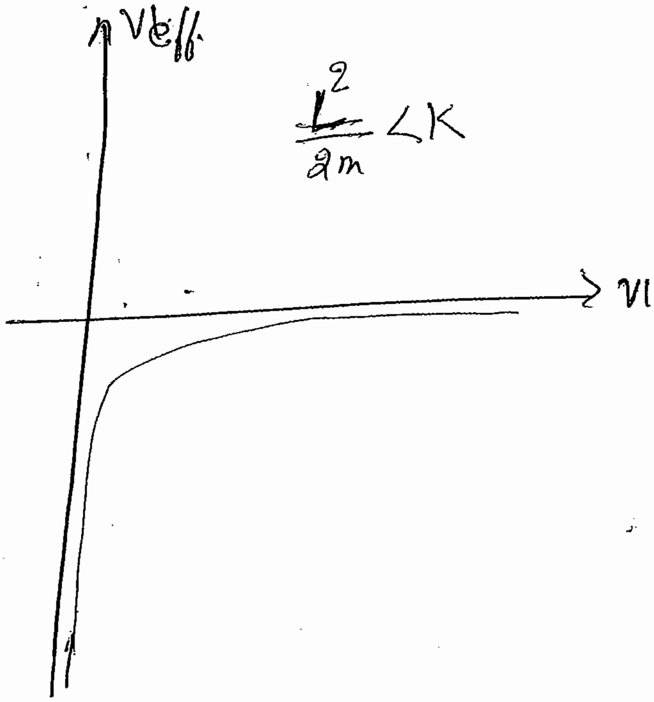
Q. $V(r) = \frac{-K}{r^2}$

$$V_{eff} = -\frac{K}{r^2} + \frac{L^2}{2mr^2}$$

$$= \frac{1}{r^2}\left[\frac{L^2}{2m} - K\right]$$

When $\frac{L^2}{2m} > K$, when $\frac{L^2}{2m} < K$





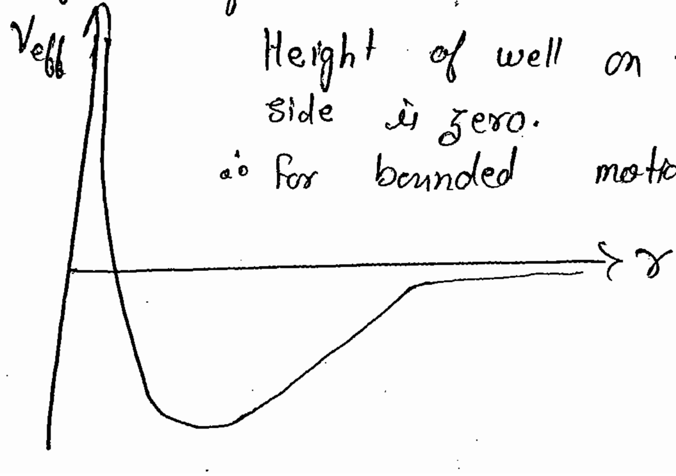
Bounded Motion

If there is a potential well in Veff graph and energy of particle lying inside the well is less than minimum hight of well then motion is bounded.

Example :-

$$V(r) = -\frac{K}{r}$$

$$V_{eff} = \frac{-K}{r} + \frac{L^2}{2mr^2}$$



Height of well on right side is zero.

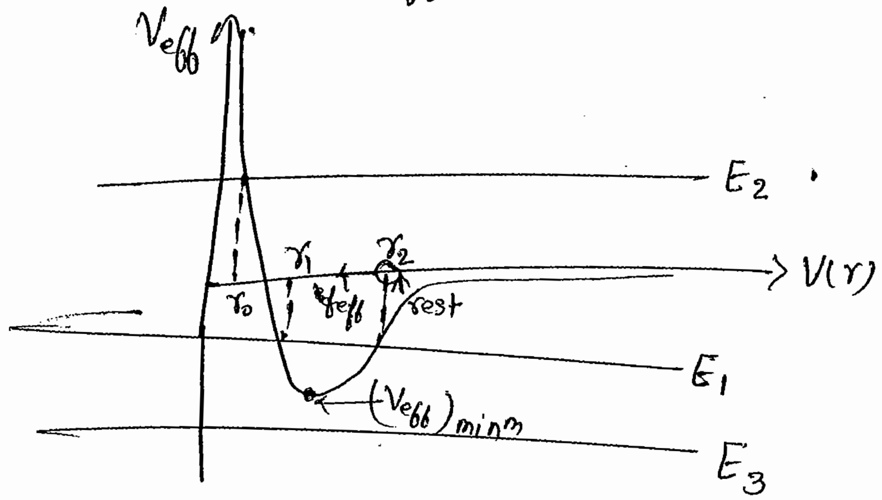
∴ For bounded motion $E < ($

$$E = \frac{1}{2} m\dot{r}^2 + V_{eff}.$$

$$E - V_{eff} = \frac{1}{2} m\dot{r}^2 \leftarrow \text{+ve or zero.}$$

$$E - V_{eff} \geq 0$$

$E \geq V_{eff} \Rightarrow$ Energy line will be above V_{eff} line.



- E_3 is not allowed.
- For energy E_1 particle moves b/w r_1 and r_2 (these are called terning point).
- for $E = E_2$ particle moves between r_0 and ∞

* Minimum allowed Energy :-

$$\boxed{E_{min} = (V_{eff})_{min}}$$

* Condition for stable orbit :-

$$\boxed{\left.\frac{\partial V_{eff}}{\partial r}\right|_{r=r_0} = 0} \quad \text{--- (I)}$$

$$\boxed{\left.\frac{\partial^2 V_{eff}}{\partial r^2}\right|_{r=r_0} > 0} \quad \text{--- (II)}$$

In General :-

If $f = kr^n$

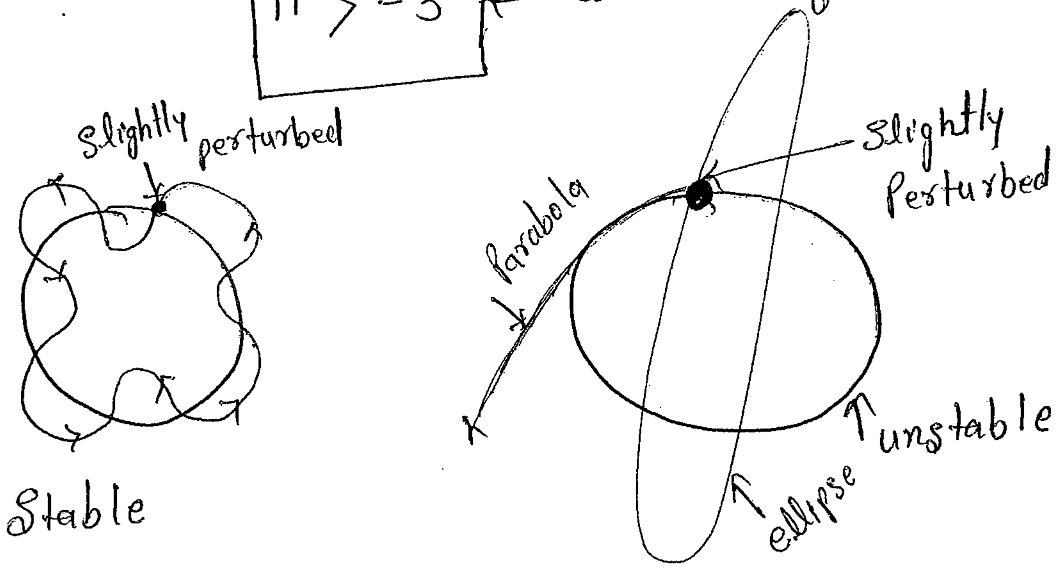
$$V(r) = -\int f(r)\,dr = \frac{-kr^{n+1}}{n+1}$$

$$V_{eff} = V(r) + \frac{l^2}{2mr^2}$$

$$V_{eff.} = -\frac{kr^{n+1}}{n+1} + \frac{l^2}{2mr^2}$$

from the above conditions ① & ② for above orbit, we get -

$\boxed{n > -3}$ ← Condition for Stable orbit.



* Condition for closed Orbit :-

After perturbation slightly the object retrace it's path after some turns. (Ultimately path should closed.).

If $f = kr^n$ then closed orbit is possible only for $n = 1$ and $n = -2$.

[Forces should be attractive]

① Hook's Low

(ii) Gravitational low. }

Q-5

Sol^n

$$f = \frac{K}{r^{3-n}} = K r^{n-3}$$

$n - 3 = 1 \Rightarrow n = 4$

$n = 3 = -2 \Rightarrow n = 1$

$\boxed{n = 4, 1}$ Ans

(*) Polar Equation of orbit under $V(r) = \frac{-K}{r}$

m_1 ○———— r ————○ m_2

$$V(r) = \frac{-G m_1 m_2}{r} = \frac{-K}{r} \qquad K = G m_1 m_2$$

$$f = \frac{-G m_1 m_2}{r^2}$$

$$\boxed{V(r) = -\int f(r)\,dr} = \frac{-K}{r}$$

$-q_1$ •———— r ————• q_2

$$V(r) = \frac{-q_1 q_2}{4\pi\epsilon_0 r} = \frac{-K}{r}$$

$$\Rightarrow \boxed{K = \frac{q_1 q_2}{4\pi \epsilon_0}}$$

Differential Equation of orbit:-

$$\frac{d^2u}{d\theta^2} + u = \frac{-mf}{l^2u^2}$$

$$f(r) = -\frac{\partial V}{\partial r} = -\frac{K}{r^2} = -Ku^2$$

$$\boxed{\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = +\frac{mk}{l^2}}$$

Sol^n of this eq^n gives -

$$\boxed{r = \frac{l^2/mk}{1 \pm \left(\sqrt{1 + \frac{2El^2}{mk^2}}\right)\cos\theta}}$$ → Polar equation of orbit

* Polar form of conic section :-

$$r = \frac{A}{1 \pm e\cos\theta}$$

Compare to get -

$$\boxed{e = \sqrt{1 + \frac{2El^2}{mk^2}}} = \text{eccentricity}$$

e depends on E (energy)

$E > 0$, $e > 1$, orbit is hyperbola
$-\frac{mk^2}{2l^2} < E < 0$, $e < 1$, orbit is ellipse.
$E = 0$, $e = 1$, orbit is parabola
$E = -\frac{mk^2}{2l^2}$, $e = 0$, orbit is circle.

only for $V(r) = -\frac{k}{r}$

Q. A particle of mass m is projected from a height $h = R$ with speed $v = \sqrt{\frac{GM}{R}}$ where M is mass of earth, m is mass of particle. What is the path of the particle? in some direc^n?

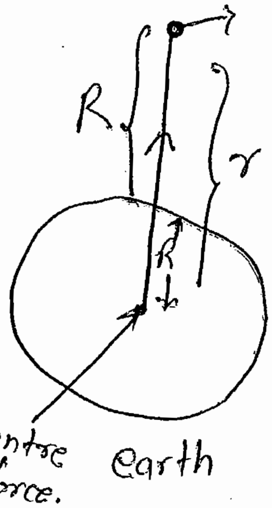
Sol^n "Note

"If a particle is thrown in radial direction it will always be straight line."

$$E = \frac{1}{2} m v^2 + V(r)$$

$$= \frac{1}{2} m \frac{GM}{R} - \frac{K}{r}$$

$$= \cancel{\frac{GMm}{2R}} - \cancel{\frac{GMm}{2R}}$$



$\therefore \boxed{E = 0}$

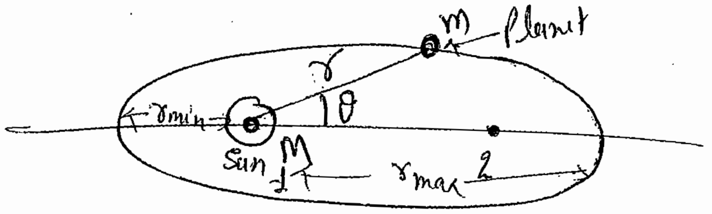
So path is parabola.

* Polar Equation of orbit :-

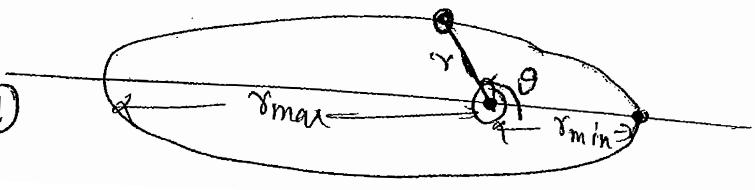
$$\boxed{r = \frac{l^2/mk}{1 \pm e\cos\theta}}$$

Elliptical Orbit :-

$$\boxed{r = \frac{l^2/mk}{1 - e\cos\theta}} \quad \text{—(I)}$$



$$\boxed{r = \frac{l^2/mk}{1 + e\cos\theta}} \quad \text{(II)}$$



r_{max} when $\theta = 0$ } in eq^n (I)
r_{min} when $\theta = \pi$

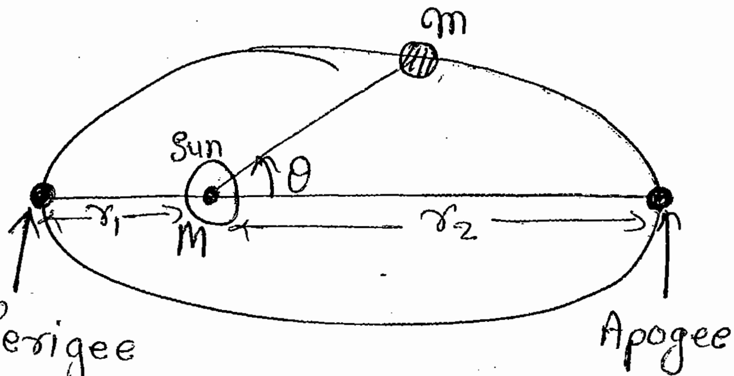
r_{max} when $\theta = \pi$ } in eq^n (II)
r_{min} when $\theta = 0$

* Energy and angular momentum of planet :-

Say:- r_1 and r_2 are given

$$V(r) = -\frac{GMm}{r}$$

Polar equation of orbit-



$$r = \frac{l^2/mk}{1-\left(\sqrt{1+\frac{2El^2}{mk^2}}\right)\cos\theta}$$

$$r_1 = \frac{l^2/mk}{1+\sqrt{1+\frac{2El^2}{mk^2}}} \quad \text{—— (I)}$$

$$r_2 = \frac{l^2/mk}{1-\sqrt{1+\frac{2El^2}{mk^2}}} \quad \text{—— (II)}$$

$$1+\sqrt{1+\frac{2EJ^2}{mk^2}} = \frac{l^2}{mk}\cdot\frac{1}{r_1} \quad \text{—— (III)}$$

$$1-\sqrt{1+\frac{2El^2}{mk^2}} = \frac{l^2}{mk}\cdot\frac{1}{r_2} \quad \text{—— (IV)}$$

(III) + (IV)

$$2 = \frac{l^2}{mk}\left(\frac{r_1+r_2}{r_1 r_2}\right)$$

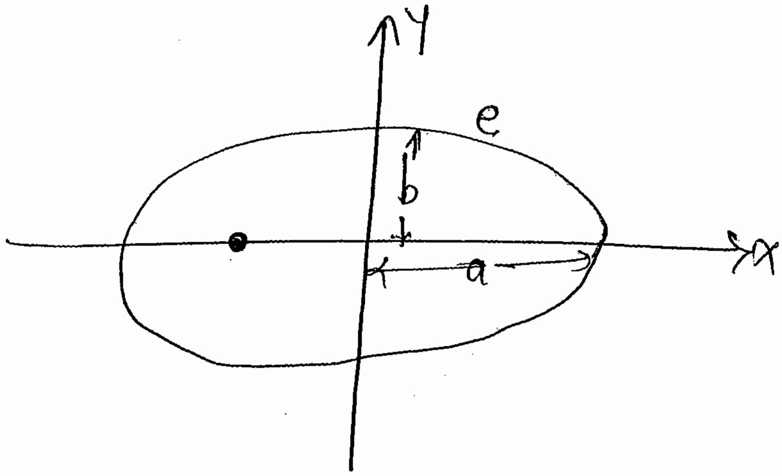
$$l = \sqrt{\frac{2 r_1 r_2}{(r_1+r_2)}}\, mk$$

$$\boxed{l = m\sqrt{\frac{2GM r_1 r_2}{r_1+r_2}}}$$

Put l in eq^n (1) to get Energy.

$$\boxed{E = \frac{-GMm}{r_1+r_2}}$$

In terms of ellipse parameter :-



$$r_1 = (1-e)a$$

$$r_2 = (1+e)a$$

$$e = \sqrt{1-\frac{b^2}{a^2}}$$

$$\boxed{r_1+r_2 = 2a}$$

$$\boxed{E = \frac{-GMm}{2a}}$$

$$r_1 r_2 = (1-e^2)a^2$$

$$= \frac{b^2}{a^2}\cdot a^2$$

$$\boxed{r_1 r_2 = b^2}$$

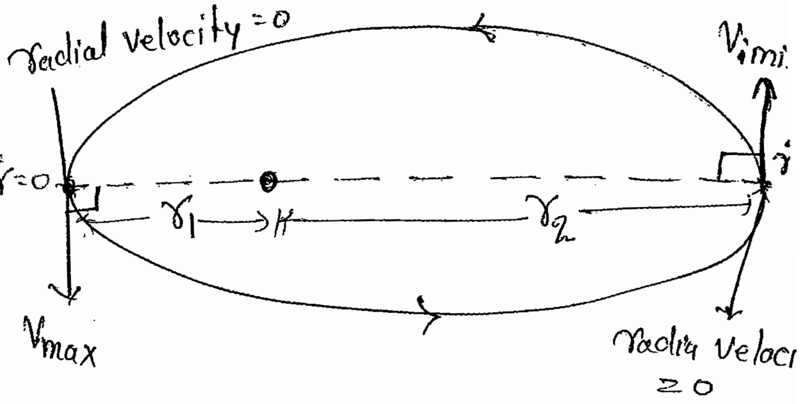
$$L = m\sqrt{\frac{\cancel{2}GMb^2}{\cancel{2}a}}$$

$$\boxed{L = m\sqrt{\frac{GMb^2}{a}}}$$ dependent on a and b.

* Maximum and Minimum Speed in elliptical orbit:-

$L = m v_{max} r_{min} = m v_{max} r_1$

$L = m v_{min} r_{max} = m v_{min} r_2$



$$L = m\sqrt{\frac{2GMr_1 r_2}{(r_1+r_2)}}$$

$$\boxed{v_{min} = \sqrt{\frac{GMr_1}{r_2(r_1+r_2)}}}$$

$$\boxed{v_{max} = \sqrt{\frac{2GMr_2}{r_1(r_1+r_2)}}}$$

$r_1 = (1-e)a$

$r_2 = (1+e)a$

$r_1 + r_2 = 2a$

$$\boxed{v_{min} = \sqrt{\frac{GM}{a}\left(\frac{1-e}{1+e}\right)}}$$

$$\boxed{v_{max} = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)}}$$

Q. Planet is moving around the sum if ratio of maximum to minimum speed is 2. what is accentricity of orbit?

Soln

$$\frac{v_{max}}{v_{min}} = \frac{\sqrt{\cancel{\frac{GM}{a}}\left(\frac{1+e}{1-e}\right)}}{\sqrt{\cancel{\frac{GM}{a}}\left(\frac{1-e}{1+e}\right)}} = 2$$

$$\frac{1+e}{1-e} = 2$$

$1+e = 2-2e$

$3e = 2-1$

$$\boxed{e = \frac{1}{3}}$$ Ans

Q. 27

Sol^n

Polar eq^n of orbit

$$r = \frac{l^2/mk}{1+e\cos\theta}$$

→ Parabola path

⟹ $E=0$, $e=1$

∵ highly elliptical

So we let it is parabola

1st case:-

$$R_0 = \frac{l^2/mk}{1+|1|}$$

$\left\{ \begin{array}{l} \cos\theta = \max^m \\ E=0,\ e=1 \end{array} \right\}$

$$R_0 = \frac{l^2}{2mk} \Rightarrow \frac{l^2}{mk} = 2R_0$$

1st Case:-

Circular orbit :-

$$\because R_0 = \frac{l^2}{2mk} \Rightarrow \frac{l^2}{mk} = 2R_0$$

$$\therefore \frac{V_{max}}{V_{min}} = \frac{1+e}{1-e}$$

$$2 = \frac{1+e}{1-e}$$

Second case :- Circular orbit

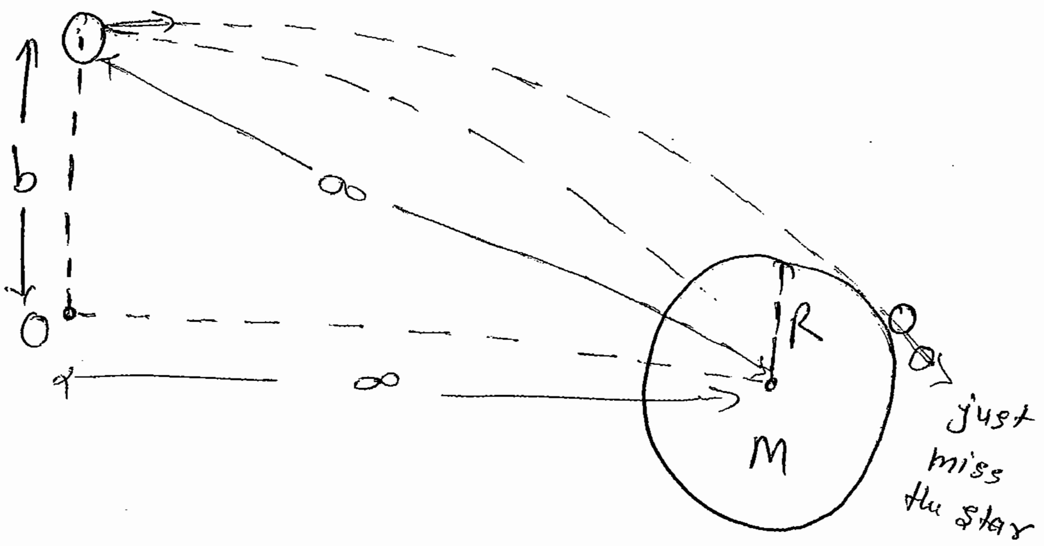
$$e=0$$

$$R_f = \frac{l^2/mk}{1+0}$$

$$\boxed{R_f = 2R_0}$$ Ans

Q.17

Sol^n

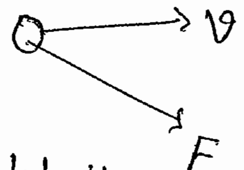


$$r_{min} = R$$

$$r = \frac{l^2/mk}{1 + e\cos\theta}$$

$$r_{min} = \frac{l^2/mk}{1+e}$$

$$\boxed{R = \frac{l^2/mk}{1+e}}$$



Velocity never cut the direc^n of force.

$$R = \frac{l^2/mk}{1 + \sqrt{1 + \frac{2El^2}{mk^2}}}$$

$$E = \frac{1}{2}mv^2 + V(r)$$

$$= \frac{1}{2}mv^2 - \frac{GMm}{\infty} = \frac{1}{2}mv^2$$

$$\boxed{L = mvb}$$

$$\boxed{K = GMm}$$

* Virial Theorem :-

It relates average value of Potential Energy and Kinetic Energy.

If $f = Kr^n$

$$\boxed{\langle K.E. \rangle = \frac{n+1}{2} \langle P.E. \rangle}$$ ← Virial Theorem

Ex –

for Gravitation :- $n = -2$

$$\boxed{\langle K.E. \rangle = -\frac{1}{2} \langle P.E. \rangle}$$

for Harmonic Oscillator :-

$$f = -Kr^1$$

$$n = 1$$

$$\boxed{\langle K.E. \rangle = \langle P.E. \rangle}$$

BA-5

Q.7 Potential energy $V = Kr^n$ then Relation b/w K.E. & P.E. is

(a) $\langle T \rangle = \langle V \rangle$ (b) $\langle T \rangle = \frac{n}{2} \langle V \rangle$ (c) $\langle T \rangle = \frac{3}{2} \langle V \rangle$

(d) $\langle T \rangle = 2 \langle V \rangle$

Soln –

$$V = Kr^n$$

$$f = -\frac{\partial V}{\partial r} = -Knr^{(n-1)}$$