$\hat{n} \to$ vector normal to the surface
$P \to$ Polarisation
Volume bound charge density

$$\boxed{\rho_b = -\vec{\nabla} \cdot \vec{P}}$$

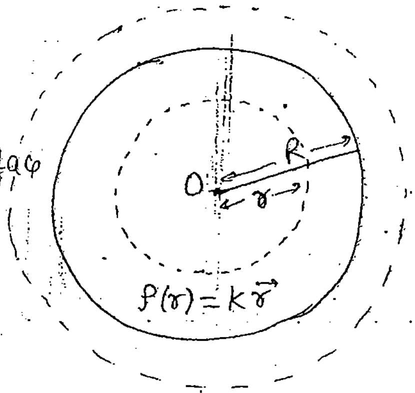
Net Electric field, inside the dielectric then elec field is $E_0 - E$ less than E_0 by the amount E_0/ϵ_r. $\left\{ \epsilon_r = \frac{E_0}{E} \right.$

$$\boxed{E_{Net} = \frac{E_0}{\epsilon_r}}$$

$\epsilon_r \to$ dielectric constant

Que. 1:- A sphere of radius R carries a polarisation & $P(r) = K \vec{r}$ where K is a constant & $\vec{r}$ is the vector from the centre.
(a) Calculate the bound charges σ_b & ρ_b
(b) Calculate the electric field inside & outside the sphere

(a) $\sigma_b = \vec{P} \cdot \hat{n}$
$= K \vec{r} \cdot \hat{n} = K r \hat{r} \cdot \hat{r}$
$= K r = \underline{KR}$ $\left[\begin{array}{l} \text{at surface} \\ r = R \end{array} \right.$



$\rho_b = -\vec{\nabla} \cdot \vec{P}$
$= -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 K \cdot r)$
$= -\frac{K}{r^2} \cdot 3r^2 = \underline{-3K}$

(b) Inside
$q_{enc} = \int \rho_b \, d\tau = -3K \cdot \frac{4\pi}{3} r^3$ (q_{enc} due to volume only)
{bcoz inside sphere is not enclosing its surface}

$$\boxed{q_{enc} = -4\pi K r^3}$$

By Gauss law, $\oint E \cdot ds = \frac{q_{enc}}{\epsilon_0}$

$$E \cdot 4\pi r^2 = \frac{-4\pi K r^3}{\epsilon_0}$$

$$\boxed{E_{in} = -\frac{K r}{\epsilon_0} \hat{r}}$$

If polarisation $\vec{P}$ is radially outward then elec. field will be opposite to it (i.e. towards inside $(-\hat{r})$)

Outside the sphere :- q_{enc} will be due to surface & volume both.

$$q_b(\sigma_b) = KR \cdot 4\pi R^2$$
$$q_b(\sigma_b) = 4\pi KR^3$$
$$\& \ q(\rho_b) = -4\pi KR^3$$
$$\text{Total } q = 4\pi KR^3 - 4\pi KR^3$$
$$\boxed{q = 0}$$
$$\text{So } \boxed{E_{out} = 0}$$

- Gauss Law in Dielectrics :- We have 2 type of bound charge density σ_b & ρ_b.

Gauss law deals with ρ_b. There are 2 types of ρ_b, ρ_b & ρ_F are bound volume charge density & free volume charge density respectively

$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$$

Here $\boxed{\rho = \rho_b + \rho_F}$

$$\Rightarrow \quad \epsilon_0 (\vec{\nabla} \cdot \vec{E}) = \rho_b + \rho_F$$
$$\epsilon_0 (\vec{\nabla} \cdot \vec{E}) = \rho_F - \vec{\nabla} \cdot \vec{P}$$
$$\nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_F$$
$$\boxed{\vec{\nabla} \cdot \vec{D} = \rho_F}$$

D → displacement vector

D, includes $\vec{E}$ & $\vec{P}$ both.

The is the differential form of Gauss law of Dielectrics

Integral form :- $\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$

$$\boxed{\oint \vec{D} \cdot d\vec{s} = q_{enc}}$$

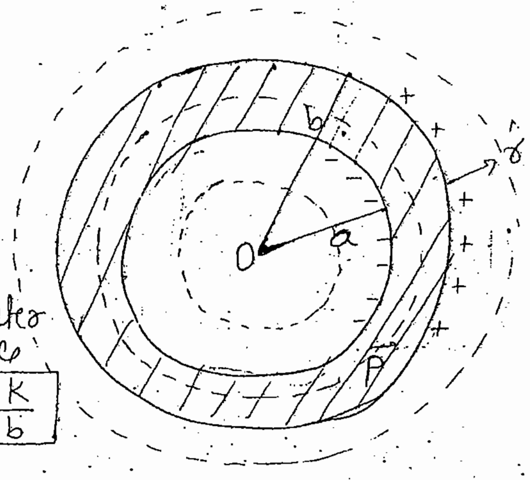
D is totally determined by the free charges.
E " " " , free & bound both

bcoz in D, bound charge is Included in it.

charge on the surface of metal or conductor → free charge

Que :- A thick spherical shell inner radius a & outer radius b is made of dielectric material with polarizati[on] $\vec{P}(r) = \frac{K}{r}\hat{r}$ where K is constant & r is the distan[ce] from the centre. There is no free charge in the region. Find the electric field in all the 3 regions:-

(i) $r<a$
(ii) $a<r<b$
(iii) $r>b$



$$\sigma_b = \vec{P}\cdot\hat{n}$$
$$= \frac{K}{r}\hat{r}\cdot\hat{r} = \frac{K}{r}$$

inner surface $\boxed{\sigma_b = -\frac{K}{a}} \neq \cancel{\frac{K}{b}}$ & on outer surface $\boxed{\sigma_b = \frac{+K}{b}}$

$$\rho_b = -\nabla\cdot\vec{P}$$
$$= -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{K}{r}\right) = -\frac{1}{r^2}\frac{\partial}{\partial r}(Kr)$$

$$\boxed{\rho_b = -\frac{K}{r^2}}$$

In $r<a$, $q_{enc} = 0$
$$E = 0$$

$a<r<b$
due to volume $q_{enc} = \int_a^r \rho_b\, d\tau = \int_a^r \frac{-K}{r^2} r^2\sin\theta\, d\theta\, d\phi$
$$= \int_a^r -\frac{K}{r^2}4\pi r^2 dr = -4\pi K[r]_a^r$$
$$= -4\pi K(r-a)$$

due to inner surface $q_{enc} = \oint -\frac{K}{a}\cdot 4\pi r^2 = -\frac{K4\pi}{a}[a^2]$
$$= -4\pi K a$$

$$q = -4\pi Kr + \cancel{4\pi Ka} - \cancel{4\pi Ka}$$
$$\boxed{q_{enc} = -4\pi Kr}$$

$$E\cdot 4\pi r^2 = -\frac{4\pi Kr}{\epsilon_0} \Rightarrow \boxed{E = -\frac{K}{\epsilon_0 r}\hat{r}}$$

$r>b$
$$q_{enc} = \underset{+4\pi Ka}{-4\pi Kb} + \underset{-4\pi Ka}{4\pi Kb} = 0$$
$$\boxed{E = 0}$$

II Method :- (i) $r<a$

$$\oint D\cdot dS = q_{enc}$$

$q_{enc}=0$ So $\boxed{D=0}$ $\Rightarrow$ $\epsilon_0 E + P = 0$

$\Rightarrow \boxed{E=0}$

(ii) $a<r<b$

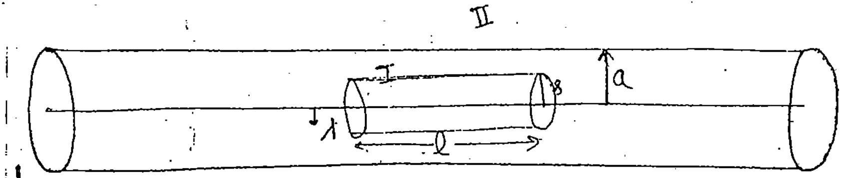
$D=0$ (No free charge in this region)

$\epsilon_0 E + P = 0 \Rightarrow \epsilon_0 E + \frac{k}{r}\hat{r} = 0$

$$\boxed{E = -\frac{k}{\epsilon_0 r}\hat{r}}$$

(iii) $r>b$ $D=0 \Rightarrow \epsilon_0 E + P = 0 \Rightarrow \boxed{E=0}$

Q.2:- A long straight wire carrying a line charge λ is surrounded by rubber insulation upto radius a. Find electric displacement D & Electric field inside & outside the rubber insulation.



Here polarization is not given & dont know about bound charge

$q_{enc} = q_f + q_b$

Use Gauss law in Dielectrics

Inside:- $\oint D\cdot ds = q_{enc}$

$q_{enc} = \lambda l$

$\therefore D\cdot 2\pi s l = \lambda l$

$$\boxed{D = \frac{\lambda}{2\pi s}\hat{s}}$$

$D = \epsilon_0 E + P$. Also $\vec{D} = \epsilon \vec{E}$. This ϵ includes P.

$\epsilon E = \frac{\lambda}{2\pi s}\hat{s}$

$E = \frac{D}{\epsilon} = \frac{D}{\epsilon_0 \epsilon_r}$

$$\boxed{E_{in} = \frac{D}{\epsilon_0 \epsilon_r} = \frac{\lambda}{2\pi \epsilon_0 \epsilon_r s}\hat{s}}$$

Outside:- q free enclosed $q_{enc} = \lambda l$

$D\cdot 2\pi s l = \lambda l \Rightarrow \boxed{D = \frac{\lambda}{2\pi s}\hat{s}}$

In free space $\epsilon_r = 1$.

so $$\boxed{E_{out} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}}$$

<u>Linear Dielectrics</u> :- are those dielectrics in which polarisation is proportional to the electric field.

$$\vec{P} \propto \vec{E}$$

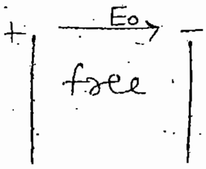
$$\Rightarrow \boxed{\vec{P} = \epsilon_0 \chi_e \vec{E}_{Macro}}$$

$\epsilon_0 \rightarrow$ permiability

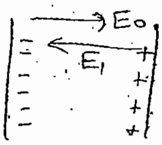
$\chi_e \rightarrow$ Electric Suceptability

$E \rightarrow$ Macroscopic elec. field

$\{ \vec{P} = \alpha \vec{E}_{local} \}$



when applied elec. field is E_0. This is local ele. field.
& if we filled dielectric b/w these plates
There are many atoms in dielectric



Then a inside elec field E_1 which is in opposit dir^n to E_0 Then total Elec field $= E_0 + E_1$ (vector)

This Ele. field is <u>Macroscopic</u>. $= E_0 - E_1$ (magnitude)

In Maxwell eq^n, the Macroscopic E-field on a indivisual atom is diffrent from average field on plates called <u>Local E.F.</u>

We have $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ Also $D = \epsilon E$

so $\epsilon \cancel{\vec{E}} = \epsilon_0 \cancel{\vec{E}} + \epsilon_0 \chi_e \cancel{\vec{E}}$

$$\epsilon = \epsilon_0(1 + \chi_e)$$

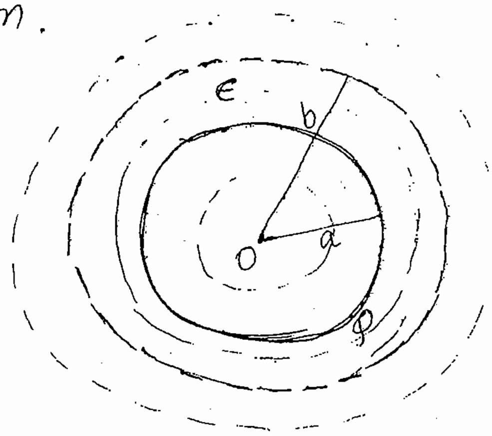
$$\boxed{\epsilon_r = 1 + \chi_e}$$

{ If we apply external E.F. on a material then How much it oppolse that E.F. will be susceptability. }

<u>Q</u> :- A metalic sphere of radius 'a' carries a charge Q. It is surrounded by a linear dielectric of permitivity ϵ upto radius b. find

(i) Polarisation (ii) bound charge densities
(iii) Displacement Vector (iv) Electric field

(v) Pot^n at the centre of the sphere
(vi) Energy of this configuration.



$$P = \epsilon_0 \chi_e E$$

$r<a$, $q_{enc} = Q \Rightarrow \boxed{D=0}$

$a<r<b$, $\oint D \cdot dS = q_{enc}$

$$D \cdot 4\pi r^2 = Q$$

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2}\hat{r}}$$

$b<r$, $\boxed{\vec{D} = \frac{Q}{4\pi r^2}\hat{r}}$

<u>Now electric field</u>, $\vec{D} = \epsilon \vec{E}$

$r<a$, $D=0 \Rightarrow \boxed{E=0}$

$a<r<b$, $\boxed{E = \frac{Q}{4\pi\epsilon r^2}\hat{r}}$

$r>b$, $\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}}$ Outside the sphere permitivity is ϵ_0.

<u>Polarisation</u>, $\vec{P} = \epsilon_0 \chi_e \vec{E}$

$r<a$, $\boxed{P=0}$

$a<r<b$, $\vec{P} = \epsilon_0(\epsilon_r - 1)\vec{E}$

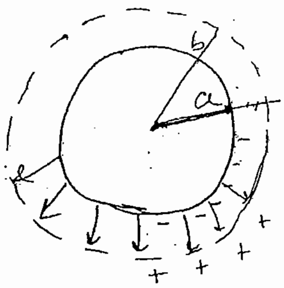
$$\boxed{\vec{P} = \frac{(\epsilon - \epsilon_0)Q}{4\pi\epsilon r^2}\hat{r}}$$

$r>b$, $\boxed{\vec{P} = 0}$ $\left\{ \begin{aligned} \vec{P} &= \epsilon_0(1-1)\vec{E} \\ &= 0 \quad [\epsilon_r = 1] \end{aligned} \right\}$

<u>Bound charge density</u>

$\sigma_b = \vec{P}\cdot\hat{n} \Rightarrow \sigma_b\Big|_{r=a} = -\frac{(\epsilon-\epsilon_0)Q}{4\pi\epsilon a^2}$

$$\sigma_b\Big|_{r=b} = +\frac{(\epsilon-\epsilon_0)Q}{4\pi\epsilon b^2}$$



$\rho_b = -\vec{\nabla}\cdot\vec{P} \Rightarrow \rho_b = -\frac{(\epsilon - \epsilon_0)Q}{4\pi\epsilon r^2} \cdot 4\pi\, \delta^3(r)$

$$\boxed{\rho_b = -\frac{(\epsilon-\epsilon_0)Q}{\epsilon}\,\delta^3(r)}$$

$\boxed{\rho_b = 0}$ $a < r < b$, .. Not enclosing the origin.

Potⁿ at the centre

$$Q_a = Q + \sigma_b|_{r=a} \times 4\pi a^2$$
$$= Q - \frac{(\epsilon-\epsilon_0)Q}{4\pi\epsilon a^2} \cdot 4\pi a^2$$
$$= Q - \frac{(\epsilon-\epsilon_0)Q}{\epsilon} = Q - Q + \frac{\epsilon_0}{\epsilon}Q$$

$$\boxed{Q_a = \frac{Q}{\epsilon_r}}$$

That's why elec. field inside the dielectric becomes $\frac{1}{\epsilon_r}$ of its value bcoz charge inside " " is Q/ϵ_r.

$$Q_b = \cancel{Q} \cancel{+} \sigma_b|_{r=b} \times 4\pi b^2$$
$$= \cancel{Q} \cancel{+} \frac{(\epsilon-\epsilon_0)Q}{\epsilon} = \cancel{Q} + \cancel{Q} - \cancel{\frac{\epsilon_0}{\epsilon}Q}$$

$$\boxed{\cancel{Q_b = 2Q \mp \frac{Q}{\epsilon_r}}}$$

Potⁿ at centre $V = \frac{1}{4\pi\epsilon_0}\frac{Q_a}{a} + \frac{1}{4\pi\epsilon_0}\frac{Q_b'}{b}$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{\epsilon_r a} + \left(\frac{\epsilon-\epsilon_0}{\epsilon}\right)\frac{Q}{b}\right]$$

$$\boxed{V_{centre} = \frac{Q}{4\pi}\left[\frac{1}{\epsilon a} + \frac{1}{\epsilon_0 b} - \frac{1}{\epsilon b}\right]}$$

By Another Method, $V_{centre} = -\int_\infty^0 \vec{E}\cdot d\vec{r}$

$$= -\int_\infty^b \vec{E}\cdot d\vec{r} - \int_b^a \vec{E}\cdot d\vec{r} - \int_a^0 \vec{E}\cdot d\vec{r}$$

$V = -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_\infty^b - \frac{Q}{4\pi\epsilon}\left[-\frac{1}{r}\right]_b^a - 0 = \frac{Q}{4\pi\epsilon_0}\frac{1}{b} + \frac{Q}{4\pi\epsilon}\frac{1}{a} - \frac{Q}{4\pi\epsilon}\frac{1}{b}$

Energy. $U = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 \, d\tau$

$$U = \frac{\epsilon_0}{2}\left[\int_0^a E_1^2 \, 4\pi r^2 dr + \int_a^b E_2^2 \, 4\pi r^2 dr + \int_b^\infty E_3^2 \, 4\pi r^2 dr\right]$$

$$U = 0 + \frac{\epsilon_0}{2}\int_a^b \frac{Q^2}{(4\pi\epsilon)^2 r^4} \cdot 4\pi r^2 dr + \frac{\epsilon_0}{2}\int_b^\infty \frac{Q^2}{(4\pi\epsilon_0)^2 r^4} \cdot 4\pi r^2 dr$$

$$= \frac{\epsilon_0}{2}\frac{Q^2}{4\pi\epsilon^2}\left[-\frac{1}{r}\right]_a^b + \frac{\epsilon_0}{2}\frac{Q^2}{4\pi\epsilon_0^2}\left[\frac{-1}{r}\right]_b^\infty$$

$$= \frac{\epsilon_0}{2}\frac{Q^2}{4\pi}\left[-\frac{1}{\epsilon^2}\left(\frac{1}{b}-\frac{1}{a}\right) - \frac{1}{\epsilon_0^2}\left[-\frac{1}{b}\right]\right]$$

$$= \frac{\epsilon_0}{2}\frac{Q^2}{4\pi}\left[-\frac{1}{\epsilon^2}\left(\frac{1}{b}-\frac{1}{a}\right) + \frac{1}{\epsilon_0^2 b}\right]$$

$$\boxed{U = \frac{\epsilon_0 Q^2}{8\pi}\left[\frac{1}{\epsilon^2 a} - \frac{1}{\epsilon^2 b} + \frac{1}{\epsilon_0^2 b}\right]}$$

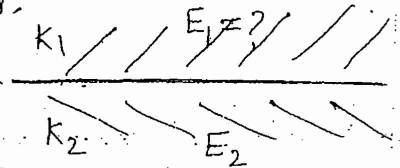
* Energy inside the Dielectric is

$$\boxed{U = \frac{1}{2}\int_{\text{all space}} \vec{D}\cdot\vec{E} \, d\tau}$$

Boundry Conditions at dielectric interface :-

Boundry comes when medium changes.
If we have 2 dielectrics having dielectric constants K_1 & K_2 ~~then~~ if Electric field is given in one region suppose E_2 & we have to find out E-field in another region E_1.
This can be done by Boundry Conditions.

dielectric Interface



Gauss law in free space

$$\oint \vec{E}\cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$$

(1) $\boxed{E^{\perp}_{above} - E^{\perp}_{below} = \frac{\sigma}{\epsilon_0}}$

Gauss law in Dielectrics

$$\oint \vec{D}\cdot d\vec{S} = q_{enc}$$

(1) $\boxed{D^{\perp}_{above} - D^{\perp}_{below} = \sigma_f}$ $\rightarrow$ free charge density

D is discontinous by the amount σ_f. $D^{\perp} \rightarrow$ Normal Comp.

(2) $\oint E \cdot dl = 0$

$$\boxed{E^{\parallel}_{above} = E^{\parallel}_{below}}$$

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

$\vec{E} = \frac{1}{\epsilon_0}[\vec{D} \pm \vec{P}]$

So $\oint E \cdot dl = \frac{1}{\epsilon_0}\oint [D - P] \cdot dl = 0$

$\oint \vec{D} \cdot \vec{dl} = \oint \vec{P} \cdot \vec{dl}$

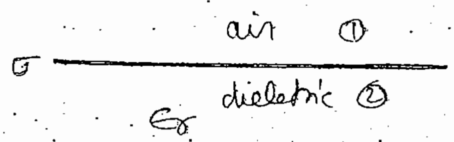
$$\boxed{D^{\parallel}_{above} - D^{\parallel}_{below} = P^{\parallel}_{above} - P^{\parallel}_{below}}$$

The tangencial comp. of $\vec{D}$ is discontinuos by the difference b/w Polarisation.

case

1. <u>Air dielectric interface</u> :- for air dielectric interface

$\sigma_f = 0$



$D^{\perp}_{above} = D^{\perp}_{below}$

$$\boxed{D_{1n} = D_{2n}}$$

$D_n \Rightarrow$ normal comp. of ~~the~~ D

Normal comp. of displacement vector & tangential comp. of E is continous but normal comp. of $\vec{E}$ & tangential comp. of D is discontinous bcoz it contain both charge density (free & bound) σ_b & σ_f

$$\boxed{E_{1n} - E_{2n} = \frac{\sigma}{\epsilon}}$$

$\sigma = \sigma_b + \sigma_f$

Tangential Component :- $E^{\parallel}_{above} = E^{\parallel}_{below}$

$$\boxed{E_{1t} = E_{2t}}$$

& $D^{\parallel}_{above} = D^{\parallel}_{below}$

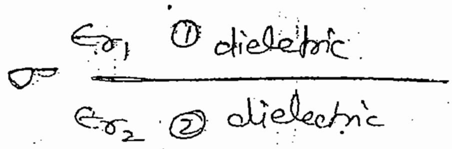
$$\boxed{D_{1t} = D_{2t}}$$

still $\sigma_f = 0$

2. <u>Dielectric - Dielectric Interface</u> :- $\sigma_f = 0$

$E_{1t} = E_{2t}$

$D_{1n} = D_{2n}$ (σ_f should be zero)



3. Air - Conductor Interface :-

This boundary contain free charge Then

$\boxed{\sigma_b \neq 0}$

air ①

σ_f

Conductor ②

$$D^{\perp}_{above} - D^{\perp}_{below} = \sigma_f$$

$$\boxed{D_{1n} - D_{2n} = \sigma_f}$$

& $\boxed{E_{1n} - E_{2n} = \frac{\sigma_f}{\epsilon_0}}$

In region ② the normal & tangential comp. of Electric field is zero. $E_{1t} = E_{2t}$ $\{E_{2t} = 0\}$

$$\boxed{E_{1t} = 0}$$

$\boxed{E_{1n} = \frac{\sigma_f}{\epsilon_0}}$ & $\boxed{D_{1t} - D_{2t} = P_{1t} - P_{2t}}$

4. Dielectric - Conductor Interface :-

Here charge density is σ.

σ_f bcoz of conductor &

σ_b " " dielectric

dielectric ①

σ

Conductor ②

$$\sigma = \sigma_b + \sigma_f$$

$$\boxed{E_{1n} - E_{2n} = \frac{\sigma}{\epsilon_0}}$$

$\boxed{E_{2n} = 0}$ $\boxed{D_{1n} - D_{2n} = \sigma_f}$

$\boxed{E_{1n} = \frac{\sigma}{\epsilon_0}}$ $\boxed{D_{1t} - D_{2t} = P_{1t} - P_{2t}}$

$$\boxed{E_{1t} = 0}$$

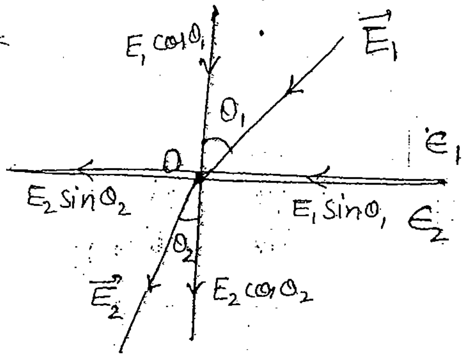
Que :- At the interface b/w 2 linear dielectrics & electric field lines bend as shown in the figure. Show that $\frac{\tan\theta_2}{\tan\theta_1} = \frac{\epsilon_2}{\epsilon_1}$. Assuming no free charge at the boundary.

For linear dielectrics

$$\vec{D} = \epsilon\vec{E}$$

This rel^n is possible only for linear dielectrics.

In boundary b/w dielectric & dielectric, there is No free charge.



$$D_{1n} = D_{2n} \quad \text{—— (i)}$$
$$E_{1t} = E_{2t} \quad \text{—— (ii)}$$

Elec. field in region ① is $\vec{E_1}$ & ② is $\vec{E_2}$ and permitivity in ① & ② are ϵ_1 & ϵ_2 then

(i) → $\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$

$\Rightarrow \epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2$ —— (1)

(ii) → $E_1 \sin\theta_1 = E_2 \sin\theta_2$ —— (2)

Devide $\frac{(2)}{(1)} \Rightarrow \frac{\tan\theta_1}{\epsilon_1} = \frac{\tan\theta_2}{\epsilon_2}$

$$\Rightarrow \boxed{\frac{\tan\theta_2}{\tan\theta_1} = \frac{\epsilon_2}{\epsilon_1}}$$

Here permitivity was given but if dielectric constai ϵ_{r_1} & ϵ_{r_2} are given then

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{\epsilon_2}{\epsilon_1} = \frac{\epsilon_0\epsilon_{r_2}}{\epsilon_0\epsilon_{r_1}}$$

$$\Rightarrow \boxed{\frac{\tan\theta_2}{\tan\theta_1} = \frac{\epsilon_{r_2}}{\epsilon_{r_1}}}$$

OR $$\boxed{\frac{\cot\theta_2}{\cot\theta_1} = \frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{\epsilon_1}{\epsilon_2}}$$

If an electric field comes from lower to higher dielectric region then it will bend away from the normal.

If $\epsilon_{r_1} = 2$, $\epsilon_{r_2} = 3$ $\Rightarrow \frac{\epsilon_{r_2}}{\epsilon_{r_1}} > 1$

$\left\{ \frac{\epsilon_{r_2}}{\epsilon_{r_1}} = \frac{3}{2} = 1.5 > 1 \right\}$

So $\tan\theta_2 > \tan\theta_1$

$$\boxed{\theta_2 > \theta_1}$$

Que :- Two extensive homogenous isotropic dielectric meet on $z=0$ plane. for $z>0$, $\epsilon_{r_1}=4$ & for $z<0$, $\epsilon_{r_2}=3$. A uniform electric field $\vec{E}$

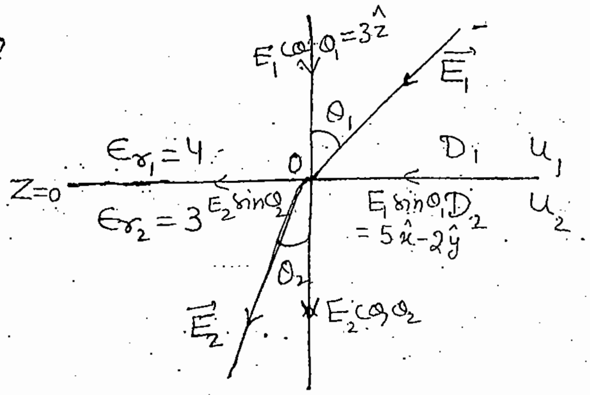
$$\vec{E_1} = 5\hat{x} - 2\hat{y} + 3\hat{z} \ \ KV/m$$

exist for $z\geq 0$. find

(a) E_2 for $z<0$ (b) D_1, $z>0$ & D_2, $z<0$

(c) The Angles, E_1 & E_2 making with the normal.

(d) Energy densities Joule/m^3 in both dielectrics.



Both are dielectrics - so there is no free charge. $\therefore \sigma_f = 0$

$$D_{1n} = D_{2n} \quad \text{——— (1)}$$

$$E_{1t} = E_{2t} \quad \text{——— (2)}$$

$$\Rightarrow \epsilon_1 E_{1n} = \epsilon_2 E_{2n} \quad \text{——— (3)}$$

vector normal to the surface is z,

then $E_{1t} = 5\hat{x} - 2\hat{y}$, $E_{1n} = 3\hat{z}$

(a) from (2) $\Rightarrow$ $E_{1t} = E_{2t} \Rightarrow \boxed{E_{2t} = 5\hat{x} - 2\hat{y}}$

(3) $\Rightarrow$ $\cancel{\epsilon_0}\epsilon_{r_1} E_{1n} = \cancel{\epsilon_0}\epsilon_{r_2} E_{2n}$

$$E_{2n} = \frac{\epsilon_{r_1} E_{1n}}{\epsilon_{r_2}} = \frac{4}{3} \times 3\hat{z}$$

$$E_{2n} = 4\hat{z}$$

So $E_2 = E_{1n} + E_{2n}$

$$\boxed{\vec{E_2} = 5\hat{x} - 2\hat{y} + 4\hat{z}} \text{ Ans}$$

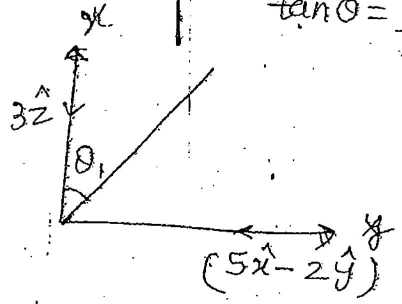
(b) $\vec{D} = \epsilon\vec{E}$

$$\vec{D_1} = \epsilon_{r_1}\epsilon_0 \vec{E_1}$$

$$\boxed{\vec{D_1} = 4\epsilon_0 \vec{E_1}}$$

& $\boxed{\vec{D_2} = 3\epsilon_0 \vec{E_2}}$ Ans (c). $\dfrac{\tan\theta_2}{\tan\theta_1} = \dfrac{\epsilon_{r_2}}{\epsilon_{r_1}} = \dfrac{3}{4}$

$\tan\theta = \frac{y}{x}$



$\tan\theta_1 = \dfrac{|5\hat{x}-2\hat{y}|}{|3\hat{z}|} = \dfrac{\sqrt{29}}{\sqrt{9}}$

$\boxed{\tan\theta_1 = \dfrac{\sqrt{29}}{3}}$

So $\tan\theta_2 = \tan\theta_1 \times \dfrac{3}{4} = \dfrac{\sqrt{29}}{3}\times\dfrac{3}{4}$

$\boxed{\tan\theta_2 = \dfrac{\sqrt{29}}{4}}$

(d) $U = \dfrac{\epsilon_0}{2}\displaystyle\int_{\text{all space}} E^2 d\tau$

{Energy density = energy per unit volume $u = \dfrac{U}{V}$}

$u_1 = \dfrac{\epsilon_0}{2}E_1^2$

$u_2 = \dfrac{\epsilon_0}{2}E_2^2$

$\Rightarrow u_1 = \dfrac{\epsilon_0}{2}\left[\sqrt{25+4+9}\right]^2 = \dfrac{\epsilon_0}{2}(38) = 19\epsilon_0$

$\boxed{u_1 = 19\epsilon_0}$

$\Rightarrow u_2 = \dfrac{\epsilon_0}{2}\left[\sqrt{25+4+16}\right]^2 = \dfrac{\epsilon_0}{2}(90)$

$\boxed{u_2 = 45\epsilon_0}$

{$E = kV$}
{$E = 10^3\, V$}

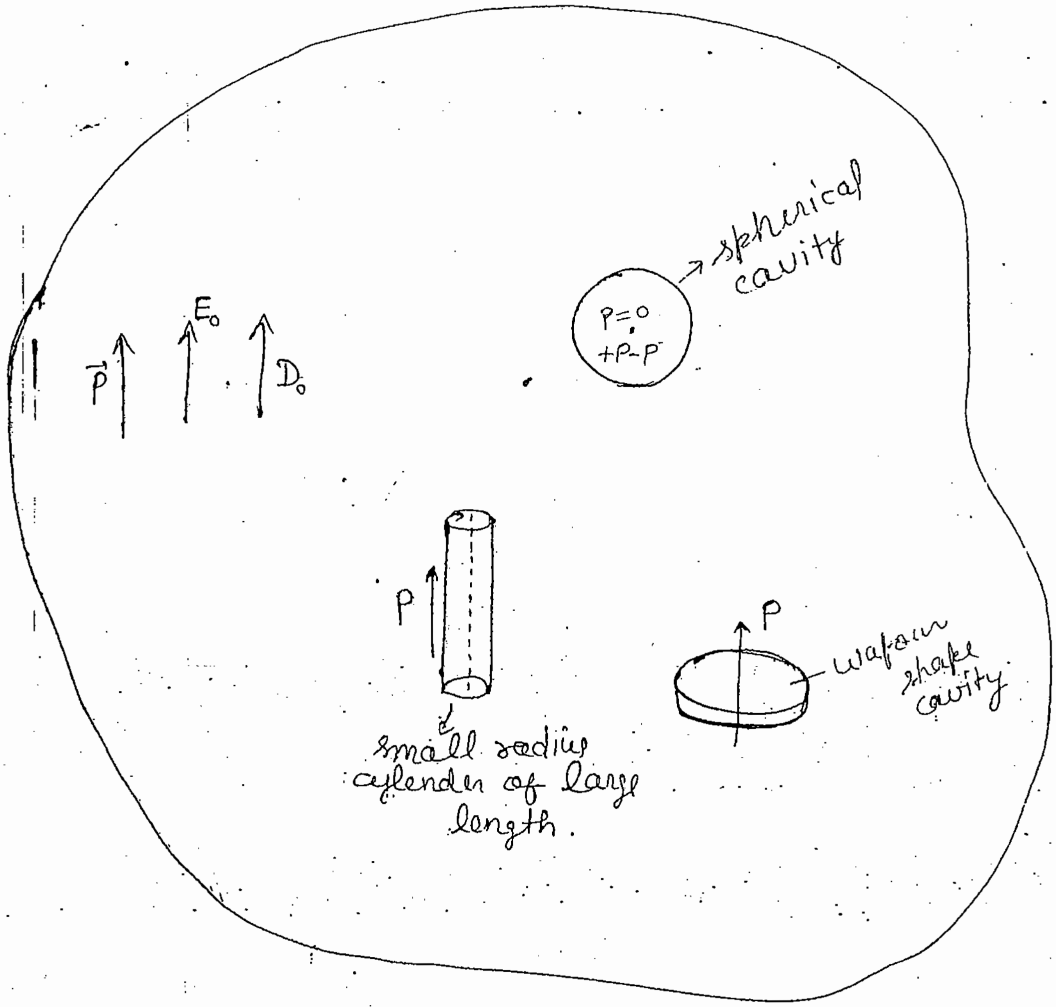
$\Rightarrow \boxed{u_1 = 19\epsilon_0 \times 10^6\ J/m^3 \quad \& \quad u_2 = 45\epsilon_0 \times 10^6\ J/m^3}$

Que :- Suppose the field inside a large piece of dielectric is E_0 such that electric displacement is D_0 s.t.
$\vec{D}_0 = \epsilon_0\vec{E} + \vec{P}$.

(a) Now a spherical cavity is hollowed out of the material. Find electric field & displacement vector inside the cavity.

(b) To the same for a long niddle shape cavity running parallel to $\vec{P}$.

(c) To the same for a thin wavoup shaped cavity $\perp$ to $\vec{P}$.



Elecbric field in spherical cavity is more as compare to Elec field in dielectrics. As we know that elec-field in free space is greater than elec field in dielectric.

<u>Inside the spherical cavity</u> $\vec{D}_0 = \epsilon_0 \vec{E} + \vec{P}$

Polarisation, $P = 0$

$+P - P = 0$

bcoz of $+P$ the elec field will be E_0

& " " $-P$ " " " " $-\frac{\vec{P}}{3\epsilon_0}$

$$\vec{E} = -\frac{\vec{P}}{3\epsilon_0}$$

So total Elec field in spherical cavity is

$$= \vec{E_0} + \frac{\vec{P}}{3\epsilon_0}$$

We have a uniformly polarised sphere, E_0 is external applied Elec field then due to external field, internal elec field will be produced $P/3\epsilon_0$.

Total $= E_0 - \frac{P}{3\epsilon_0} < E_0$

$$\vec{D_0} = \epsilon_0 \vec{E_0} + \vec{P} \qquad \Rightarrow (\epsilon_0 E_0 = D_0 - P)$$

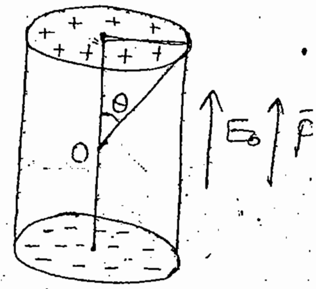
$$\vec{D} = \epsilon_0 \vec{E} \qquad (\text{Put } \vec{E})$$

$$\vec{D} = \epsilon_0 \vec{E_0} + \frac{\vec{P}}{3}$$

$$\vec{D} = D_0 - \vec{P} + \frac{\vec{P}}{3} \Rightarrow \boxed{\vec{D} = \vec{D_0} - \frac{2\vec{P}}{3}}$$

Niddle shape Cavity

When external elec. field E_0 is applied then it will produce polarisation $\vec{P}$ inside it then total elec. field



$$\boxed{\vec{E} = \vec{E_0} - \frac{\vec{P}}{\epsilon_0}(1-\cos\theta)}$$

Elec. field inside the dielectric is less than E_0

for Niddle shape cavity

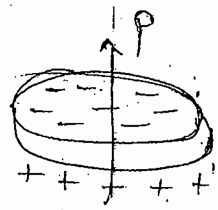
$$\boxed{\theta = 0^\circ}$$

So $\vec{E} = \vec{E_0} - \frac{\vec{P}}{\epsilon_0}(1-1)$

$$\Rightarrow \boxed{\vec{E} = \vec{E_0}}$$

$$\vec{D} = \epsilon_0 \vec{E}$$
$$= \epsilon_0 \vec{E_0} - P + P$$
$$\boxed{\vec{D} = \vec{D_0} - \vec{P}}$$

Wafeur shape $\boxed{\theta = \pi/2}$

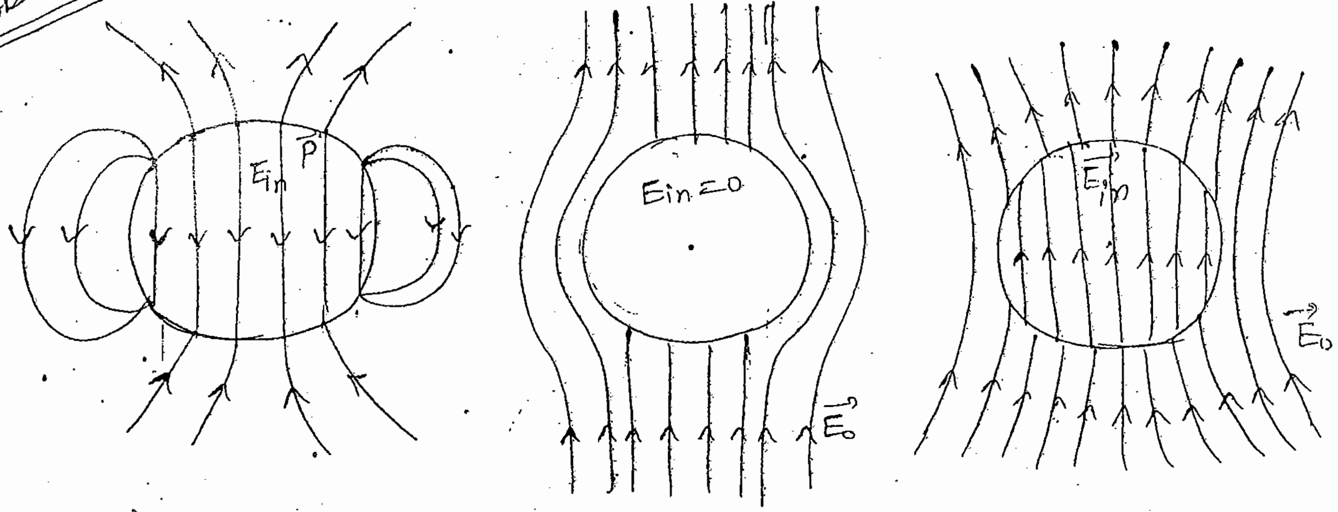


$$\vec{E} = \vec{E_0} + \frac{\vec{P}}{\epsilon_0}$$

$\left\{ P = 0 \Rightarrow \underset{E_0}{+P} - \underset{-\frac{P}{\epsilon_0}}{P} = 0 \right\} \rightarrow$ by superposition Principal

$$\vec{D} = \epsilon_0 \vec{E}$$
$$= \epsilon_0 \vec{E_0} + P$$
$$\boxed{\vec{D} = \vec{D_0}}$$

Problem :-



(i) $\vec{P}$ uniformly polarised sphere

$$\vec{E}_{in} = -\frac{\vec{P}}{3\epsilon_0}$$

(ii) Conducting sphere in uniform elec. field

$$E_{in} = 0$$

(iii) sphere filled with homogeneous linear dielectric

$$\vec{E}_{in} = \frac{3}{\epsilon_r + 2}\vec{E}_0$$

Que :- The space b/w plates of parallel plate capacitor is filled with two slabs of linear dielectric material. Each slab has thickness a. Slab 1 has dielectric const. of 2. slab 2 has a dielectric const. of 1.5. Free charge density on the top plate is $+\sigma$ & the bottom plate is $-\sigma$. Find

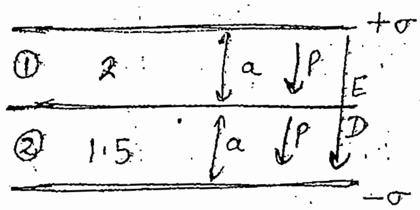
(a) $\vec{D}$ in each slab
(b) $\vec{E}$ in each slab
(c) $\vec{P}$ in each slab
(d) V b/w the plates
(e) location & amount of bound charges

$+\sigma$ & $-\sigma$ are free charge density bcoz plates of parallel plates capacitor is made up of metal. & on metal surface free charges are found & bound charge density is zero.

(1), $\oint D \cdot dS = q_{enc}$.

$D \cdot A = \sigma A$

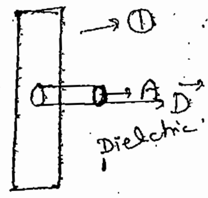
$$\boxed{D_1 = \sigma}$$



Amount of bound charges in slab (1) & (2) are different bcz dielectric constant is diffrnt.

But amount of free charges is same. & D is determined by free charges. so

$$\boxed{D_1 = D_2 = \sigma}$$



flux will pass throug only by the cross-sectio which is in dielectric.

Electric field :- It is determined by both bound & free charges.

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

$$\boxed{\vec{E_1} = \frac{\sigma}{2\epsilon_0}(-\hat{z})}$$

$$\boxed{\vec{E_2} = \frac{2\sigma}{3\epsilon_0}(-\hat{z})}$$

Polarisation :- $\vec{P} = \epsilon_0 \chi_e \vec{E}$

$$\vec{P} = \epsilon_0 (\epsilon_r - 1)\vec{E}$$

$$\boxed{\vec{P_1} = \frac{\sigma}{2}(-\hat{z})}$$

$$\boxed{P_2 = \frac{\sigma}{3}(-\hat{z})}$$

D_1 & D_2 are same but E_1 & E_2 and P_1 & P_2 are different. so E_1 & P_1 and E_2 & P_2 adjest so that D_1 & D_2 same.

Potn diff :-

$$V = V_1 + V_2$$

$$V = \frac{\sigma a}{2\epsilon_0} + \frac{2\sigma a}{3\epsilon_0}$$

$$= \frac{\sigma a}{\epsilon_0}\left[\frac{1}{2} + \frac{2}{3}\right]$$

$$\boxed{V = \frac{7\sigma a}{6\epsilon_0}}$$

(e) There are 4 surfaces :- Upper & lower surfaces of dielectric (1) & (2).

Upper surface of Upper dielectric $\sigma_b = -\vec{P}\cdot\hat{n} = -\frac{\sigma}{2}$

lower " " " " $\sigma_b = +\frac{\sigma}{2}$

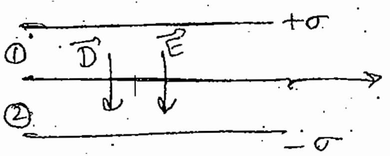
Upper surface of lower dielectric $\sigma_b = -\frac{\sigma}{3}$

lower " " " " $\sigma_b = +\frac{\sigma}{3}$

In dielectric (1), $\rho_{b_1} = 0$. { bcoz P is uniform }

" (2), $\rho_{b_2} = 0$ { " " " " }

*



At this interface :-

(a) both E & D are continous.

(b) " " " " dis "

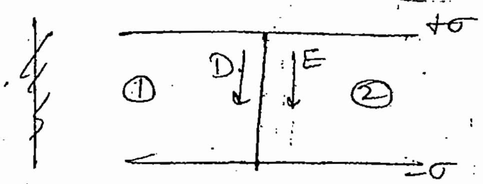
(C) E is continous, D is discontinous

✓(d) E is discontinous, D is continous

Dir^n of $\vec{D}$ & $\vec{E}$ → Normal to the surface. Normal comp. of D is continous

In Dielectric Interface → free charges are zero.

*

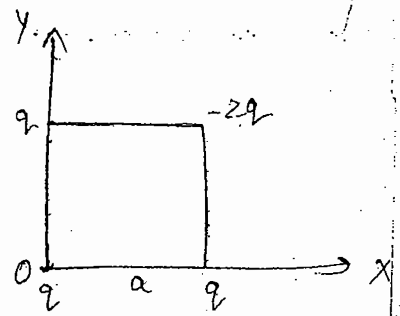


Dir^n of $\vec{D}$ & $\vec{E}$ → tangential to the surface then

✓(C) → correct.

tangential comp. of E is continous.

(34) Taking O → origin



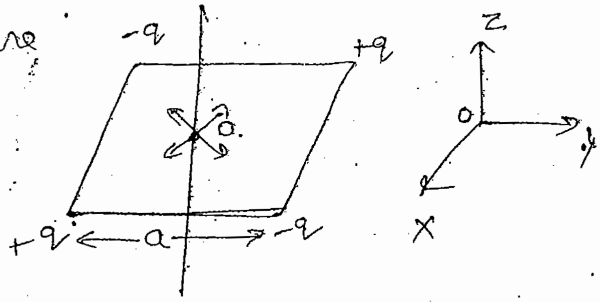
$\vec{p} = 0 + q(a)\hat{x} + qa\hat{y} + (-2q)(a)\hat{x} + (-2q)(a)\hat{y}$

$= -qa\hat{i} - qa\hat{j}$

Assignment (Level-I)

①. (d) ✓

Que :- How many equilibrium points are there?



(a) 0

(b) 1

(c) 5

✓(d) ∞

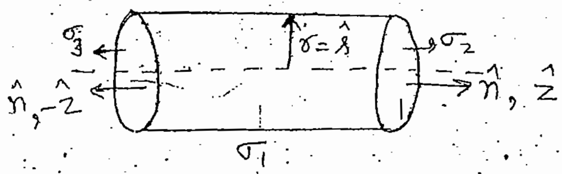
There are ∞ points at which E field is zero. If there are all +q charges on corner then there will be only 1 point at which $E = 0$. i.e. Equilibrium

⑤. $\vec{P} = P_0 \vec{r}$

$\sigma_b = \vec{P} \cdot \hat{n}$

$\sigma_1 = P_0 R$

$\sigma_2 = \sigma_3 = 0$ $\quad (\hat{s} \cdot \hat{z} = 0)$

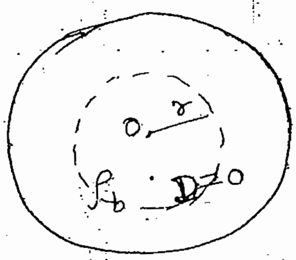


⑪ $\vec{P} = K r^2 \hat{r}$

$\rho_b = -\vec{\nabla} \cdot \vec{P} = -\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \cdot K r^2)$

$\boxed{\rho_b = -4Kr}$

⑫



$q_b = \int \rho_b \, d\tau$

$= -4K \int_0^d r \, 4\pi r^2 \, dr$

$= -4K [r^4]_0^d \pi = -4K\pi d$

$\oint E \cdot ds = \frac{+q}{\epsilon_0} \Rightarrow E \cdot 4\pi d^2 = -\frac{4K\pi d}{\epsilon_0}$

$\boxed{\vec{E} = -\frac{Kd^2}{\epsilon_0} \hat{r}}$

OR Another Method :- $D = 0$ (there is No free charges so $D = 0$)

$D = \epsilon_0 E + P \Rightarrow 0 = \epsilon_0 E + P$

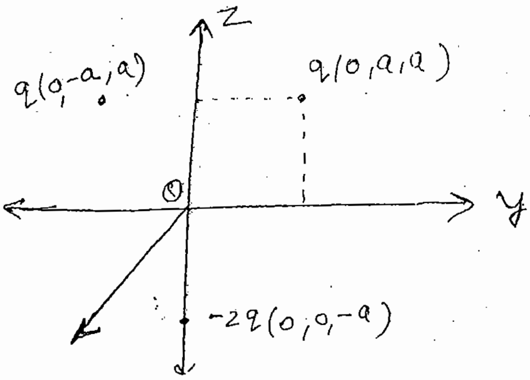
$\Rightarrow E = -\frac{\vec{P}}{\epsilon_0}$

$\boxed{\vec{E} = -\frac{Kd^2}{\epsilon_0} \hat{r}}$

(16) Monopole Mom = 0

So dipole mom. is independent on choice of origin.

Take O as origin



<u>Dipole Mom</u> →

$$p = qa\hat{y} + qa\hat{z} + q(-a)\hat{y} + qa\hat{z} + (-2q)(-a)\hat{z}$$

$$\boxed{p = 4qa\hat{z}}$$

(17) (22) $\phi = \phi_0(x^2+y^2+z^2)$

$$\nabla^2\phi = -\rho/\epsilon_0 \Rightarrow \rho = -\epsilon_0(\nabla^2\phi)$$
$$= -\epsilon_0\phi_0(2+2+2) = -6\epsilon_0\phi_0$$

(41) $E = 0, \; r < a$

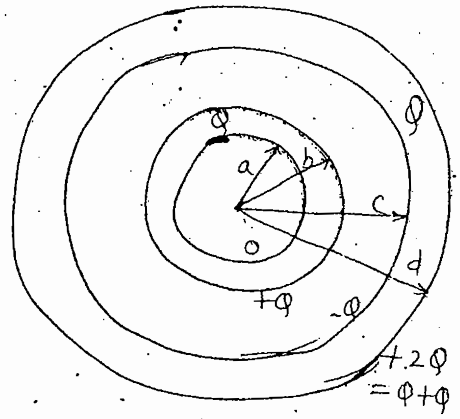
$E = 0, \; a < r < b$

(bcoz no charge can exist in conducting sphere)

$E = \frac{Q}{4\pi\epsilon_0 r^2}, \; b < r < c$

$E = 0, \; b < r < c$

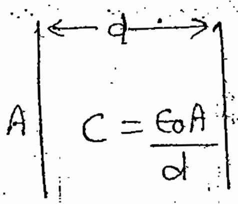
$E = \frac{2Q}{4\pi\epsilon_0 r^2}, \; r > d$



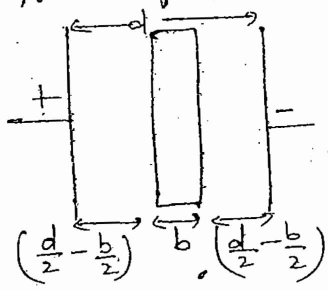
(43) charge density of outer surface of inner shell = charge density of outer surface of outer shell

$$\frac{Q}{4\pi b^2} = \frac{2Q}{4\pi d^2} \Rightarrow \boxed{d = \sqrt{2}\, b}$$

(50)



Now If we put a metal of b thickness



→ This is a series combination, bcoz in parallel combi all the capacitors at at same path.

$$C_1 = \frac{\epsilon_0 A}{\left(\frac{d}{2} - \frac{b}{2}\right)} = \frac{2\epsilon_0 A}{(d-b)} \quad \& \quad C_2 = \frac{2\epsilon_0 A}{(d-b)}$$

for series combination,
resultant capacitance $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\Rightarrow \frac{1}{C} = \frac{d-b}{2\epsilon_0 A} + \frac{d-b}{2\epsilon_0 A}$$

$$\frac{1}{C} = \frac{d-b}{\epsilon_0 A} \Rightarrow \boxed{C = \frac{\epsilon_0 A}{d-b}}$$

As compare to previous case, this ~~ca~~ value of C is decrease by a finite value.
If full capacitor is filled with metal then $C \geq \infty$.

(87) $\vec{E}_{in} = \frac{3}{\epsilon_r + 2}\vec{E}_0$ (free space $\to E_0$ inside field = ?, permittivity ϵ)

$$= \left(\frac{3\epsilon_0}{\epsilon + 2\epsilon_0}\right)\vec{E}_0$$

In ~~Outer~~ our prob. external elec field $\vec{E}$ inside & permitivity is ϵ_0. Replace ϵ_0 by ϵ.

$$E_{in} = \left(\frac{3\epsilon}{\epsilon_0 + 2\epsilon}\right)E$$

(90) Energy $\overset{\leftarrow}{W} = \frac{1}{2}CV^2$
This much work done req. to store the charge.
for spherical capacitor $C = 4\pi\epsilon_0 r$. $(r = 2)$

$$W = \frac{1}{2} 4\pi\epsilon_0 \underline{\underline{r}}\, \underline{\underline{V}}^2$$

$$= \frac{1}{2} \times \frac{1}{9\times 10^9} 2\times (3000)^2 = \frac{1}{2} \frac{1}{9\times 10^9} 2\times 9\times 10^6$$

$$= 10^{-3}\ J$$

(92) $\vec{E} = P\left[xy\,\hat{i} + \left(\frac{1}{2}x^2 + y^2\right)\hat{j}\right]$

$$V = -\int_{(0,0)}^{(1,2)} \vec{E}\cdot d\vec{l} \qquad (d\vec{l} = dx\,\hat{x} + dy\,\hat{y})$$

$$= -P\int \left[xy\,dx + \left(\tfrac{1}{2}x^2 + y^2\right)dy\right]$$

Let $y = 2x \Rightarrow dy = 2dx$

$$V = -P\int_0^1 \left[2x^2 + \left(\tfrac{1}{2}x^2 + 4x^2\right)2\right]dx = -11P\int_0^1 x^2 dx$$

$$V = -11P\left(\frac{x^3}{3}\right)_0^1 = -\frac{11P}{3}$$

{ − sign means pot^n is lower at final position }

(93) Monopole $= 0$

dipole mom $= Q(-b)\hat{x} + Q(b)\hat{x} + (-Q)(-a)\hat{y} + (-Q)(a)\hat{y}$

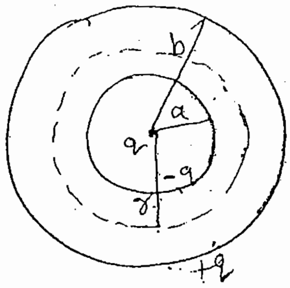
$= 0$

So Q. Mom. $\rightarrow$ dominate.

(96) $a < r < b$.

$$V = \cancel{\frac{q}{4\pi\epsilon_0 r}} - \cancel{\frac{q}{4\pi\epsilon_0 r}} + \frac{q}{4\pi\epsilon_0 b}$$

$$V = \frac{q}{4\pi\epsilon_0 b}$$



(107) $\vec{E} = \vec{E_1} + \vec{E_2}$

$u_1 = \frac{\epsilon_0}{2} E_1^2$ & $u_2 = \frac{\epsilon_0}{2} E_2^2$

$$u = \frac{\epsilon_0}{2} E^2 = \underbrace{\frac{\epsilon_0}{2} E_1^2 + \frac{\epsilon_0}{2} E_2^2}_{u_1 + u_2} + \underbrace{\epsilon_0 E_1 E_2}_{\text{extra}}$$

$= \frac{\epsilon_0}{2}(E_1+E_2)^2 \Rightarrow \ne u_1 + u_2$

i.e. Energy does not follow superposition.

Assignment- Level-II

(7) $\vec{P} = (5z^2+7)\hat{k}$

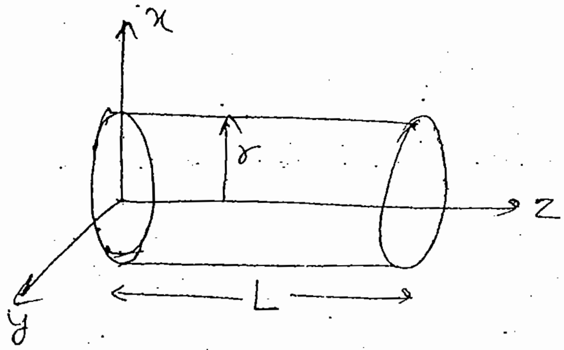
$\rho_b = -\vec{\nabla}\cdot\vec{P} = -10z$

$q_b = \int \rho_b \, d\tau$

$$= -\int_0^r \int_0^L \int_0^{2\pi} 10z \, s \, ds \, d\phi \, dz$$

$$= -10 \frac{L^2}{2} \cdot \frac{r^2}{2} \cdot 2\pi$$

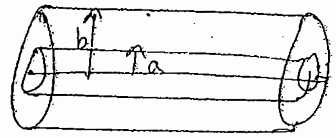
$\boxed{q_b = -5\pi r^2 L^2}$ (d) ✓



(22) $+\frac{\partial V_{out}}{\partial r} = -\frac{\sigma}{\epsilon_0}$ $\quad V_{out} = -E_0\left(1 - \frac{R^3}{r^3}\right) r\cos\theta$

$\sigma = \epsilon_0 \frac{\partial V}{\partial r} = +\epsilon_0 E_0 (-3$

(33) capacitance per unit length



$$\frac{C}{l} = \frac{2\pi\epsilon_0}{\ln(b/a)}$$

if dielectric is filled then $\epsilon_0 \to \epsilon$

$$\frac{C}{l} = \frac{2\pi\epsilon_0\epsilon_r}{\ln(b/a)}$$

$$\frac{C}{l} = \frac{4\pi\epsilon_0}{2\ln(b/a)} = \frac{3.5}{9\times10^9\times2\times\ln 2} \quad F/10^{-3}\,km$$

(75) $p\uparrow \; \uparrow\vec{E} \; \downarrow\vec{p}$

$$W = \Delta U = U_f - U_i = pE + pE = 2pE$$

$U = -\vec{p}\cdot\vec{E}$

$U_f = -pE\cos 180$

$U_i = -pE\cos 0$

(92)
$$\rho(r) = \frac{A}{r}e^{-kr}$$

$$\nabla^2 V = -\rho/\epsilon_0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = -\frac{1}{\epsilon_0}\frac{A}{r}e^{-kr}$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = -\frac{A}{\epsilon_0}re^{-kr}$$

$$r^2\frac{\partial V}{\partial r} = -\frac{A}{\epsilon_0}\left[-\frac{r}{K}e^{-kr} - \frac{1}{K^2}e^{-kr}\right]$$

$$\frac{\partial V}{\partial r} = -\frac{A}{\epsilon_0}\left[-\frac{1}{Kr}e^{-kr} - \frac{1}{K^2r^2}e^{-kr}\right]$$

$$= \frac{A}{\epsilon_0}\left[\frac{1}{Kr}e^{-kr} + \frac{1}{K^2r^2}e^{-kr}\right]$$

$$V = \frac{A}{\epsilon_0 K}\left[\int\frac{1}{r}e^{-kr}dr + \int\frac{1}{K}\frac{1}{r^2}e^{-kr}dr\right]$$

$$V = \frac{A}{\epsilon_0 K}\left[-\frac{1}{Kr}e^{-kr} - \cancel{\int\frac{1}{Kr^2}e^{-kr}dr} + \cancel{\int\frac{1}{Kr^2}e^{-kr}dr}\right]$$

$$\boxed{V = -\frac{A}{\epsilon_0 K^2}\frac{1}{r}e^{-kr}}$$

(b) ✓

(105) $\vec{E} = \alpha(1-e^{-r/R})\frac{\hat{r}}{r^2}$

$\rho = \nabla\cdot E$

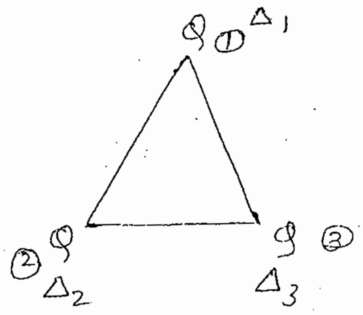
$q =$

(107)

$$\Delta_1 = 0$$

$$\Delta_2 = \frac{Q^2}{4\pi\epsilon_0 a^2}$$

$$\Delta_3 = \frac{2Q^2}{4\pi\epsilon_0 a^2} = \frac{Q^2}{4\pi\epsilon_0 a^2} + \frac{Q^2}{4\pi\epsilon_0 a^2}$$



$$\Delta_1 : \Delta_2 : \Delta_3 = 0 : 1 : 2$$

(109)

$$\vec{D} = (2y^2 + z)\hat{i} + 4xy\,\hat{j} + x\hat{k}$$

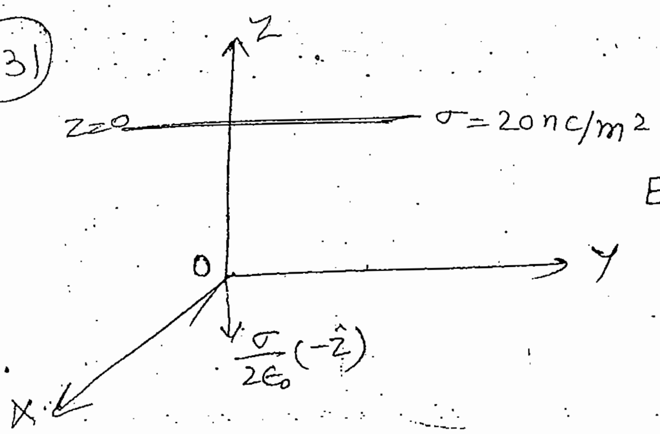
$$\vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\vec{\nabla}\cdot\vec{D} = 4x \Rightarrow \rho_f = 4x$$

$$q_f = \int \rho_f \, d\tau = \int_0^1\int_0^1\int_0^1 4x \, dx\,dy\,dz$$

$$= 1 \times 1 \times 4\left(\frac{x^2}{2}\right)_0^1 = \underline{2\ C}$$

(131)



$$\sigma = 20\ nC/m^2 = 20 \times 10^{-9}\ C/m^2$$

$$E = \frac{\sigma}{2\epsilon_0} = \frac{2\pi\sigma}{4\pi\epsilon_0} = \frac{2\pi \times 20 \times 10^{-9} \times 9 \times 10^9}{}$$

$$= -360\pi\hat{z}\ V/m$$

(145)

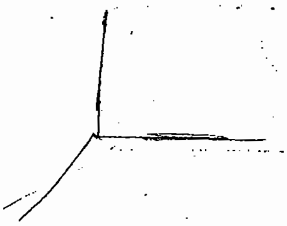
$$\frac{\epsilon_1}{\epsilon_0} = 2; \quad \frac{\epsilon_2}{\epsilon_0} = 5, \quad E_1 = \underbrace{2i - 3j}\,+\underbrace{5k}$$

$$D_1 = \epsilon_1 E_1$$

$$= 2\epsilon_0(2i - 3\hat{j} + 5k) = \epsilon_0(4\hat{i} - 6\hat{j} + 10\hat{k})$$

$$D_{1n} = D_{2n}$$

$$\boxed{D_{2n} = 10\,\epsilon_0\,\hat{z}}$$



$$E_{1t} = E_{2t}$$

$$E_{2t} = 2\hat{x} - 3\hat{y}$$

$$D_{2t} = \epsilon_2 E_{2t}$$

$$= \epsilon_0(2\hat{x} - 3\hat{y})\,5$$

$$D_2 = D_{2t} + D_{2n}$$

$$D_2 = \epsilon_0(10\hat{i} - 15\hat{j} + 10\hat{k})$$

(146) $a = 0.5 \times 10^{-10}\, m$, $E = 30 \times 10^{5}\, V/m$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{qd}{a^3} \Rightarrow d = \frac{4\pi\epsilon_0 a^3 E}{q} = \frac{0.125 \times 10^{-30} \times 30 \times 1}{9 \times 10^{9} \times 1.6 \times 10^{-19}}$$

$$= 2.6 \times 10^{-16}$$

(147) $p = \alpha E \Rightarrow \alpha = \frac{p}{E} = \frac{ed}{E}$

$$\alpha = 4\pi\epsilon_0 a^3 = \frac{0.125 \times 10^{-30}}{9 \times 10^{9}}$$

Magnetostatics

Here we have STATIC Magnetic field (not changing with time)

static electric field produces from charge at rest. (Not moving). And its condition is

$$\nabla \times E = 0$$

similarly to produce magnetic field we need current. i.e. ~~To pro~~ Static magnetic field can be produced from current.

When Magnetic field not changing with time - Magnetostatic i.e. Current is not a func^n of time.

The current which is not changing with time → Steady current.

When flow of current is steady then it is steady current

Defination of Steady Current :- The current flowing smoothly without pilling up charge anywhere called Steady Current.

If Charge accumulate then M.f will not be static.

If wire is uncharged. At The point in space, there is mag. field but not electric field will present.

Inside the wire, if current flows then electric field will defenitely present.

If a charge is moving with constant velocity v then there will be static mag field & also electric field. elec. field & mag. field in this case do not depand on each other

If v is changing with time then it is accelerated motion. then both elec. & mag. field depand on each other. (both depands on time)

If a wire is connected to a battery then a steady current will flow in wire (dc current)

Magnetic force :- Mag. force on a charge particle q moving with velocity v in mag. field B then

$$\boxed{\vec{F} = q(\vec{v} \times \vec{B})}$$

In Electric field, We have force $\vec{F} = q\vec{E}$

Magnetic force is also called Lorentz force.

Magnetic forces do no work, they only change the dirⁿ of motion of charged particle.

If a charge particle q moving with v in $\vec{B}$ then work done in the distance travelled dr is

$$\boxed{dW = \vec{F} \cdot d\vec{r}}$$

$$dW = \vec{F} \cdot \frac{dr}{dt} dt = q(\vec{v} \times \vec{B}) \cdot \vec{v}\, dt$$

$\vec{v} . dt \rightarrow$ distance travelled

$v \times B$ will be the $\perp$ vector to both $v \times B$.

so $\boxed{dW = 0}$

i.e. Power accociated by mag. force is zero.

These type of forces are called Fictitous forces.

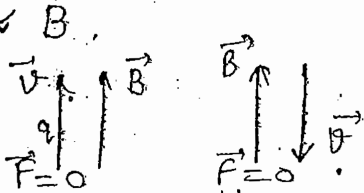
If in any problem, mag. force do work then there → by changing the mag. field there gennate mag. force. & then In that case mag. field do the work, not the mag. force.

Path of the charged particle in mag. field :- Suppose a charge particle q moving with velocity $\vec{v}$ in $\vec{B}$ then force on it $\vec{F} = q(\vec{v} \times \vec{B})$

then magnitude of force will depend on angle b/w $\vec{v}$ and $\vec{B}$ as $|\vec{F}| = q\, vB \sin\theta$

$\theta \rightarrow$ angle b/w v & B.

Case 1 :- If $\theta = 0, \pi$



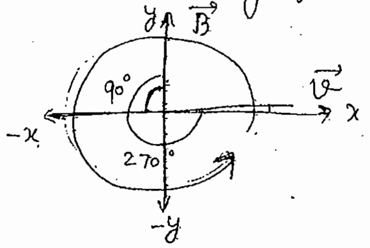
$F = 0$

charged particle It will move undeflected if v & B are parallel or antiparallel. [undeflected means straight line]

Case 2. $\theta = \perp (90°)$, $\frac{\pi}{2}$, $\frac{3\pi}{2}$

then charged particle will follow circular path & rotate in a plane which is perpendicular to the mag. field.

If a charge particle move in a circle. i.e. v & B are $\perp$ to each other (angle b/w them is $\frac{\pi}{2}$ or $\frac{3\pi}{2}$) Then its motion is called <u>Cyclotron motion</u>.



If particle do rotational motion then there will be 2 forces on it. → Lorentz & centripital force.

$$qvB = \frac{mv^2}{r}$$

Momentum $\boxed{p = mv = qBr}$ ——— (A)

Now frequency of rotation (Angular freq.):-

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Particle should travell in a circle so

$$T = \frac{2\pi r}{v}$$

$$\therefore \omega = \frac{2\pi v}{2\pi r} \Rightarrow \omega = \frac{v}{r}$$

Compare the value of $\frac{v}{r}$ with eq. (A),

ang. freq. $\boxed{\omega = \frac{qB}{m}}$

freq. $f = \frac{\omega}{2\pi} \Rightarrow \boxed{f = \frac{qB}{2\pi m}}$

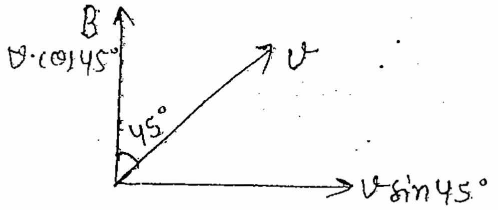
Time $T = \frac{1}{f} \Rightarrow \boxed{T = \frac{2\pi m}{qB}}$

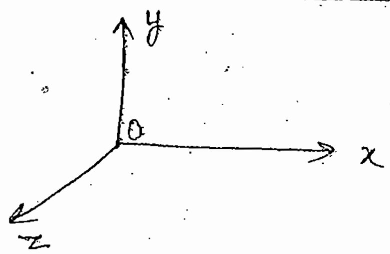
This time is required by a particle to complete one rotation of a circle.

<u>Case 3</u>:- When $\theta \neq 0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}$

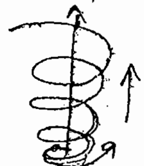
Suppose angle $\theta = 45°$

If a charge particle enter in mag field $\vec{B}$ so that it $\vec{v}$ makes angle 45° with $\vec{B}$.



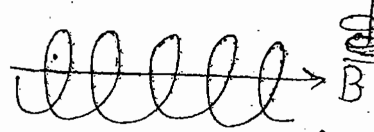


$v \sin\theta$ comp. will tend to rotate the charge particle and
$v\cos\theta$ " " " " move this charge upward.



This is called Hallical Path.

$\vec{B}$ will lie along the axis of hellix.

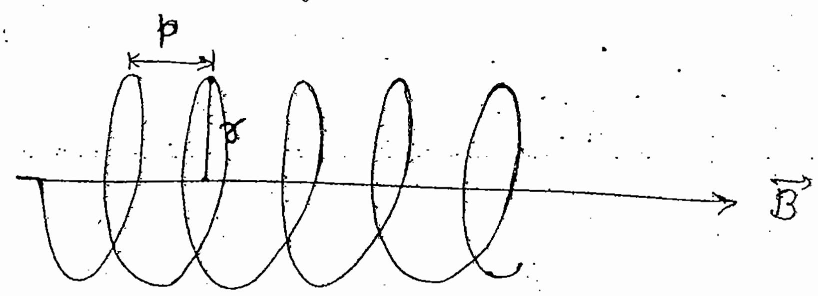


Ques. 1 :- An e^- enter in a uniform mag. field with magnitude 0.3 tesla, at an angle of 45° w.r.to mag. field.

M.K.S. Unit of Mag. field → Tesla — $1T = 1$ weber/m^2
C.G.S. " " " → Gauss

$\boxed{1T = 10^4 G}$ → Relation b/w both units.

Determine the radius r & pitch p of the electron's helical path. Assuming its speed is 2×10^6 m/s.



p → pitch
r → radius

$\theta = 45°$, $v = 2 \times 10^6$ m/sec

$v_{\perp} = v\sin 45° = 2 \times 10^6 \times \frac{1}{\sqrt{2}} = \sqrt{2} \times 10^6$

$v_{\parallel} = v\cos 45° = 2 \times 10^6 \times \frac{1}{\sqrt{2}} = \sqrt{2} \times 10^6$

$$v_{\parallel} = v_{\perp} = \frac{\sqrt{2}}{\sqrt{2}} \times 10^6 \text{ m/sec}$$

$v_{\perp}$ is responsible for circular motion.
$v_{\parallel}$ " " " forward "

So Rotational motion is determined by $v_{\perp}$.
Translational " " " $v_{\parallel}$.

$$q v_\perp \times B = \frac{m v_\perp^2}{r}$$

$$\Rightarrow r = \frac{m v_\perp}{qB} = \frac{9.1 \times 10^{-31} \times \sqrt{2} \times 10^{\pm 6}}{1.6 \times 10^{-19} \times 0.3}$$

$$r = 26.81087 \times 10^{-6}$$

$$\boxed{r = 26.8\ \mu m}$$

To calculate the pitch, Ist Calculate time for making a circle,

$$\Delta t = \frac{2\pi r}{v_\perp}$$ {bcoz circular motion $v_\perp$ is responsible}

Pitch $p = v_{\parallel} \Delta t$

$= v_{\parallel} \times \frac{2\pi r}{v_\perp}$ ($v_\perp = v_{\parallel}$ in this Ques.)

$p = 2\pi r = 2 \times 3.14 \times 26.8 = 168.304$

$$\boxed{p = 169\ \mu m}$$

<u>Note</u>:- • pitch of a helical path is constant in uniform magnetic field while it is not constant in Non uniform mag. field.

$p \propto t$ & $t \propto \frac{1}{B}$

so $p \propto \frac{1}{B}$

If non uniform mag. field will increase then pitch decrease.

- force will be $F = q v_\perp B$, $F \neq q v_{\parallel} B$

<u>Motion of the charge particle in both Electric field & magnetic field</u>:- We have a charge particle q at rest.

In present case $E \perp B$

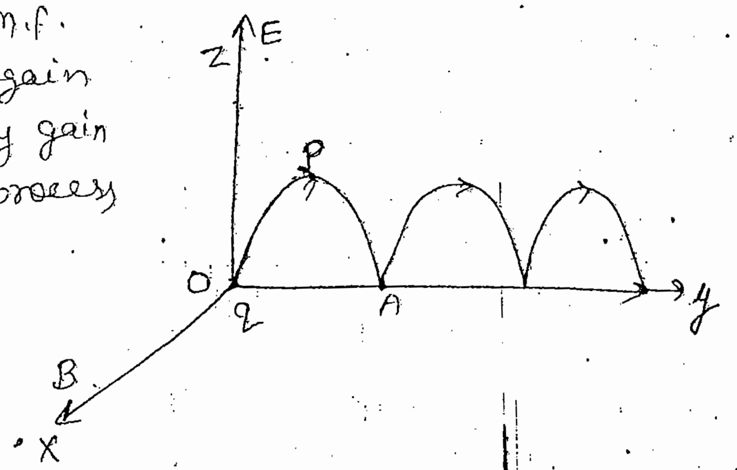
$\vec{E}$ is along z-axis.

$\vec{B}$ " " x-axis.

<u>charge is at rest placed at origin.</u>

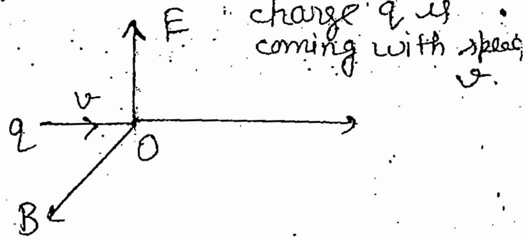
When charge particle is at rest $\vec{E}$ will work on it, & it will start motion. As $v \uparrow$, $B \uparrow$

At P, it lose its velocity so m.f. decreases & at A, vel. = 0. Again E.F. will act on it, velocity gain & process will go on, the process is repeated.



The path followed by the charged particle will be Cycloid.

- If charge particle moving with velocity $\vec{v}$ then Electric field push the charge in upward dir^n
  Mag. " " " " downward "
  so both are opposite.
  Generally strength of electric field dominant over speed.



$E \perp B$,

$$qE = qvB$$
$$E = vB$$
$$\boxed{v = \frac{E}{B}}$$

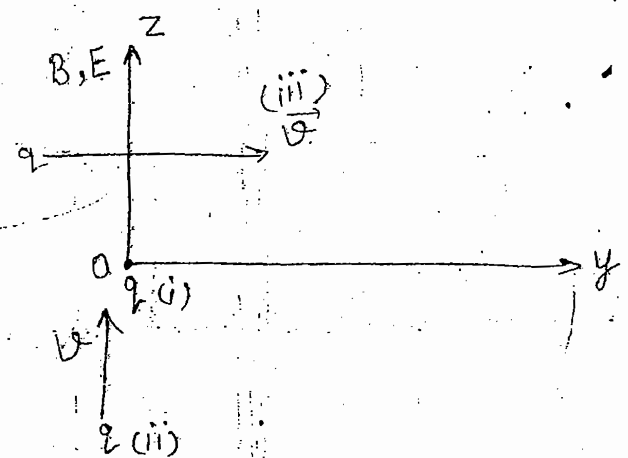
If strength of both $\vec{E}$ & $\vec{B}$ are same then it will move undeflected i.e. in straight line.

- If E & B are parallel

(i) If q is at rest then path will be straight line.

(ii) If q is moving in z-dir^n (∥ dir^n) then path is also straight line.

(iii) If q is moving in x or y dir^n ($\perp$ dir^n) then path will be spiral or helical.



$v \perp$ to $\vec{E}$ & $\vec{B}$, v along y dir^n & $\vec{E}$, $\vec{B}$ along z-dir^n

$\vec{E}$ push the charge particle in upward dir^n & $\vec{B}$ make the path helical in upward $\hat{z}$ dir^n.

Currents :-

~~Line~~ Current :- $\boxed{I = \frac{q}{t} = \lambda v}$

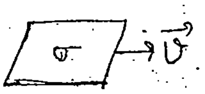
$\lambda \to$ line charge density , $v \to$ velocity

If any line charge λ moving with v velocity then current in it will be $I = \lambda v$

Surface Current :- If a surface charge moving with v then $\vec{K} = \sigma \vec{v}$, $\vec{K} \to$ surface current

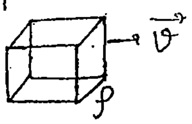
it is a vector quantity.

If any +ve charge particle moving with $\vec{v}$ then the dir^n of current will be in the dir^n of motion of q.



$$\boxed{\vec{K} = \sigma \vec{v} = \frac{I}{l_1}}$$ (current/unit length)

Volume Current :- If volume charge ρ move with v then produce vol. current.



$$\boxed{\vec{J} = \rho \vec{v} = \frac{I}{a_1}}$$ (current/unit area)

Continous force in terms of three Currents :-

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$= \int dq \left[\frac{d\vec{l}}{dt} \times \vec{B}\right]$$

$$= \int \frac{dq}{dt} \, d\vec{l} \times \vec{B} \qquad I = \frac{dq}{dt}$$

$$\boxed{F = I \int d\vec{l} \times \vec{B}}$$

force on a current carring wire,

for a surface current,

$$\vec{F} = \int \sigma da \, (\vec{v} \times \vec{B})$$

$$= \int da \, (\sigma \vec{v} \times \vec{B})$$

$$\boxed{\vec{F} = \int_s (\vec{K} \times \vec{B}) \, da}$$

This is force on a surface element having current density $\vec{K}$
Similarly for Volume,

$$\vec{F} = \int \rho \, d\tau \, (\vec{v} \times \vec{B})$$

$$\boxed{\vec{F} = \int_V (\vec{J} \times \vec{B}) \, d\tau}$$

This is force on a volume current.

$\vec{K} = \frac{I}{l_\perp}$ current/unit length perpendicular to the flow of current is Surface current.

$\vec{J} = \frac{I}{a_\perp}$ current/unit area perpendicular to the flow of current is Volume current.

Que :- (a) If Current I is uniformly distributed over a wire of circular cross-section with radius a, Find the volume current density $\vec{J}$.
(b) If Current in the wire is proportional to the distance from the axis $J = Ks$. Find the total current. K is any constant.

(a) $J = ?$

$$\vec{J} = \frac{I}{a_\perp} = \frac{I}{\pi a^2}$$

(Area $\perp$ to flow of current $a_\perp$ = area of circular cross section of radius a)

(b) $J = Ks$

$I = ?$ $\quad J = \frac{I}{a_\perp}$

This current density is Non-uniform so

$$I = \int J \cdot da_\perp$$

$$= \int_0^{2\pi}\int_0^a Ks \cdot s \, ds \, d\phi$$

$$= K \left(\frac{s^3}{3}\right)_0^a 2\pi$$

wire is a cylender. So dir^n of flow of current is z dir^n along the length.

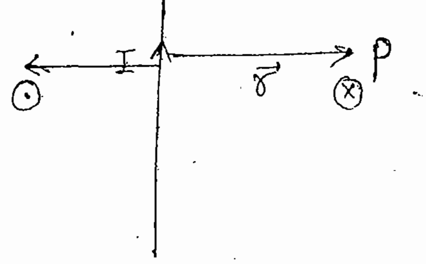
$$\boxed{I = \frac{2\pi K a^3}{3}}$$

Biot-Savert Law :-

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I(d\vec{l} \times \hat{r})}{r^2}$$

The $\vec{dl}$ is a vector along the dir^n of current flow.

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I(d\vec{l} \times \vec{r})}{r^3}$$



- dir^n of $\vec{B}$ = $d\vec{l} \times \vec{r}$

⊗ → into the page
⊙ → out of the page

Wires are of cylendrical shape
so dir^n of current is $\hat{z}$ &
dir^n of Mag. field will be $\hat{\phi}$.

Mag. field curl arround the wire.

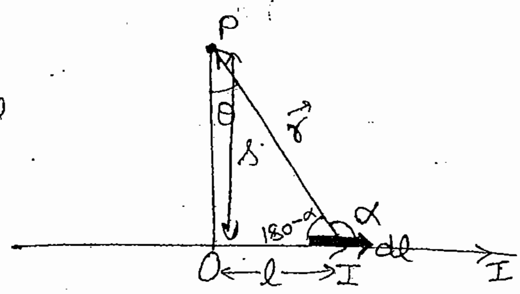
If the dir^n of current is $\hat{\phi}$ then dir^n of mag. field will be $\hat{z}$.

In Solanoid → Current is in $\hat{\phi}$ & Mag. field is in $\hat{z}$.

12/8/2012 **Application of Biot-Savert law :-**

Q. Find the mag. field at a distance s from a straight wire carrying a steady current I.

) Take an small element dl.
dir^n of element dl will be in the dir^n of flow of current.



$$dB = \frac{\mu_0}{4\pi} I \frac{\vec{dl} \times \vec{r}}{r^3}$$

Complete mag. field for wire

$$B = \int dB = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \vec{r}}{r^3}$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{dl\, r \sin\alpha}{r^3}$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{dl \sin\alpha}{r^2}$$

r, α → unknown

$$\cos\theta = \frac{s}{r}$$

$$r = \frac{s}{\cos\theta}$$

$$90 + 180 - \alpha + \theta = 180 \quad \text{(Total angles in } \Delta = 180)$$

$$\boxed{\alpha = 90° + \theta}$$

$$\therefore \sin\alpha = \sin(90+\theta) = \cos\theta$$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{\cos^2\theta}{s^2} \times \cos\theta \, dl$$

$\tan\theta = \dfrac{l}{s}$

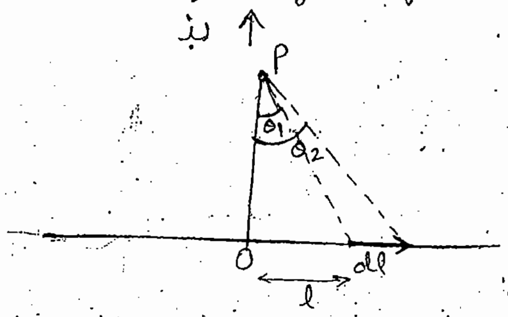
$l = s\tan\theta$

$dl = s\sec^2\theta \, d\theta$

$$= \frac{\mu_0 I}{4\pi}\int \frac{\cos^2\theta}{s^2}\cos\theta \, s\sec^2\theta \, d\theta$$

$$= \frac{\mu_0 I}{4\pi s}\int_{\theta_1}^{\theta_2}\cos\theta \, d\theta$$

If length of element is ↑



$$\boxed{B = \frac{\mu_0 I}{4\pi s}\left[\sin\theta_2 - \sin\theta_1\right]}$$

This is the mag-field of finite wire.

Dir^n :- ⊙ i.e. $[\hat{\phi}]$

<u>for Infinitly long wire</u> :-

then $\theta_1 = -90°$, $\theta_2 = +90°$

$$\boxed{B = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$$

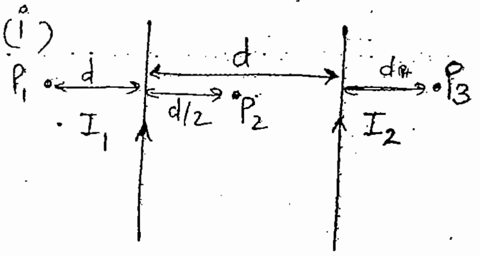
$\hat{\phi} \rightarrow$ Circumfrantial vector

for a infinitly long wire, we have electric field $E = \dfrac{\lambda}{2\pi\epsilon_0 s}$

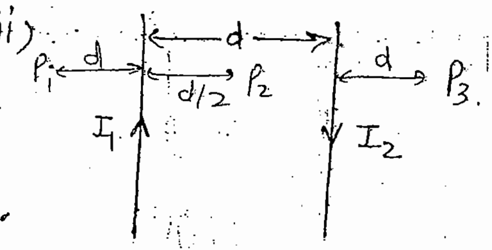
Here $\mu_0 \leftrightarrow \dfrac{1}{\epsilon_0}$ & $I \leftrightarrow \lambda$

<u>Q.</u> Find the force per unit length in two parallel wire arrangement as shown in the figure.

(i)



(ii)



Also Find the mag. field at point P_1, P_2 & P_3.

$$B = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

force on wire 2 will be due to mag. field of wire 1.

$$\therefore \vec{F_2} = I_2 \int d\vec{l_2} \times \vec{B_1} \quad \text{(force on wire 2)}$$

force per unit length is $\vec{f_2} = \vec{I_2} \times \vec{B_1}$

$\vec{B_1} \rightarrow$ mag. field on wire 1 at the position of 2.

$$\vec{B_1} = \frac{\mu_0 I_1}{2\pi d} \text{ (into the page)}$$ dir^n of ~~wir~~ $\vec{B}$ on wire(2) $\rightarrow \otimes$
in region of wire (1) $\rightarrow \odot$

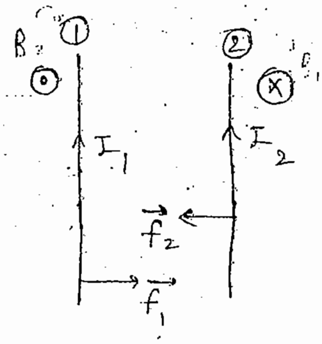
$$\boxed{\vec{f_2} = \frac{\mu_0 I_1 I_2}{2\pi d}}$$ (dir^n $\rightarrow$ to the ~~length~~ left)

force/unit length $\propto I_1$ & I_2
$\propto \frac{1}{d}$

<u>force on wire 1</u>,

$$\vec{F_1} = I_1 \int d\vec{l_1} \times \vec{B_2}$$

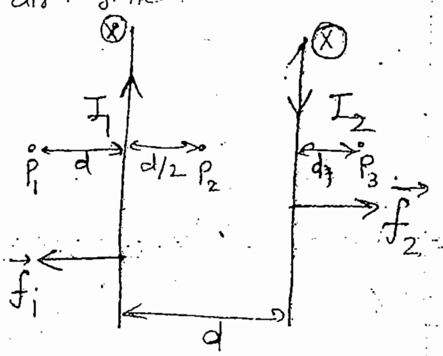
$$\vec{f_1} = \vec{I_1} \times \vec{B_2}$$



$$B_2 = \frac{\mu_0 I_2}{2\pi d} \text{ (out of page)}$$

$$\boxed{\vec{f_1} = \frac{\mu_0 I_1 I_2}{2\pi d}}$$ (to the right)

If current is in same dir^n (for two ~~wi~~ wires) then force is <u>attractive</u>. This attraction is due to magnetic force.



~~Q~~ (ii) Magnitude of force per unit length is same. Only dir^n will be different.

dir^n of Mag field on wire (2) due to (1) is $\otimes$

" " " (1) " (2) " $\otimes$

If Currents are in opposite dir^n the force is Repulsive

$$\vec{f_1} = \frac{\mu_0 I_1 I_2}{2\pi d} \quad \text{(to the left)}$$

$$\vec{f_2} = \frac{\mu_0 I_1 I_2}{2\pi d} \quad \text{(to the right)}$$

At P_1, Mag. field at any point is vector sum of $\vec{B} = \vec{B_1} + \vec{B_2} + \cdots$

$\vec{B_1} \rightarrow \odot$, $\vec{B_2} \rightarrow \otimes$

$$\vec{B_1} = \frac{\mu_0 I_1}{2\pi d} \text{ (out of the page)}$$

$$\vec{B_2} = \frac{\mu_0 I_2}{2\pi(2d)} \text{ (into the page)}$$

Out mag. field is more than into mag. ~~field~~ so resultant mag. field will be in out of the page dirn.

$$\vec{B} = \vec{B_1} + \vec{B_2} \Rightarrow \vec{B} = \frac{\mu_0}{2\pi d}\left[I_1 - \frac{I_2}{2}\right]$$

If current in 2 wires are same then

$$\boxed{\vec{B}(P_1) = \frac{\mu_0 I}{4\pi d}} \text{ (out of page)}$$

At P_2, $\vec{B_1} \rightarrow \otimes$, $\vec{B_2} \rightarrow \otimes$

$$\vec{B_1} = \frac{\mu_0 I_1}{2\pi(d/2)}$$

$$\vec{B_2} = \frac{\mu_0 I_2}{2\pi(d/2)}$$

Resultant Mag. field

$$B = \frac{\mu_0}{2\pi}\left[\frac{1}{d/2}(I_1 + I_2)\right]$$

$$B = \frac{\mu_0}{2\pi}\frac{2}{d}(I_1 + I_1)$$

If $I_1 = I_2 = I$ then $\boxed{B(P_2) = \frac{2\mu_0 I}{\pi d}}$ (into the page)

At P_3, $B_1 \rightarrow \otimes$, $B_2 \rightarrow \odot$

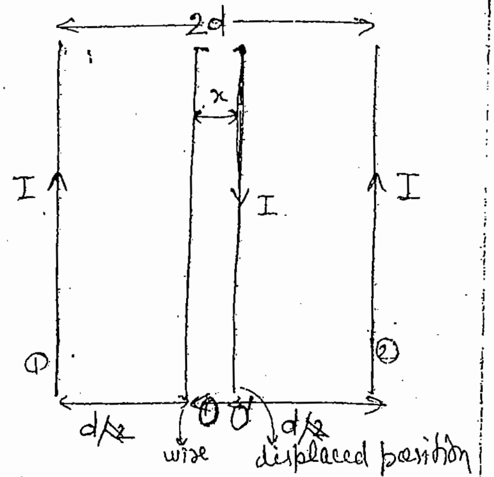
$$B_1 = \frac{\mu_0 I_1}{2\pi(2d)}, \quad B_2 = \frac{\mu_0 I_2}{2\pi(d)}$$

$$B(P_3) = \frac{\mu_0}{2\pi d}\left[-\frac{I_1}{2} + I_2\right]$$

$$I_1 = I_2 = I$$

$$\boxed{B(P_3) = \frac{\mu_0}{4\pi d}} \text{ (out of page)}$$

Q. We have a 3 wire arrangement. Current in each wire is I. Mass per unit length of the middle wire is m. Earlier it was placed at mid point. Now it is displaced by x. Calculate its freq. of oscillation.



$F \propto x$

Here, force / unit length so

$f \propto x$

If wire is placed at mid point then resultant force is zero.

But when it is displaced then force will be Non-zero.

At Present position,

$$\vec{B_1} = \frac{\mu_0 I}{2\pi(d+x)} \text{ (into the page)}, \quad \vec{B_2} = \frac{\mu_0 I}{2\pi(d-x)} \text{ (out of page)}$$

{ B_1 is field at ① due to mid
B_2 " " ② "

Resultant Mag. field, $\vec{B} = \vec{B_1} + \vec{B_2}$

$$B = \frac{\mu_0 I}{2\pi}\left[\frac{1}{(d-x)} - \frac{1}{(d+x)}\right] \text{ (out of page)}$$

$$B = \frac{\mu_0 I}{2\pi}\left[\frac{d+x-d+x}{d^2-x^2}\right]$$

Here x^2 is very small. This wire will perform Simple harmonic motion (if x in not small then wire will not perform S.H.M.) so neglect x^2

$$\boxed{B = \frac{\mu_0 I x}{\pi d^2}} \text{ (out of page)}$$

After displament the motion of wire will be Simple Harmonic as it starts to oscillate.

force/unit length $f = I \times B$

$$f = \frac{\mu_0 I^2 x}{\pi d^2}$$

$\boxed{f \propto x}$ i.e. S.H.M.

$$F = m\frac{d^2x}{dt^2} \quad \& \quad f = \frac{m}{l}\frac{d^2x}{dt^2}$$

Here mass per unit length is m. So $\boxed{f = m\frac{d^2x}{dt^2}}$

$$f = Kx$$

$$\frac{\mu_0 I^2 x}{\pi d^2} = Kx \Rightarrow \boxed{K = \frac{\mu_0 I^2}{\pi d^2}}$$

Both f & K are force per unit length

So freq. of Oscillation, $\omega = \sqrt{\frac{K}{m}}$

$$\omega = \sqrt{\frac{\mu_0 I^2}{\pi d^2 m}}$$

$$\boxed{\omega = \frac{I}{d}\sqrt{\frac{\mu_0}{\pi m}}}$$

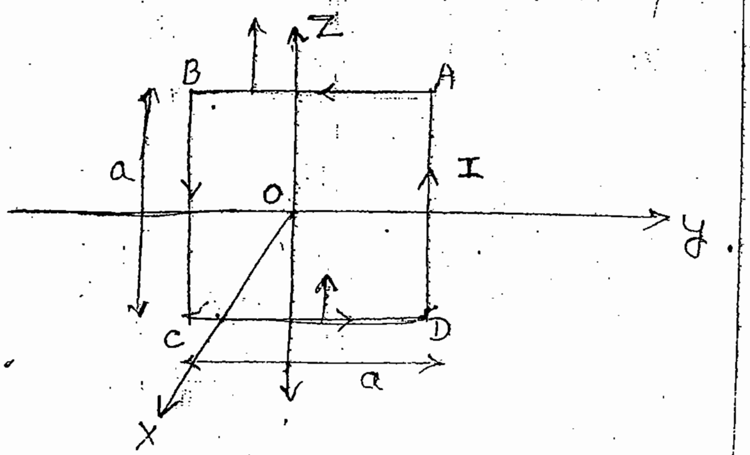
$$\boxed{\text{Linear frequency} = \frac{\omega}{2\pi} = \frac{I}{2\pi d}\sqrt{\frac{\mu_0}{\pi m}}}$$

$$\text{Time} = \frac{1}{\text{freq.}} \Rightarrow \boxed{T = \frac{2\pi d}{I}\frac{1}{\sqrt{\mu_0/\pi m}}}$$

Q. A Mag. field in some region is given by $\boxed{\vec{B} = kz\hat{x}}$ where k is constant. Find the force on a square loop of sides a lying in the y-z plane & centered at the origin if it carries a current I in the counter clockwise dir^n when you look down the x-axis.

$$F = \int I d\vec{l} \times \vec{B}$$

$$= I\int d\vec{l} \times \vec{B}$$



for AB wire, Mag. field

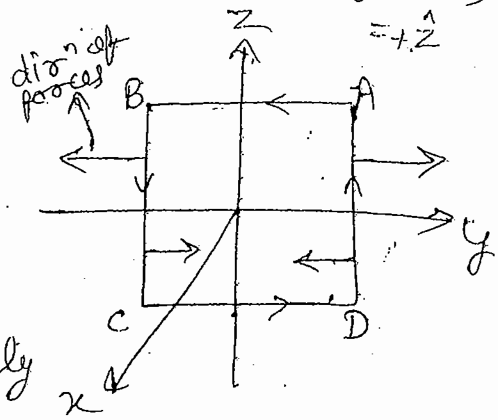
$$B = \frac{ka}{2}\hat{x}$$

$$\boxed{\vec{F_{AB}} = \frac{IKa^2}{2}\hat{z}}$$ $(-\hat{y}\times\hat{x} = \hat{z})$

On Wire CD, Mag. field $B = \frac{ka}{2}(-\hat{x})$

$$\boxed{\vec{F}_{CD} = \frac{Ika^2}{2}\hat{z}}$$

$+\hat{y} \times (-\hat{x}) = +\hat{z}$



On: AD upper half, Dir^n of $\vec{B} \rightarrow \hat{x}$
On BC " " " $\rightarrow \hat{x}$
On AD lower half Dir^n of $\vec{B} \rightarrow -\hat{x}$
BC " " " $\rightarrow -\hat{x}$

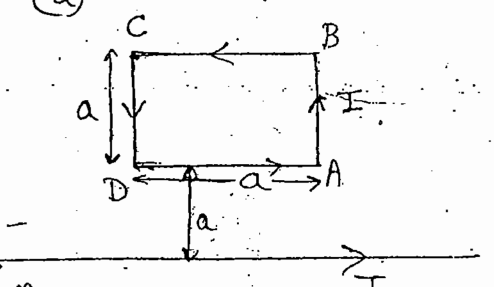
forces on AD & BC are oppositely directed So

$\vec{F}_{AD} = \vec{F}_{BC} = 0$ [Dir^n of mag. field is opposite, +z or −z]

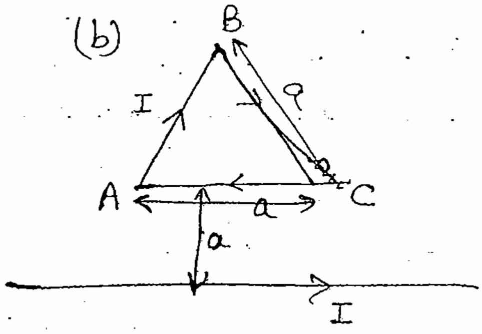
Resultant force $\vec{F} = \left(\frac{IKa^2}{2} + \frac{IKa^2}{2}\right)\hat{z} \Rightarrow \boxed{\vec{F} = IKa^2\hat{z}}$

If $\vec{B} = K\hat{x}$ then Resultant $F = 0$
Becoz dir^n of of mag. field on upper & lower part is same here & in previous case it is different.

Q. :- (a)



(b)

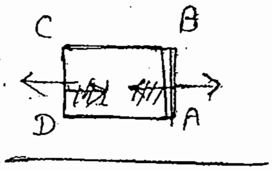


due to this wire, there will be field on each wire of loop

(a) Find the force on a squar loop placed near an infinte straight wire. Both carries current I as shown in figure

(b) find the force on a equilatoral triangle loop placed near infinite straight wire. Both carry current I.

(a) Dir^n of $\vec{B}$ for AB = ⊙
CD = ⊙



$\vec{F} = I\int d\vec{l} \times \vec{B}$

$F_{AB} = F_{CD} = 0$

$$F = I \int dl \times B$$

$$F_{DA} = I \frac{\mu_0 I a}{2\pi a} \qquad \vec{B} = \frac{\mu_0 I}{2\pi a} \quad \odot$$

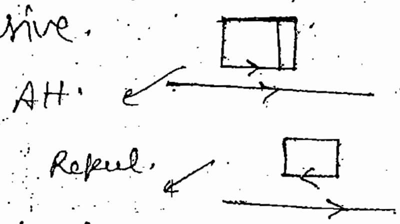
$$f_{DA} = \frac{\mu_0 I^2}{2\pi} \quad (\text{downward})$$

$$F_{BC} = \frac{\mu_0 I^2 a}{2\pi(2a)} \Rightarrow F_{BC} = \frac{\mu_0 I^2}{4\pi} \quad (\text{upward}) \qquad \boxed{B \odot}$$

Resultant force $\boxed{\vec{F} = \frac{\mu_0 I^2}{4\pi}}$ (downward)

If current in DA wire & below wire is same then attractive and if opposite the repulsive.

So This force is Attractive.



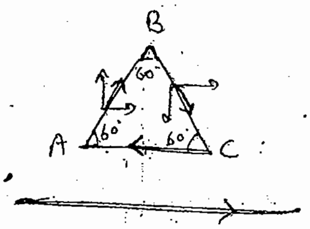
Q.(ii) Now wires are not parallel & perpendicular.

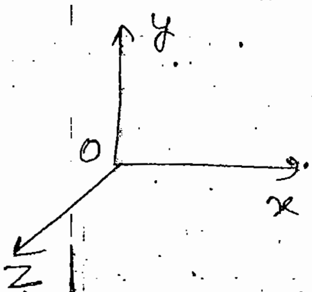
This loop is in x-y plane.

$$\vec{dl}_{AB} = dx\,\hat{x} + dy\,\hat{y}$$

$$\vec{dl}_{BC} = dx\,\hat{x} - dy\,\hat{y}$$

$$\vec{dl}_{CA} = -dx\,\hat{x}$$





$$\vec{dl} = dl_{AB} + dl_{BC} = 2dx\,\hat{x}$$

$$\sin 60° = \frac{h}{a}$$

$$\frac{\sqrt{3}}{2} = \frac{h}{a}$$

$$h = \frac{\sqrt{3}}{2}a$$

Mag. field for part ABC, $\vec{B} = \frac{\mu_0 I}{2\pi y}\hat{z}$

$$d\vec{F}_{ABC} = I(\vec{dl} \times \vec{B})$$

$$d\vec{F}_{ABC} = \frac{\mu_0 I^2}{2\pi y} 2\,dx\,(-\hat{y})$$

Total force $\vec{F}_{ABC} = \int d\vec{F}_{ABC} = \frac{\mu_0 I^2}{\pi} \int \frac{dx}{y}(-\hat{y})$

$$\tan\theta = \frac{y}{x} \Rightarrow \tan 60° = \frac{y}{x} \Rightarrow \sqrt{3} = \frac{y}{x} \Rightarrow y = \sqrt{3}x$$

Convert y into x bcoz in integration y is variable & integration is over x.

$$\vec{F}_{ABC} = \frac{\mu_0 I^2}{\pi}\int_{a/\sqrt{3}}^{a/\sqrt{3}+a/2} \frac{dx\,(-\hat{y})}{\sqrt{3}\,x}$$

$$= \frac{\mu_0 I^2}{\pi\sqrt{3}}\int_{a/\sqrt{3}}^{\frac{a}{\sqrt{3}}+\frac{1}{2}} \frac{dx}{x}\,(-\hat{y})$$

$$= \frac{\mu_0 I^2}{\pi\sqrt{3}} \ln\left(\frac{\frac{a}{\sqrt{3}}+\frac{a}{2}}{\frac{a}{\sqrt{3}}}\right)(-\hat{y})$$

$$\boxed{\vec{F}_{ABC} = \frac{\mu_0 I^2}{\pi\sqrt{3}} \ln\left(1+\frac{\sqrt{3}}{2}\right)(-\hat{y})} \simeq 0.13$$

from going $A \to B \to C$, x changes by a distance, a

{ y changes distance a to B. if we convert in y }

As $y = \sqrt{3}x \Rightarrow x = \frac{y}{\sqrt{3}}$

when $y = a$, $x = a/\sqrt{3}$

$y = a + \frac{\sqrt{3}}{2}a$, $x = \frac{(a+\frac{\sqrt{3}}{2}a)}{\sqrt{3}}$

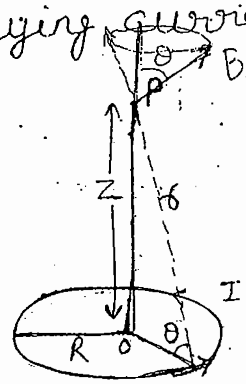
for CA Wire, $\vec{B} = \frac{\mu_0 I}{2\pi y}\hat{z}$ (out of the page)

So force $\boxed{\vec{F}_{CA} = \frac{\mu_0 I^2}{2\pi}(\hat{y})} \simeq 0.5$

{ $\frac{\mu_0 I}{2\pi a}$, $I\int d\vec{l}\times\vec{B}$, $\frac{\mu_0 I^2 a}{2\pi a}\hat{y}$ }

$\vec{F}_{CA}$ is greater than $\vec{F}_{ABC}$ so Resultant force is in $\hat{y}$ dir^n. This force will be <u>Repulsive</u>.

<u>Q.</u> Find the magnetic field at a distance z above the centre of a circular loop of radius R carrying current I.



We Use Biot-Savart law.

Take a small element $d\vec{l}$.

dir^n of Mag. field $\to d\vec{l}\times\vec{r}$

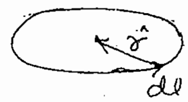
Inclination of $\vec{r}$ w.r.to this plane is θ.

Here r is not in this plane while it is tilted by an angle θ

So mag. field will also tilt by an angle θ.

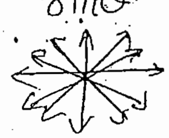
i.e. $\vec{r}$ tilt from horizontal $=$ $\vec{B}$ tilt from vertical

{ if we have to find $\vec{B}$ at centre then $\vec{r}$ will be & dir^n of $\vec{B}$ will be $\hat{z}$ }



If we break the components of B then

$\sin\theta$ comp. will cancel out bcoz $\sin\theta$ comp. is rotating



Only $\cos\theta$ is responsible for $\vec{B}$.

$\cos\theta$ comp. adds up so it is along upward dir^n.

So Net Mag. field will be in z-dirn.

$$B_z = B\cos\theta$$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{\vec{dl}\times\vec{r}}{r^3} = \frac{\mu_0 I}{4\pi}\int \frac{dl\, r\sin\theta}{r^3}$$

$\theta = 90°$ { $\vec{r}$ makes angle with horizontal natu $\vec{dl}$. It is 90° }

$$B = \frac{\mu_0 I}{4\pi}\int \frac{dl}{r^2}$$

$$\cos\theta = \frac{R}{r} = \frac{R}{(R^2+z^2)^{1/2}}$$

$$\therefore B_z = \frac{\mu_0 I}{4\pi}\int \frac{dl}{(R^2+z^2)}\,\frac{R}{(R^2+z^2)^{1/2}} = \frac{\mu_0 I R}{4\pi}\int \frac{dl}{(R^2+z^2)^{3/2}}$$

$$= \frac{\mu_0 I R \times 2\pi R}{4\pi(R^2+z^2)^{3/2}}$$

$$\boxed{B_z = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}}$$

Limiting Cases :-

(1) At the centre of the loop ;- $z = 0$

$$B_{centre} = \frac{\mu_0 I}{2R}$$

(2) $z >> R$. very far from the loop :- neglect R^2 bcoz z is very large

$$B_z = \frac{\mu_0 I R^2}{2z^3} \times \frac{2\pi}{2\pi}$$

$$B_z = \frac{\mu_0 I\, 2\pi R^2}{4\pi z^3}$$

At far distances, Mag. field $\propto \frac{1}{z^3}$ $\qquad z \rightarrow$ distance

(3) If loop contains N turns :-

$$B_z = N B_z$$

(bcoz current will become N times)

$$\boxed{B_z = \frac{N\mu_0 I R^2}{2(R^2+z^2)^{3/2}}}$$

Find the value of z at which mag. field is maximum :-

$$\frac{dB}{dz} = 0 \qquad B = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}$$

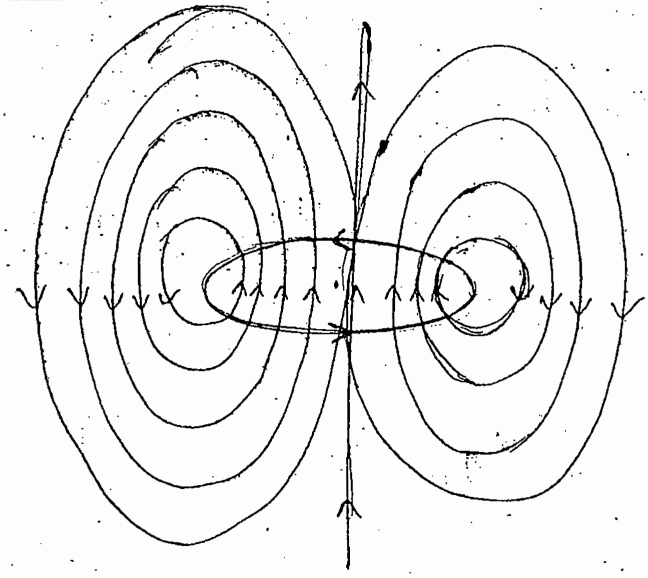
$$\frac{\mu_0 I R^2}{2} \cdot -\frac{3}{\cancel{2}} \frac{\cancel{2} z}{(R^2+z^2)^{5/2}} = 0$$

$$\Rightarrow \quad z = 0$$

i.e. At the centre Mag field will be maximum

Maxi. field $B = \dfrac{\mu_0 I R^2}{2R^3} \Rightarrow \boxed{B_{max} = \dfrac{\mu_0 I}{2R}}$

Magnetic field lines :-

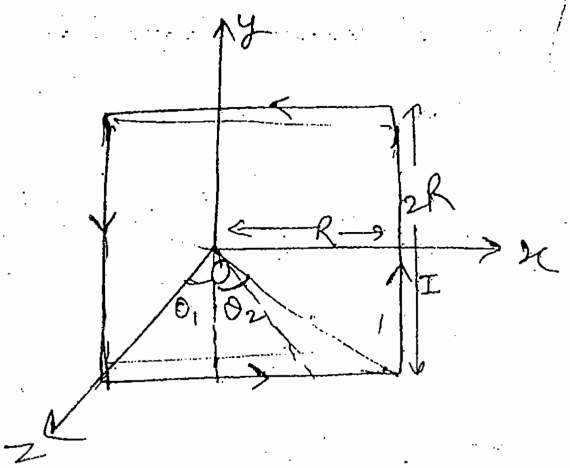


Q.(a) Find the magnetic field at the centre of a square loop which carries steady current I. R be the distance from centre to sides

(b) Find the field at the centre of a N side polygon carring a steady current I.

for all the wires dir^n of B is outside.

It is additive so Mag field at centre will be 4 times of mag. field due to one wire.



Mag. field for a finite wire

$$B = \frac{\mu_0 I}{4\pi s}[\sin\theta_2 - \sin\theta_1]$$

for lower wire;

$$B = \frac{\mu_0 I}{4\pi R}\left[\sin 45^\circ - (-\sin 45^\circ)\right]$$

$$= \frac{\mu_0 I}{4\pi R} 2\sin 45^\circ = \frac{\mu_0 I}{4\pi R} 2\times\frac{1}{\sqrt{2}}$$

$$\vec{B} = \frac{\mu_0 I\sqrt{2}}{4\pi R}\hat{z}$$

for all the sides i.e. for square loop

B = 4 times of (B for one wire)

$$\boxed{\vec{B} = \frac{\sqrt{2}\,\mu_0 I}{\pi R}\hat{z}}$$

(b) general formula :-

Mag. field for n-side polygon is

$$\boxed{B = \frac{n\mu_0 I}{2\pi R}\sin\left(\frac{\pi}{n}\right)}$$

$I \rightarrow$ Current

$n \rightarrow$ sides

$R \rightarrow \perp$ distance from the mid point of wire

Now check it for Square loop

$$\vec{B} = \frac{4\mu_0 I}{2\pi R}\sin\left(\frac{\pi}{4}\right) = \frac{2\mu_0 I}{\pi R}\frac{1}{\sqrt{2}}$$

$$\vec{B} = \frac{\sqrt{2}\,\mu_0 I}{\pi R}\hat{z}$$

Circle is ∞ sides polygon so $n \rightarrow \infty$

$$B = \frac{n\mu_0 I}{2\pi R}\cdot\frac{\pi}{n} \qquad \theta \rightarrow 0 \qquad \text{when } \theta = 0,\ \sin\theta \simeq \theta$$

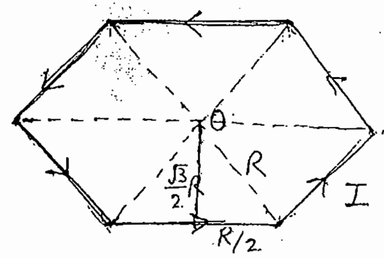
$$\boxed{B = \frac{\mu_0 I}{2R}}$$

* We can not apply this formula for rectangle. This is valid only for regular polygon, & rectangle is not a regular polygon.

Q. Find the mag. field at the centre of a regular hexagon as shown in figure..

It is placed in y-z plane.

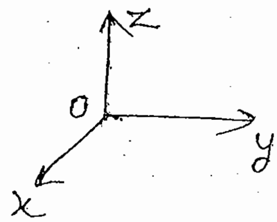
~~for square loop n = 6~~



Mag. field for n side polygon

$$B = \frac{n \mu_0 I}{2\pi R} \sin\left(\frac{\pi}{n}\right)$$

n = 6 for Hexagon



$$B = \frac{6 \mu_0 I}{2\pi\left(\frac{\sqrt{3}}{2}R\right)} \sin\frac{\pi}{6} = \frac{2\,3 \mu_0 I}{\sqrt{3}\pi R}\,\frac{1}{2}$$

$$B = \frac{\sqrt{3}\,\mu_0 I}{\cancel{2}\,\pi R}(+\hat{z})$$

~~Rad~~ distance R in the formula is the ⊥ distance from centre to wire

Mag. field due to all the wires is outward so dir^n z $+\hat{z}$

so $\boxed{B = \frac{\sqrt{3}\mu_0 I}{\pi R}(+\hat{z})}$

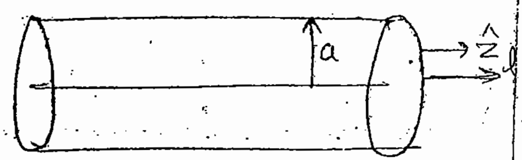
Q. A Current I flows down a wire of radius a.

(a) If it is uniformly distributed over the surface. What is surface current density K.

(b) If it is distributed in such a way that the volume current density is inversly proportional to the distance from the axis then find J.

$[\vec{K} = \sigma \vec{v}]$

(a) $K = \frac{I}{l_\perp}$



$l_\perp$ length ⊥ to the flow of Current $= 2\pi a$ (circumference of cross section of radius a)

$$\boxed{K = \frac{I}{2\pi a}\hat{z}}$$

(b) $J \propto \frac{1}{s}$ (distance from axis → s, vector away from axis → s)

$$J = \frac{k}{s}$$

$$\int \vec{J} \cdot d\vec{S}_\perp = I$$

$$\int_0^a \int_0^{2\pi} \frac{K}{s} \cdot s \, ds \, d\phi \, \hat{z} = I$$

area element $\Rightarrow$ $d\vec{S}_\perp = s \, ds \, d\phi \, \hat{z}$

$$\int_0^{2\pi} \int_0^a K \, ds \, d\phi = I$$

$$K \, (s)_0^a (2\pi) = I \Rightarrow I = 2\pi K a$$

$$K = \frac{I}{2\pi a}$$

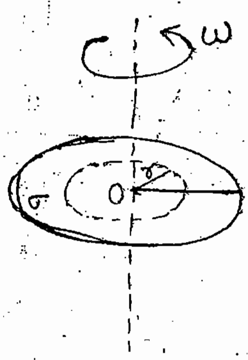
$$\vec{K} = \frac{I}{2\pi a} \hat{z}$$

So $\boxed{J = \frac{I}{2\pi a s} \hat{z}}$ Ans

Qu :- If phonograph records carries a uniform density of static electricity is σ. If it rotates at angular velocity ω. What is the current density K at a distance r from the centre.

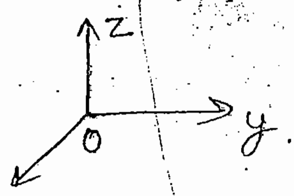
dir^n of $\omega \rightarrow \hat{z}$

Curl the fingers in the dir^n of rotation.



Surface current density,

$\vec{K} = \sigma \vec{v}$ ——— (1)



dir^n of $\vec{K}$ will be same as of $\vec{v}$.

$\vec{v} = \vec{\omega} \times \vec{r}$

$= \omega r [\hat{z} \times \hat{r}]$ $\quad (\vec{r} = r\hat{r})$

$\vec{v}$ will be tangential but ω will be $\perp$ to the plane

$\boxed{\vec{v} = \omega r \hat{\phi}}$

dir^n of $\vec{v}$ will be $\hat{\phi}$.

$\theta = 90°$.

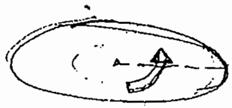
{ $\cdot$ In We write $\hat{r}$ in cartisian then
$z \times r = \sin\theta \, \hat{\phi}$ & $\theta = 90°$ so $\hat{\phi}$ }

$$\vec{K} = \sigma \vec{v}$$

$$\Rightarrow \boxed{\vec{K} = \sigma \omega r \hat{\phi}}$$

If R is the radius then current $I = ?$

$$K = \frac{I}{l_\perp}$$



dirⁿ ⊥ to the flow of current $\rightarrow \hat{r}$
element $= dr$

Total current $I = \int K \, dl$

$$I = \int_0^R \sigma \omega r \, dr$$

$$\boxed{I = \frac{\sigma \omega R^2}{2}}$$

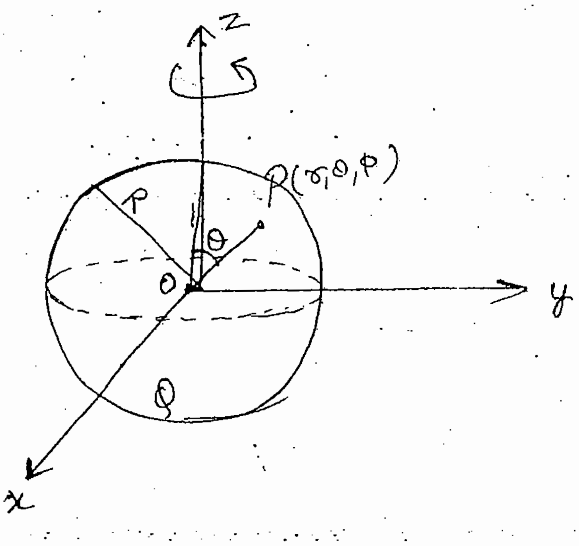
Q. A uniformly charged solid sphere of radius R & total charge Q is centred at the origin & spining at a constant angular velocity ω about z-axis. find the current density J at r, θ, ϕ within the sphere.

$\vec{J}$ at r, θ, ϕ.

$$\vec{J} = \rho \vec{v}$$

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3}$$

As the sphere is uniformly charged otherwise we can not write like this.



dirⁿ of $\omega \rightarrow \hat{z}$

$$\vec{\omega} = \omega \hat{z}$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$= \omega r (\hat{z} \times \hat{r})$$

$$= \omega r \sin\theta \, \hat{\phi}$$

$$\hat{z} \times \hat{r} = (\cos\theta \, \hat{r} - \sin\theta \, \hat{\theta}) \times \hat{r}$$

$$= -\sin\theta (\hat{\theta} \times \hat{r})$$

$$= \sin\phi \, \hat{\phi}$$

$$\vec{J} = \rho \omega r \sin\theta \, \hat{\phi}$$

$$\vec{J} = \frac{Q}{\frac{4\pi R^3}{3}} \omega r \sin\theta \, \hat{\phi}$$

Total Current = ?

$$I = \int \vec{J}\cdot d\vec{S}$$

$$dS_\phi = r\,dr\,d\theta$$

$$I = \int_0^R \int_0^\pi \rho\omega r \sin\theta \; r\,dr\,d\theta$$

$$I = \rho\omega \frac{R^3}{3}\left(-\cos\theta\right)_0^\pi = \frac{\rho\omega R^3}{3}(1+1)$$

$$I = \frac{2\omega\rho R^3}{3}$$

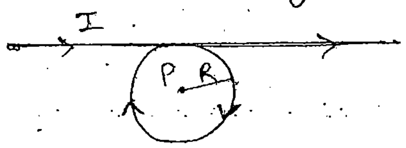
$$I = \frac{2}{3}\omega \cancel{R^3}\frac{Q}{\frac{4}{3}\pi \cancel{R^3}} = \frac{Q\omega}{2\pi}$$

$$\boxed{I = \frac{Q\omega}{2\pi}}$$

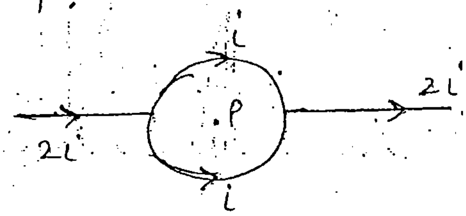
This total current is due to volume charge density. There may be some current at surface also but there is not given σ & A. If given then Total current will be due to surface current density + vol. current density.

Q. Find the mag. field at point P.

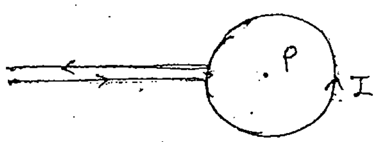
(i)



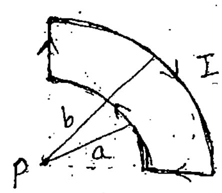
(ii)



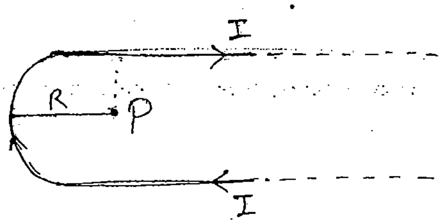
(iii)



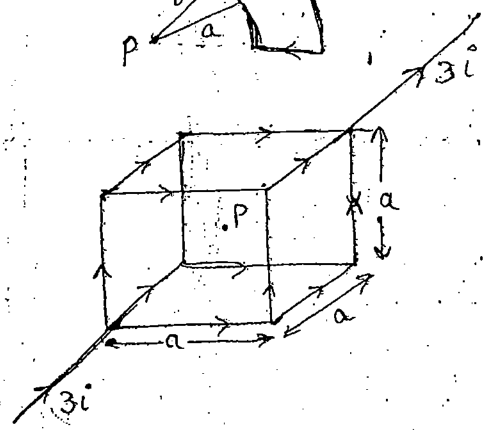
(iv)



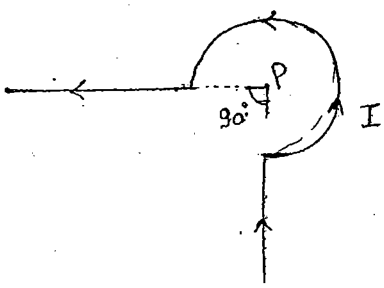
(v)



(vi)



(vii)



(i) Mag. field of circular loop $= \frac{\mu_0 I}{2R}$ (inward) (bcoz of circle)

In the ∞ wire, $\vec{B} = \frac{\mu_0 I}{2\pi R}$ (inward)

$$\text{Total Mag. field} = \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R}$$

$$\boxed{B_P = \frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right)}$$

Due to $\perp$ bend in wire, Mag. field $\uparrow$ Otherwise in straight wire it is $\mu_0 I/2\pi R$.

(ii) Due to lower half $= \frac{\mu_0 I}{4R}$ out of page

upper " $= \frac{\mu_0 I}{4R}$ into the page

for Half circle $\vec{B} = \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right)$

$$\text{Total } \vec{B} \text{ at } P = \frac{\mu_0 I}{4R} - \frac{\mu_0 I}{4R}$$

$$\boxed{B = 0}$$

Due to the wires no contribution. So Total $B = 0$

$\left(B = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \vec{r}}{r^3}\right)$ $\angle$ b/w dl & $r = 0$

(iii) Dir^n of $\vec{B}$ = outward (for circle)

$$\vec{B} = \frac{\mu_0 I}{2R}$$

Due to wire $\Rightarrow$ $B = 0$

$$\text{Total } \boxed{B = \frac{\mu_0 I}{2R}}$$

(iv) $\frac{1}{4}$th part of circle

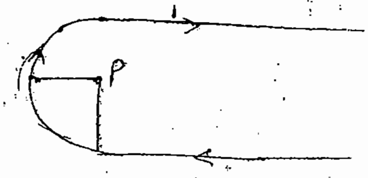
for a radius part $B = \frac{1}{4}\frac{\mu_0 I}{2a} = \frac{\mu_0 I}{8a}$ (out of page)

" b " $B = \frac{\mu_0 I}{8b}$ (Into ")

Contribution of B is only due to curved path. due to other 2 path $B = 0$ ($\angle$ b/w $\hat{r}$ & surface $= 0$)

$$\text{Total } \boxed{B = \frac{\mu_0 I}{8}\left[\frac{1}{a} - \frac{1}{b}\right]} \text{ (out of page)}$$

(v) It is a combination of half circle & 2 semi ∞ wire.
Due to all 3 paths dir^n of $\vec{B}$ is into the page.



$$\theta_2 = 90°$$
$$\theta_1 = 0$$

~~for lower~~ wire, $B = \frac{\mu_0 I}{4\pi R}[\sin\theta_2 - \sin\theta_1]$

$$= \frac{\mu_0 I}{4\pi R}$$

point P is from mid point of wire

for upper half ∞ wire,

$$B = \frac{\mu_0 I}{4\pi R}$$

So Total B due to both wires $= \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4\pi R} = \frac{\mu_0 I}{2\pi R}$

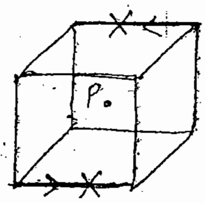
Due to curved wire $\vec{B} = \frac{\mu_0 I}{4R}$

So Total Mag. field at P, $B = \frac{\mu_0 I}{4R} + \frac{\mu_0 I}{2\pi R}$

$$\boxed{B = \frac{\mu_0 I}{2R}\left[\frac{1}{2} + \frac{1}{\pi}\right]}$$

Mag. field Depends on the shape of wire

(vi) Same Current flow in all the wires of same type as the resistance of wires are same.
So Mag. field $= 0$



(In these 2 surfaces B is same, distance from P is same, only dir^n ~~is~~ of B is different so they cancell out.)

for Digonally opposite sides, $\vec{B}$ have opposite dir^n so B will cancel out.

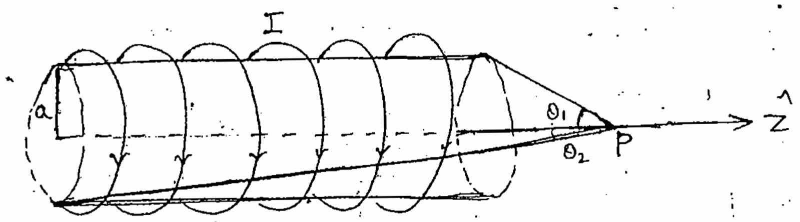
So $\boxed{\text{Total } B = 0}$

(vii) Circular path $\rightarrow \frac{3}{4}$ circle

$$B = \frac{3}{4} \times \frac{\mu_0 I}{2R} \Rightarrow B = \frac{3\mu_0 I}{8R} \text{ (out of page)}$$

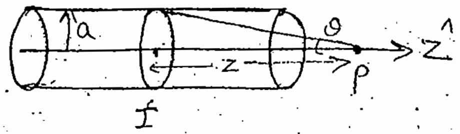
Due to wires $\Rightarrow B = 0$

Q. Find the mag. field at a point on the axis of a tightely wound solanoid consisting of a small n turnes/unit length & carrying current I. Express your answer in terms of θ_1 & θ_2 (finite Solanoid).



The mag-field of a single circle is

$$B = \frac{\mu_0 I a^2}{2(a^2+z^2)^{3/2}}$$



There are many circles bounded tightely ~~by~~ with each other.

Total no. of turns $= n$, length dz.

- total no. of turns in length $dz = n\,dz$
- Total current in this thickness $= I\,n\,dz$

from Biot - Savart law,

$$dB = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\vec{r}}{r^3}$$

$$\cancel{dB} = \frac{\mu_0 \cancel{I}\, n\, \cancel{dz}}{\cancel{4\pi}}$$

$$dB = \frac{\mu_0 a^2 n I\, dz}{2(a^2+z^2)^{3/2}}$$

dir^n of B $\rightarrow \hat{z}$

$$B = \int dB$$

$$= \frac{\mu_0 a^2 n I}{2}\int \frac{dz}{(a^2+z^2)^{3/2}}$$

$$= \frac{\mu_0 a^2 n I}{2}\int \frac{-a\,\mathrm{cosec}^2\theta\, d\theta}{a^3(1+\cot^2\theta)^{3/2}}$$

$$= \frac{\mu_0 \cancel{a^2} n I}{2}\int \frac{-\cancel{a}\,\mathrm{cosec}^2\theta\, d\theta}{\cancel{a^3}\,\mathrm{cosec}^3\theta}$$

$$= \frac{\mu_0 n I}{2}\int_{\theta_1}^{\theta_2} -\sin\theta\, d\theta$$

Rel^n b/w θ & z

$$\tan\theta = \frac{a}{z}$$

$$z = a\cot\theta$$

$$dz = -a\,\mathrm{cosec}^2\theta\, d\theta$$

As $dz \uparrow$, there will be two ends. & limits from θ_1 to θ_2

$$B = \frac{\mu_0 n I}{2} \left[\cos\theta \right]_{\theta_1}^{\theta_2}$$

$$\boxed{B = \frac{\mu_0 n I}{2} \left[\cos\theta_2 - \cos\theta_1 \right] \hat{z}}$$

Limiting Cases :-

(i) Mag. field of a Infinite Solanoid :-

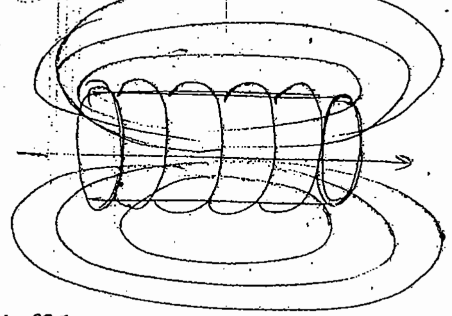
$\theta_1 = 180°$

$\theta_2 = 0°$

$$\boxed{\vec{B} = \mu_0 n I \hat{z}}$$

Mag. field inside the solanoid is constant.

for a finite solanoid ; mag. field inside the solanoid is still $\vec{B} = \mu_0 n I \hat{z}$



(ii) Outside the solanoid :-

$B_{out} = 0$

Practically B at pt. P is not zero but very weak. so theoritically we assume it is zero.

Bcoz mag. field lines of upper & lower are opposite.

for a short Solanoid, $\theta_1 = \theta_2$ so $B = 0$

(iii) At End point :- $\theta_2 = 0$; $\theta_1 = 90°$

$$B_{end} = \frac{\mu_0 n I}{2}$$

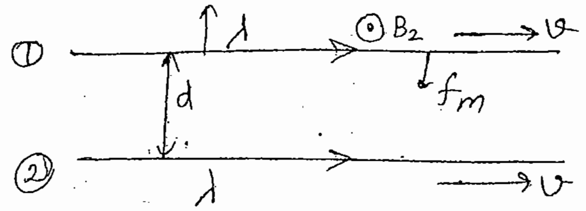
Mag. field at the end of solanoid is half of the field ~~at~~ inside the solanoid.

- Dirn of Wire & solanoid are interchangable.

dirn of m.f. $\rightarrow \hat{z}$; current $\rightarrow \hat{\phi}$ } solanoid

$I \rightarrow \hat{z}$; m.f. $\rightarrow \hat{\phi}$ } Wire

Q. Suppose 2 ∞ line charges λ a distance d apart moving at a constant speed v. Calculate the value of v (speed) in order for mag. attraction to balance electrical repulsion.



If λ & λ are +ve charges moving with velocity v, there must be the electrical repulsion.

But current is flowing in same dir^n (current will be in same dir^n as v) so there will be mag. attraction.

Electric force $\vec{F_e} = q\vec{E}$

wires are ∞ line charges. So force per unit length

$$f_e = \lambda E$$

force/unit length for wire (1) $\quad f_{e_1} = \lambda_1 E_2$

$$f_{e_1} = \frac{\lambda_1 \lambda}{2\pi\epsilon_0 d} = \frac{\lambda^2}{2\pi\epsilon_0 d} \quad \text{(upward)} \quad \text{—— (1)}$$

same force will be on wire (2) but in opposite dir^n but we are interested only in magnitude.

Magnetic force $\quad F_m = q(\vec{v} \times \vec{B})$

$$f_{m_1} = \lambda_1 (\vec{v_1} \times \vec{B_2}) \quad \text{(downward)}$$

$$B_{2_1} = \frac{\mu_0 I}{2\pi d} \;(\text{out of page}) = \frac{\mu_0 \lambda v}{2\pi d}$$

$I = \frac{q}{t} = \lambda$

$\theta = 90°$

$$f_{m_1} = \lambda_1 v_1 B_2$$

$$f_{m_1} = \frac{\mu_0 \lambda^2 v^2}{2\pi d} \quad \text{—— (2)}$$

These 2 forces must balance each other so

$$\frac{\mu_0 \cancel{\lambda^2} v^2}{\cancel{2\pi d}} = \frac{\cancel{\lambda^2}}{\cancel{2\pi}\epsilon_0 \cancel{d}}$$

$$v^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$\Rightarrow v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\mu_0 = 4\pi \times 10^{-7} \Rightarrow v = \frac{1}{\sqrt{4\pi\epsilon_0 \times 10^{-7}}} = \frac{1}{\sqrt{\frac{1}{9\times 10^9} \times 10^{-7}}}$$

$$v = 3 \times 10^8 \ m/c$$

$$v = c \ m/s.$$

- If $v < c$ then electric force will dominent.
  $v\uparrow$, $F_m\uparrow$

AMPERE's Law :- This law is valid only in Magnetostatic

In Electrostatics → Coulomb law ⟷ Biot-Savert law

Gauss law ⟷ Ampere's law

$$\boxed{\oint B \cdot dl = \mu_0 I_{enc}} \Rightarrow$$ Integral form of Ampere's law.

By using Stokes theorm, closed line integral can be written as open surface integral.

$$\Rightarrow \int_S (\vec{\nabla} \times \vec{B}) \cdot d\vec{S} = \mu_0 \int_S \vec{J} \cdot d\vec{S}$$

$$\Rightarrow \int_S (\nabla \times B - \mu_0 J)\, dS = 0$$

$$\int_S \quad \boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}} \Rightarrow$$ Differential force

Compare with Electrostatics :-

$\nabla \cdot E = \rho / \epsilon_0$	$\nabla \cdot B = 0$
$\nabla \times E = 0$	$\nabla \times B = \mu_0 J$

Curl $B \neq 0$

So Mag. field is not a Conservative field, ~~and~~

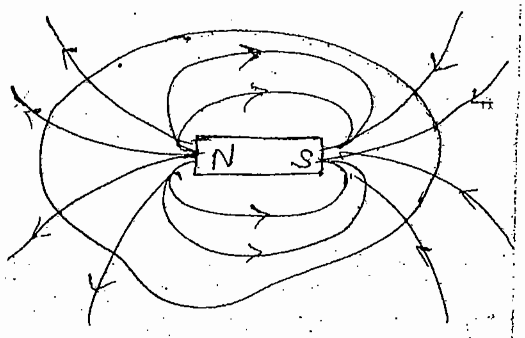
Physical significance of $\nabla \cdot B = 0$ is Non existance of magnetic monopole.

Integral form of $\nabla \cdot B = 0$ is

$$\boxed{\oint_S \vec{B} \cdot d\vec{S} = 0}$$ (by using Gauss law)

It is also called
<u>Gauss law in Magnetostatic</u> $\Rightarrow$ $\vec{\nabla} \cdot \vec{B} = 0$ or $\oint_S \vec{B} \cdot d\vec{S} = 0$

If we have a close surface & put a magnet into this closed surface then <u>mag-flux passing through close surface</u> <u>this $= 0$.</u>



Mag. field lines originates at North pole & terminates at south pole.

$\boxed{\text{line enter in south pole = line leaving North pole}}$

Net no. of field lines coming or going $= 0$

So Magnetic monopole do not exist.

Theoritically Mag. monopole exist but No experimental proof till now.

<u>Note</u>: If We want to find out the mag. field from Ampere's law than there should be symmetry in problem.

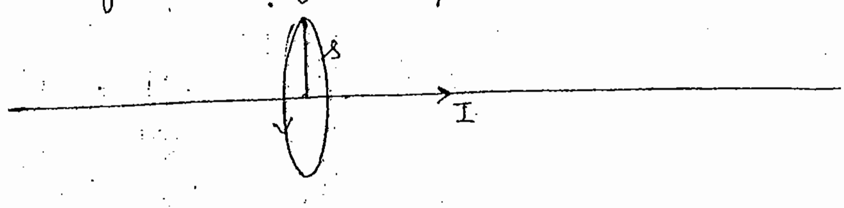
<u>Que</u>:- Find the magnetic field at a distance s from a long straight wire carrying a steady current I.

(If we have a short wire then the dir^n of $\vec{B}$ will be complex i.e. there will be complexity. $\xrightarrow{I}$ $\cdot P$ dir^n of $\vec{B}$ at P $\rightarrow$ outward

but $\xrightarrow{I}$ $\cdot P$ $\rightarrow$ complexity. (at P in this case, we can't define dir^n of $\vec{B}$ short wire

So wire should be long or infinite.

Sketch a Amprical loop s.t. point should be lie on the circumference of loop.



$I_{enc} = I$

Ampere's law $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$

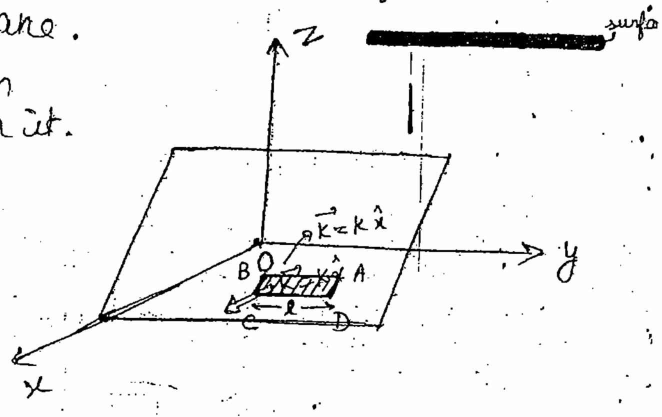
$$B\cdot 2\pi s = \mu_0 I$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$$

Find the mag field of an infinite uniform surface current $\vec{K} = K\hat{x}$ flowing over x-y plane.

Amperical loop should be choosen s.t. current ~~must~~ pass through it.

surface is very thin.



$$\oint \vec{B}\cdot\vec{dl} = \int_{AB} + \int_{BC} + \int_{CD} + \int_{DA}$$

If we put to many wires together then they make a surface.

$$\int_{AB} \vec{B}\cdot\vec{dl} = B\cdot l\cos 0 = Bl$$

$$\int_{BC} \vec{B}\cdot\vec{dl} = B\cdot l\cos 90 = 0$$

$$\int_{CD} \vec{B}\cdot\vec{dl} = B\cdot l\cos 0 = Bl$$

$$\int_{DA} \vec{B}\cdot\vec{dl} = B\cdot l\cos 90 = 0$$

Right hand
Palm on surface
Thumb in the dirn of current
fingure will tell the dirn of $\vec{B}$

i.e. for upper surface dirn of $\vec{B} \Rightarrow -\hat{y}$

lower surface $\Rightarrow +\hat{y}$

So $\quad Bl + 0 + Bl + 0 = \mu_0 I_{enc}$

$$K = \frac{I}{l_\perp} \Rightarrow \frac{I}{l} = K \Rightarrow I = Kl$$

so $\quad 2B\cancel{l} = \mu_0 K\cancel{l} \Rightarrow B = \frac{\mu_0 K}{2}$

$$\boxed{\begin{aligned}\vec{B} &= \frac{\mu_0 K}{2}(-\hat{y}), \quad z>0\\ \vec{B} &= \frac{\mu_0 K}{2}(+\hat{y}), \quad z<0\end{aligned}}$$

(upper surface)

(lower surface)

This mag field is <u>independent of distance</u>.

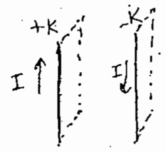
If we replace $\mu_0 \to \frac{1}{\epsilon_0}$ & $K \to \sigma$ then, we get

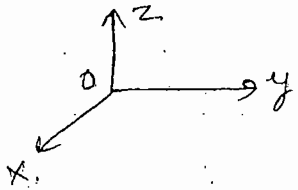
$$\vec{E} = \frac{\sigma}{2\epsilon_0}(+\hat{z}), \quad z>0$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0}(-\hat{z}), \quad z<0$$

so $\vec{E}$ & $\vec{B}$ are Equivalent.

Q. If we have 2 parallel surfaces one carries surface current $+K$ & another carries surface current $-K$. Find the mag. field in region I, II, III with dir^n.





I ($+K\hat{z}$)	II ($-K\hat{z}$)	III
$\frac{\mu_0 K}{2}\hat{x}$	$-\frac{\mu_0 K}{2}(+\hat{x})$	$-\frac{\mu_0 K}{2}\hat{x}$
$-\frac{\mu_0 K}{2}\hat{x}$	$-\frac{\mu_0 K}{2}\hat{x}$	$+\frac{\mu_0 K}{2}\hat{x}$

In (I) region, $\vec{B} = \cancel{\frac{\mu_0 K}{2}\hat{x}} - \cancel{\frac{\mu_0 K}{2}}\hat{x}$

$\vec{B} = 0$

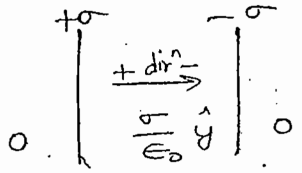
(III) region, $\vec{B} = 0$

(II) region, $\vec{B} = -\frac{\mu_0 K}{2}\hat{x} - \frac{\mu_0 K}{2}\hat{x}$

$$\boxed{\vec{B} = \mu_0 K(-\hat{x})}$$

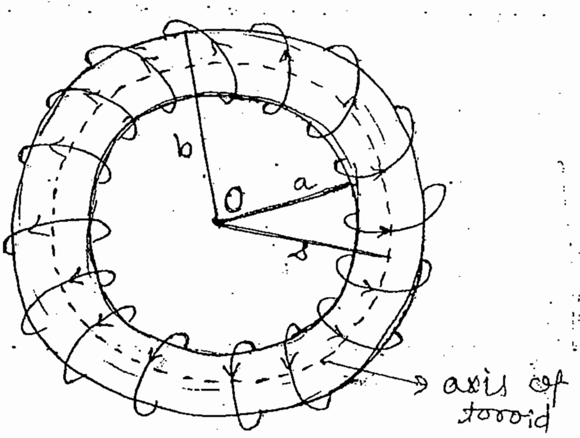
i.e. If 2 plates have equal & opposite surface current then mag. field in b/w plates is Non-zero. & outside is zero.

similar as Ele. field



Q. Mag. field of a toroidal coil [Toroid is endless solenoid]

If Toroid contains N no. of total turns then Magnetic field inside the toroidal coil is



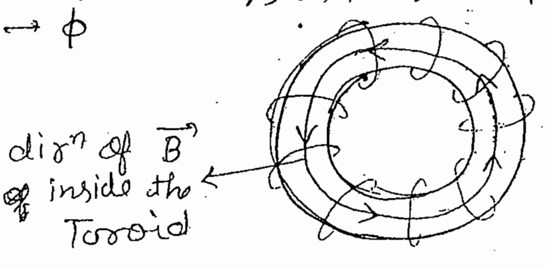
$$\boxed{\vec{B}_{in} = \frac{\mu_0 N I}{2\pi s}\hat{\phi}}, \quad a < s < b$$

$$\boxed{\vec{B}_{out} = 0}, \quad s > b \text{ or } s < a$$

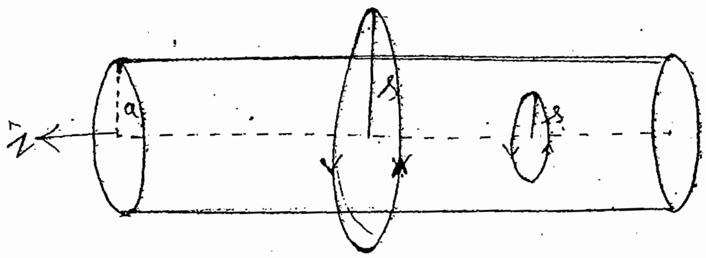
s is measured from origin

Mag. field of a solenoid is along $\rightarrow \hat{z}$

but " " " toroid " $\rightarrow \hat{\phi}$



Q. A steady current I flows down a long cylindrical wire of radius a. Find the mag. field both inside & outside the wire if (a) Current is uniformly distributed over the outside surface of the wire (b) current is distributed in such a way that $J \propto s$.



(a) $I_{enc} = 0$ (Inside)

$$\oint B \cdot dl = \mu_0 I$$

$$\Rightarrow \boxed{B_{in} = 0}$$

Outside :- $I_{enc} = I$

$$\oint B_{out} \cdot dl = \mu_0 I \Rightarrow B_{out} \cdot 2\pi s = \mu_0 I$$

$$\boxed{\vec{B}_{out} = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$$

(b) $J \propto s$

$J = Ks$

$$I = \int \vec{J} \cdot d\vec{s} = \int_0^{2\pi}\int_0^a Ks \, s\,ds\,d\phi$$

$$I = K\left(\frac{s^3}{3}\right)_0^a (2\pi) = K\frac{a^3}{3}2\pi$$

$$\boxed{K = \frac{3I}{2\pi a^3}}$$

$$\boxed{J = \frac{3Is}{2\pi a^3}}$$

Inside :- $I_{enc} = \int_0^{2\pi}\int_0^s J \cdot ds = \int_0^{2\pi}\int_0^s \frac{3I}{2\pi a^3} s \cdot s\,ds\,d\phi$

$$I_{enc} = \frac{Is^3}{a^3}$$

$$\oint B_{in} \cdot dl = \mu_0 I_{enc} \Rightarrow B_{in} \cdot 2\pi s = \mu_0 \frac{Is^3}{a^3}$$

$$\boxed{B_{in} = \frac{\mu_0 I s^2}{2\pi a^3}\hat{\phi}}$$

Outside :- $I_{enc} = I$

$$\boxed{\vec{B}_{out} = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$$

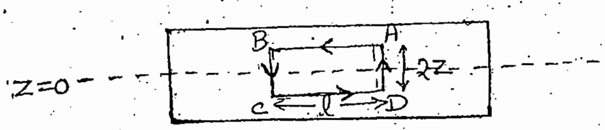
[same as part a]

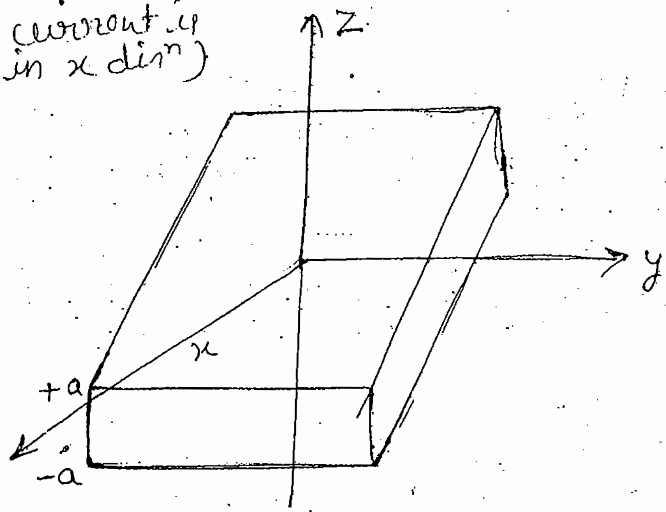
Note :- If $J \propto s^n$ then $B_{in} \propto s^{n+1}$, $B_{out} \propto \frac{1}{s}$

$\rho \propto s^n$
$E_{in} \propto s^{n+1}$, $E_{out} \propto \frac{1}{s}$

Q. A thick slab extending from $z=-a$ to $z=+a$ carries a uniform volume current $\vec{J} = J\hat{x}$. find the magnetic field as a fun^n of z both inside & outside the slab.

$\vec{J} = J\hat{x}$ (i.e. current is in x dir^n)





$\vec{B} \rightarrow -\hat{y}$ for $z>0$

$\vec{B} \rightarrow \hat{y}$ $z<0$

Inside Current enclosed by the loop

$I = JA_{\perp}$

$$\boxed{I = J2zl}$$

Inside Mag. field

$$\oint \vec{B}\cdot d\vec{l} = \int_{AB} B\cdot dl + \int_{BC} B\cdot dl + \int_{CD} B\cdot dl + \int_{DA} B\cdot dl$$

$$= Bl + 0 + Bl + 0 = 2Bl$$

$\oint B\cdot dl = \mu_0 I_{enc}$

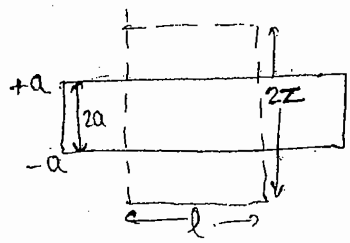
$\cancel{2}B\cancel{l} = \mu_0 J\cancel{2}z\cancel{l} \Rightarrow B = \mu_0 J z$

$$\boxed{\begin{aligned}\vec{B}_{in} &= \mu_0 J z(-\hat{y}), \quad z>0 \\ \vec{B}_{in} &= \mu_0 J z(+\hat{y}), \quad z<0\end{aligned}}$$

So Mag. field inside is linearly $\uparrow$. (as it depends on dist^n z)

Outside $I = J\cdot 2al$

$\oint B\cdot dl = 2Bl$

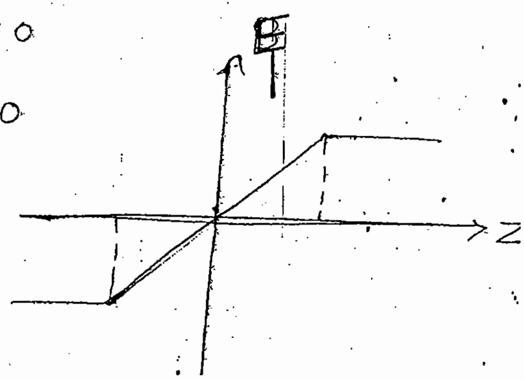


$$2Bl = \mu_0 \cdot J\, 2al$$

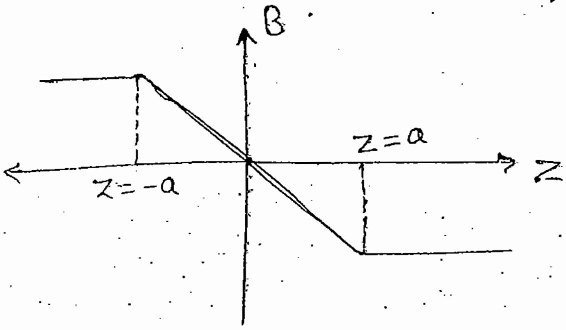
$$B_{out} = \mu_0 J a \quad \text{(constant)}$$

Outside mag. field is constant.

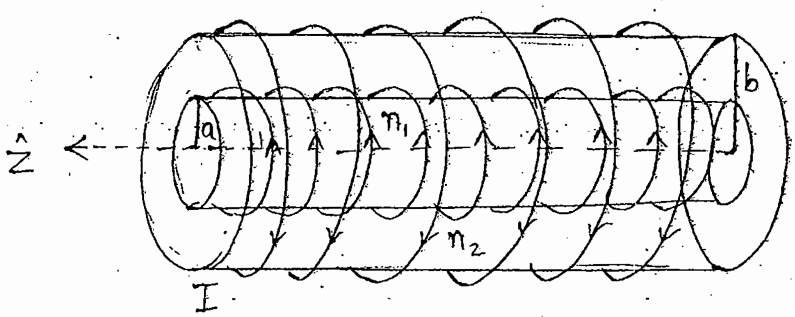
$$\boxed{\begin{aligned} B_{out} &= \mu_0 J a(-\hat{y}) \\ B_{out} &= \mu_0 J a(+\hat{y}) \end{aligned}} \quad \begin{aligned} &, z > 0 \\ &, z < 0 \end{aligned}$$



Plot :- Similar as Electric field.



Q. :- Two long Co-axial solenoids each carry current I but in opposite dirn as shown in the figure. The inner solenoid radius a has n_1 turns per unit length and outer one radius b has n_2 turns per unit length. Find mag. field in 3 regions.



(i) $s < a$
(ii) $a < s < b$
(iii) $s > b$

(i) $s < a$, region is inside both the solenoid.

for inner solenoids, $S_1 \rightarrow \vec{B} = \mu_0 n_1 I \hat{z}$

outer $S_2 \rightarrow \vec{B} = -\mu_0 n_2 I \hat{z}$

$$\boxed{\vec{B} = \mu_0 (n_1 - n_2) I \hat{z}}$$

(ii) $a < s < b$. $\quad B = 0$ for S_1

$B = \mu_0 n_2 I(-\hat{z})$ for S_2

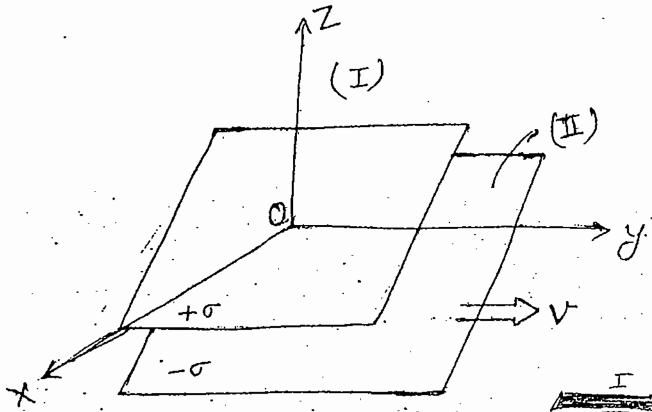
$$\boxed{\vec{B} = \mu_0 n_2 I(-\hat{z})}$$

(iii) $s > b$ $\quad B_{total} = 0$ for both S_1 & S_2 as $I_{enc} = 0$

Q. A large parallel plate capacitor with uniform surface charge σ ~~on~~ on the upper plate & $-\sigma$ on lower plate moving with constant speed v. (a) find $\vec{B}$ b/w the plates & outside the plates.

(b) Find the mag. force per unit area (pressure) on the upper plate with dirn

(c) What should be the value of v ~~to balance~~ such that magnetic ~~attraction~~ repulsion balances the electrical attraction.



(a) $\vec{K}_{upper} = \sigma v \hat{y}$

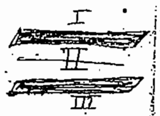
$\vec{K}_{lower} = \sigma v(-\hat{y})$

Due to upper plate $\vec{B}$ in region

I $\rightarrow$ $\frac{\mu_0 K}{2}\hat{x}$

II $\rightarrow$ $-\frac{\mu_0 K}{2}\hat{x}$

III $\rightarrow$ $-\frac{\mu_0 K}{2}\hat{x}$



for $+\sigma$, dirn of K is same as v
for $-\sigma$, opposite to v

Due to lower plate, $\vec{B}$ in

I $\rightarrow$ $-\frac{\mu_0 K}{2}\hat{x}$, II $= -\frac{\mu_0 K}{2}\hat{x}$, III $= \frac{\mu_0 K}{2}\hat{x}$

So In region (I), $\boxed{\vec{B} = 0}$, In (III), $\boxed{\vec{B} = 0}$

In Region (II), $\vec{B} = -\frac{\mu_0 K}{2}\hat{x} - \frac{\mu_0 K}{2}\hat{x}$

$\vec{B} = -\mu_0 K\hat{x} = \mu_0 K(-\hat{x})$

$$\boxed{\vec{B} = \mu_0 \sigma v(-\hat{x})}$$

Magnitude of $K = \sigma v$

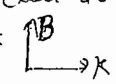
(b). Magnetic force, $\vec{F} = q(\vec{v}\times\vec{B})$

In terms of surface current $\vec{F}_m = \int(\vec{K}\times\vec{B})\, d\vec{a}$

Mag. force/unit area $\vec{f}_{mu} = (\vec{K}_U \times \vec{B}_L)$

mag. repulsion $\leftarrow$ $\boxed{\vec{f}_{m\,(upper)} = \frac{\mu_0 \sigma^2 v^2}{2}(+\hat{z})}$

dirn of B due to σ =



dirn of $\vec{K}_U \rightarrow \hat{y}$, dirn of $B_L \rightarrow -\hat{x}$

so $f_m \rightarrow y \times (-x) = +\hat{z}$

||ly f_m' for lower plate $\vec{f}_m = \frac{\mu_0 \sigma^2 v^2}{2}(-\hat{z})$.

(c) $\vec{F} = q\vec{E}$

$$f_{e_{\text{upper}}} = \sigma_u \vec{E}_\ell = \sigma \frac{\sigma}{2\epsilon_0}(-\hat{z})$$

$$\frac{\mu_0 \sigma^2 v^2}{2} = \frac{\sigma^2}{2\epsilon_0}$$

$$\Rightarrow v^2 = \frac{1}{\epsilon_0 \mu_0} \Rightarrow \boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c}$$

The speed for which d remains constant (when attractive force = repulsion forces) (d → distance b/w 2 plates)

Magnetic Vector Potential ($\vec{A}$): As for electrostatic field,

$$\boxed{\vec{\nabla} \times \vec{E} = 0} \Rightarrow \vec{E} = -\vec{\nabla} V$$

$V \rightarrow$ scalar potⁿ.

{ If we put the V in $\nabla \times E$ then it still 0 bcz curl (grad) = 0

& $\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}} \Rightarrow \boxed{\nabla^2 V = -\rho/\epsilon_0}$ → Poission's eqⁿ in electrostatics.

But here, $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ & $\vec{\nabla} \cdot \vec{B} = 0$

If $\vec{\nabla} \cdot \vec{B} = 0$ then B can be written as

$$\boxed{\vec{B} = \vec{\nabla} \times \vec{A}} \quad \text{as div (curl)} = 0$$

Put the value of B in $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$; we get

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J}.$$

In Magnetostatic, We choose a conditions that

$$\vec{\nabla} \cdot \vec{A} = 0$$

Hence $\boxed{\nabla^2 A = -\mu_0 \vec{J}}$

This is called Poission's eqⁿ in Magnetostatics.

* If $\vec{B}$ is given & we have to find $\vec{A}$ then

$$\boxed{\vec{A} = \frac{1}{2}(\vec{B} \times \vec{r})}$$

This is conditionally true. Its condⁿ is $\vec{B}$ should be uniform i.e. $\boxed{\vec{\nabla} \times \vec{B} = 0}$

i.e. If $\vec{\nabla} \times \vec{B} = 0$ only then we can use this formula of $\vec{A}$

where $\vec{r}$ is a position vector.

$$\nabla^2 \vec{A} = -\mu_0 \vec{J} \quad \text{in Magnetostatic}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{I\,d\vec{l}}{r}$$

$$= \frac{\mu_0}{4\pi} \int \frac{\vec{K}\,da}{r}$$

$$= \frac{\mu_0}{4\pi} \int \frac{\vec{J}\,d\tau}{r}$$

These formulas are applicable only if current is not extended to ∞.

e.g. If we have a ∞ current carrying wire then these formulas can not be applicable.

$$\nabla^2 V = -\rho/\epsilon_0 \quad \text{in E.S.}$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda\,dl}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \int \frac{\sigma\,da}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \int \frac{\rho\,d\tau}{r}$$

These formulas are valid only if charge is localised i.e. charge is not extended to ∞.

* Dirⁿ of A is ⊥ to dirⁿ of B always. generally it matches with dirⁿ of current (parallel or antiparallel). If I in $\hat{x}$ dirⁿ then $\vec{A}$ can be in $\hat{x}$ or $-\hat{x}$ but can never be in $\hat{y}$ or $\hat{z}$.

Q. :- Find the mag. vector potential of an ∞ solenoid with n turns per unit length, radius R & current I

Here current is extended to ∞ i.e. Not localised. So can't use above formula to find $\vec{A}$.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \quad \text{(Ampere's law)}$$

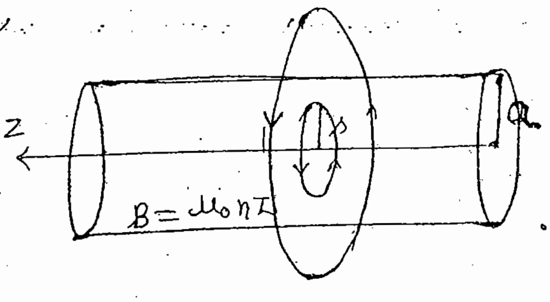
Mag flux passing through any surface S

$$\phi_m = \int_S \vec{B}\cdot d\vec{S} = \int_S (\nabla \times \vec{A})\cdot d\vec{S}$$

$$= \oint \vec{A}\cdot d\vec{l} \quad \text{(from Stokes's theorem)}$$

Mag. field inside the solenoid

$$B = \mu_0 n I$$



$$\phi_m = B\cdot \pi s^2$$

$$= \mu_0 n I\cdot \pi s^2 = \oint \vec{A}\cdot d\vec{l}$$

$$\mu_0 n I \pi s^2 = A\cdot 2\pi s$$

$$\Rightarrow \boxed{\vec{A}_{in} = \frac{\mu_0 n I s}{2}\hat{\phi}}$$

Outside :-

$$\therefore \phi_m = B.\pi a^2$$
$$= \mu_0 n I \cdot \pi a^2 = \oint \vec{A}\cdot d\vec{l}$$
$$= A_{out} \cdot 2\pi s$$
$$\Rightarrow \vec{A}_{out} = \frac{\mu_0 n I a^2}{2s}\hat{\phi}$$

Mag. vector pot^n inside & outside the solenoid = Non-zero
while Mag. field inside → Non-zero, outside → zero.

& $\vec{A}_{out} \propto \frac{1}{s}$ dir^n matches with current

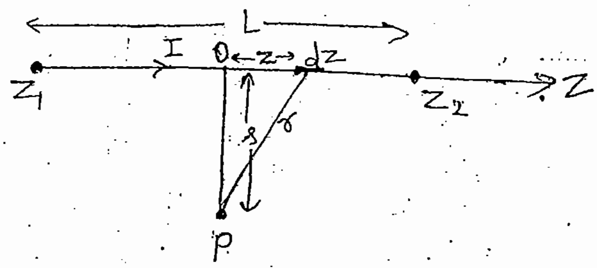
Note :- It is Not necessary that if $\vec{B} = 0$ then $\vec{A} = 0$

17/8/2012

Q. Find the magnetic vector potential of a finite segment of straight wire carrying current I. Put the wire along z-axis from z_1 to z_2.

We have to find mag. vector pot^n at point P which is at a distance s from wire.

This is a finite wire



so $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I\,dl}{r}$

Take a small element, its length is dz which is at a distance z from mid point of wire O.

$$\vec{A} = \frac{\mu_0}{4\pi}\int_{z_1}^{z_2} \frac{I\,dz}{(s^2+z^2)^{1/2}} = \frac{\mu_0}{4\pi}\int_{-l/2}^{l/2} \frac{I\,dz}{(s^2+z^2)^{1/2}}$$

This is standard integral. It gives

$$\vec{A} = \frac{\mu_0 I}{4\pi} \ln\left[z + \sqrt{z^2+s^2}\right]_{-l/2}^{l/2}$$

$$\vec{A} = \frac{\mu_0 I}{4\pi} \ln\left[\frac{l/2 + \sqrt{\frac{l^2}{4}+s^2}}{-l/2 + \sqrt{\frac{l^2}{4}+s^2}}\right]$$

$$\vec{A} = \frac{\mu_0 I}{4\pi} \ln\left[\frac{l/2 + (l^2/4+s^2)^{1/2}}{-l/2 + (l^2/4+s^2)^{1/2}}\right]$$

This is mag. vector pot^n at P from mid point of wire.

⇒ If wire is very very long $L >> s$

$$\vec{A} = \frac{\mu_0 I}{4\pi} \ln\left[\frac{\cancel{L/2}\left[1+\left(1+\frac{4s^2}{L^2}\right)^{1/2}\right]}{\cancel{L/2}\left[-1+\left(1+\frac{4s^2}{L^2}\right)^{1/2}\right]}\right]$$

$$\vec{A} = \frac{\mu_0 I}{4\pi} \ln\left[\frac{1+1+\frac{2s^2}{L^2}}{\cancel{-1}+\cancel{1}+\frac{2s^2}{L^2}}\right] = \frac{\mu_0 I}{4\pi} \ln\left[\frac{\cancel{2}+\frac{\cancel{2}s^2}{L^2}}{\frac{2s^2}{L^2}}\right]$$

$$\vec{A} = \frac{\mu_0 I}{4\pi} \ln\left[\frac{L^2}{s^2}+1\right]$$

$L >> s$ then $\frac{L}{s} >> 1$ & $\frac{L^2}{s^2} >>>> 1$ so neglect 1 as compare to L^2/s^2 so

$$\vec{A} = \frac{\mu_0 I}{4\pi} \ln\left(\frac{L}{s}\right)^2$$

$$\boxed{\vec{A} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{L}{s}\right) \hat{z}}$$ (dir^n same as current)

This is the mag. vector pot^n of a very long wire at pt. P which is at a distance s from mid point O of wire.

Q. Find the current density corresponding to a vector pot^n $\vec{A} = k\hat{\phi}$ where k is constant in cylendrical co-ordinates.

$J = ?$

$\vec{A}$ ↓ B ↓ J | $\vec{V}$ ↓ E ↓ ρ

$\vec{A} = K\hat{\phi}$

$\{\nabla^2\vec{A} = -\mu_0\vec{J}$ (it may give wrong ans)$\}$

$\cancel{\frac{1}{s^2}\frac{\partial^2}{\partial\phi^2}A_\phi} = -\cancel{\mu_0}\vec{J} \Rightarrow \cancel{\frac{1}{s^2}\frac{\partial^2}{\partial\phi^2}(K\phi)} = \cancel{-\mu_0 J}$

$\cancel{\frac{1}{s^2}}$ first find $\vec{B}$ & then $\vec{J}$.

$$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_s & sA_\phi & A_z \end{vmatrix}$$

$$= \frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & sA_\phi & 0 \end{vmatrix} = \frac{1}{s}\left[\frac{\partial}{\partial s}(sA_\phi)\hat{z} - \frac{\partial A_\phi}{\partial z}\hat{s}\right]$$

$$= \frac{1}{s}\left[\frac{\partial}{\partial s}(sK)\hat{z}\right]$$

$$\vec{B} = \frac{1}{s} k \hat{z} \quad \text{or} \quad \boxed{\vec{B} = \frac{k}{s}\hat{z}}$$

We have $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

$$\vec{J} = \frac{1}{\mu_0}(\vec{\nabla} \times \vec{B})$$

$$\vec{J} = \frac{1}{\mu_0}\frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ B_s & sB_\phi & B_z \end{vmatrix} = \frac{1}{\mu_0}\left[-\frac{\partial}{\partial s}(B_z)\hat{\phi} + \frac{1}{s}\frac{\partial}{\partial \phi}(B_z)\hat{s}\right]$$

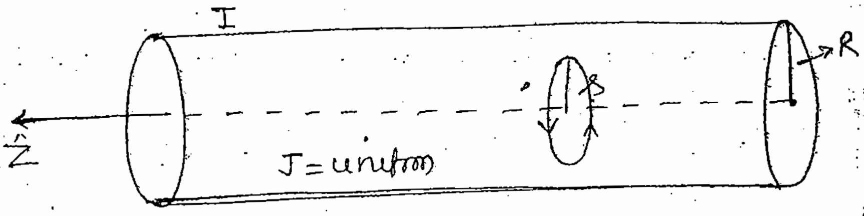
$$= \frac{1}{\mu_0}\left[-\frac{\partial}{\partial s}(B_z)\hat{\phi}\right]$$

$$\vec{J} = -\frac{1}{\mu_0}\frac{\partial}{\partial s}\left(\frac{k}{s}\right)\hat{\phi} = \frac{1}{\mu_0}\frac{k}{s^2}\hat{\phi}$$

$$\boxed{\vec{J} = \frac{k}{\mu_0 s^2}\hat{\phi}}$$

Q. Find the mag. vector potn inside & outside the infinite wire if its radius is R and total current I is uniformly distributed over the cross-section. Assume that vector potn vanishes on the surfaces of the wire.

We have a thick wire of radius R & current I is flowing ~~I~~ in z dirn.



If wire is infinite then we can not apply directly

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I\,dl}{r}$$ (This can be used only if current is localised)

When current extended ~~to~~ itself to ∞ then we'll find A through B.

$$\oint \vec{A}\, d\vec{l} = \phi_m = \int_s \vec{B}\cdot d\vec{S}$$

• Inside

To find $\vec{B}_{in}$, take a Ampirical loop inside the wire ($\vec{J}$ = uniform) & By Ampere's law

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$J = I/A_\perp = I/\pi R^2$ { current is distributed over the whole cross-section

Current enclose by the loop

$$I_{enc} = \int_0^s J \cdot ds = \oint J \cdot \pi s^2 = \frac{I}{\pi R^2} \pi s^2$$

$$\boxed{I_{enc} = \frac{I s^2}{R^2}}$$

So $\oint B \cdot dl = \mu_0 I_{enc}$

$$B \cdot 2\pi s = \frac{\mu_0 I s^2}{R^2}$$

$$\boxed{\vec{B}_{in} = \frac{\mu_0 I s}{2\pi R^2} \hat{\phi}}$$

for a uniformely charged ∞ thick wire, then mag. field inside is $\propto s$. i.e. if $\vec{J} =$ uniform then $\boxed{B_{in} \propto s}$

• Outside :-

$$I_{enc} = \int_0^R J \cdot ds = \frac{I}{\pi R^2} \pi R^2$$

$$I_{enc} = I$$

So $\quad B \cdot 2\pi s = \mu_0 I$

$$\boxed{\vec{B}_{out} = \frac{\mu_0 I}{2\pi s} \hat{\phi}}$$

Now find Mag. Vector potn,

$$\oint \vec{A} \cdot d\vec{l} = \int_S \vec{B} \cdot d\vec{s} = \phi_m$$

We can not use this formula bcoz $\vec{B}$ is in $\hat{\phi}$ dirn & Amperical loop ~~and~~ is also in $\hat{\phi}$ dirn so no flux will pass through the loop. So

We use $\vec{B} = \vec{\nabla} \times \vec{A}$

$$\vec{B} = \frac{1}{s} \begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_s & sA_\phi & A_z \end{vmatrix}$$

$$B\hat{\phi} = \cancel{\tfrac{1}{s}}\left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right) \hat{\phi}$$

$\frac{\partial A_s}{\partial z} \rightarrow$ Mag. vector potⁿ is in s dirⁿ & variation with z.

$\frac{\partial A_z}{\partial s} \rightarrow$ " " " " " "

If current is along z then mag. vector potⁿ must be in z-dirⁿ. $\frac{\partial A_s}{\partial z} = 0$

$$B\hat{\phi} = \left(0 - \frac{\partial A_z}{\partial s}\right)\hat{\phi}$$

$$B_{in} = -\frac{\partial}{\partial s} A_{in} \Rightarrow -\frac{\partial}{\partial s} A_{in} = \frac{\mu_0 I s}{2\pi R^2}$$

$$A_{in} = \int \frac{\mu_0 I s}{2\pi R^2}\, ds + c$$

$$A_{in} = -\frac{\mu_0 I}{2\pi R^2} \frac{s^2}{2} + c$$

Now apply boundary Condⁿs -

At $s = R$ (at surface) $A = 0$

$$\Rightarrow \quad c = \cancel{0}\ \frac{\mu_0 I}{4\pi R^2} R^2$$

So $$\boxed{\vec{A}_{in} = \frac{\mu_0 I}{4\pi R^2}(R^2 - s^2)\,\hat{z}}$$

Now, $$B_{out} = -\frac{\partial}{\partial s} A_{out} = \frac{\mu_0 I}{2\pi s}$$

$$A_{out} = -\frac{\mu_0 I\, ds}{2\pi s}$$

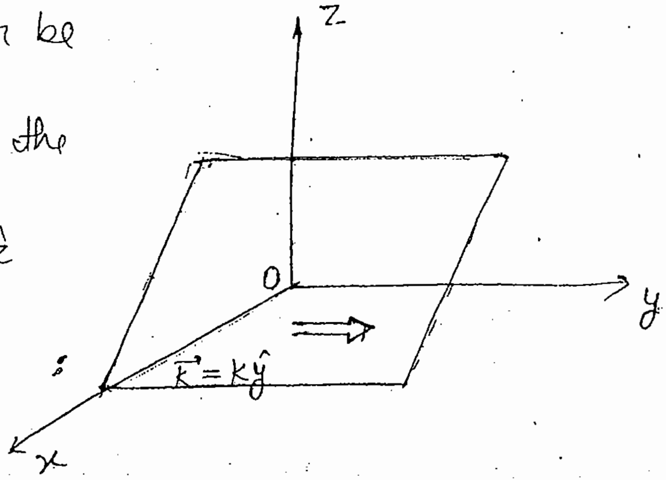
$$A_{out} = -\frac{\mu_0 I}{2\pi} \ln(s) + c$$

At $s = R \Rightarrow A = 0 \Rightarrow c = \frac{\mu_0 I}{2\pi} \ln(R)$

So $$\boxed{\vec{A}_{out} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{R}{s}\right)\hat{z}}$$

Q. Find the mag. vector potⁿ above & below the plane having uniform surface current, $\vec{K} = K\hat{y}$ flowing over the x-y plane & ∞ dimension.

dirⁿ of mag. field can never be ⊥ʳ to the plane (x-y plane)($\hat{z}$)
Also can not be parallel to the dirⁿ of current. ($\hat{y}$)
So dirⁿ of $\vec{B}$ can't be $\hat{z}$ or $\hat{y}$. So it may be $+\hat{x}$ or $-\hat{x}$.



$$\vec{B} = \frac{\mu_0 K}{2}\hat{x} \quad (z>0)$$

$$\vec{B} = \frac{\mu_0 K}{2}(-\hat{x}) \quad (z<0)$$

We have to find $A = ?$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{x}\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]$$

if current is in y-dirⁿ so there will be no variation of A_z with y.

$$\vec{B} = \hat{x}\left(0 - \frac{\partial A_y}{\partial z}\right)$$

$z>0$, $$\frac{\mu_0 K}{2} = -\frac{\partial}{\partial z}A_y$$

$$A = \frac{-\mu_0 K z}{2} + C$$

Assuming the B-C, at $z=0$ (at surface) $A=0$
$\Rightarrow C=0$

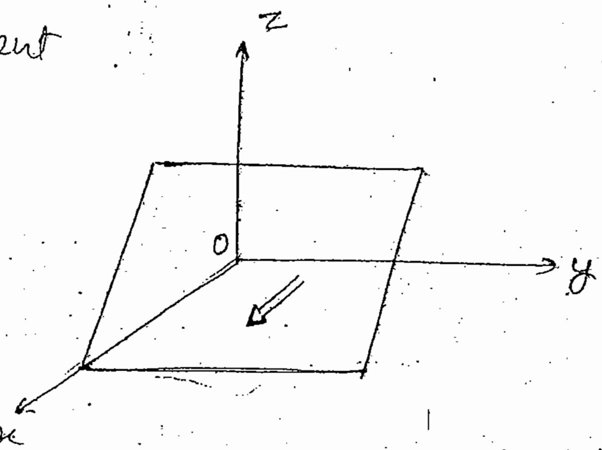
So $$\boxed{\vec{A} = -\frac{\mu_0 K z}{2}\hat{y}} \quad (z>0)$$

$z<0$, $$\boxed{\vec{A} = \frac{\mu_0 K z}{2}\hat{y}} \quad (z<0)$$

Magnetic Boundary Conditions
OR
B.C.s on Mag. field :-
Whenever electric field crosses surface charge σ it suffers a discontinuity.

Illy, whenever ~~surfa~~ mag. field crosses surface current K it suffers discontinuity.

We have a surface current
$\vec{K} = K\hat{x}$



$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \Leftarrow \oint \vec{E}\cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0} \Rightarrow$

$$E^{\perp}_{above} - E^{\perp}_{below} = \frac{\sigma}{\epsilon_0}$$

$\vec{\nabla}\times\vec{E} = 0 \Leftarrow \oint \vec{E}\cdot d\vec{l} = 0$

$$\Rightarrow E^{\parallel}_{above} = E^{\parallel}_{below}$$

Similarly, for Mag. field,

$\vec{\nabla}\cdot\vec{B} = 0 \Leftarrow \oint \vec{B}\cdot d\vec{s} = 0 \Rightarrow \boxed{B^{\perp}_{above} = B^{\perp}_{below}}$

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \Leftarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow \boxed{B^{\parallel}_{above} - B^{\parallel}_{below} = \mu_0 K}$

Conclusion - Tangential component of electric field is continous but Normal comp. of Mag. field is Continous. Also, Normal comp. of $\vec{E}$ is discontinous by the amount $\frac{\sigma}{\epsilon_0}$ & tangential " " $\vec{B}$ " " " " " $\mu_0 K$.

In Electrostatic, scalar potn V & In Magnetostatic vector potn is $\vec{A}$.

$$V_{above} = V_{below} \qquad \boxed{A_{above} = A_{below}}$$

$$\frac{\partial V_{above}}{\partial n} - \frac{\partial V_{below}}{\partial n} = \frac{-\sigma}{\epsilon_0} \qquad \boxed{\frac{\partial A_{above}}{\partial n} - \frac{\partial A_{below}}{\partial n} = -\mu_0 K}$$

$n \rightarrow$ vector normal to the surface

$\frac{\partial V_{above}}{\partial n} \rightarrow$ E. field normal to the surface.

18/8/2012

Multipole Expansion of $\vec{A}$:-

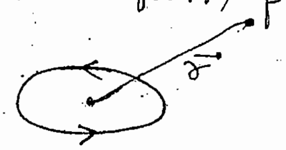
$$A(r) = \frac{\mu_0 I}{4\pi}\Big[\underbrace{\frac{1}{r}\oint dI}_{\text{Monopole}} + \underbrace{\frac{1}{r^2}\oint r\cos\theta\, dI}_{\text{dipole}} + \underbrace{\frac{1}{r^3}\oint (r')^2\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) dI}_{\text{Quadroupal}} + \cdots\Big]$$

$\oint dI = 0$ So Monopole term $= 0$

Dipole term is given by $\boxed{\vec{A}_{dipole} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}}$

where m is magnetic dipole mom (or magnetic moment) & $\vec{r}$:

If we have a current loop (it will behave like a dipole) & we have to find mag. vector pot^n at point P then $\vec{r}$ be the distance from centre of dipole to the point P



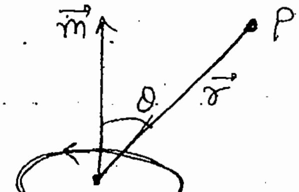
Magnetic Mom. = Current X area

$$\boxed{\vec{m} = I\,\vec{a}}$$

Its dir^n will be the dir^n of $\vec{a}$.

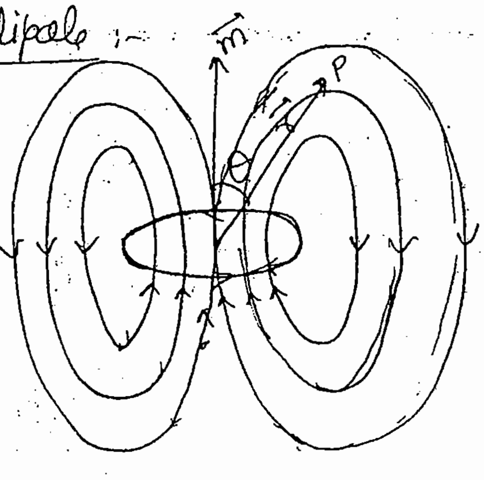
(Curl the fingure in the dir^n of current & thumb will tell the dir^n of mag. moment.)

$$\boxed{\vec{A}_{dip} = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi}}$$



As Electric dipole mom always directed along z-dir^n. Here loop will be in x-y plane & mag. mom. $\vec{m}$ is along z-axis & it will make θ angle with $\vec{r}$.

Field lines of Mag. dipole :-



$\vec{m} = m\hat{z}$

dir$^n \Rightarrow \vec{m}\times\hat{r}$

$\hat{z}\times\hat{r} = \sin\theta$

i.e. dir^n of $\vec{A}$ $= \hat{\phi}$

dir^n of $\vec{A}$ matches with dir^n of current.

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$B = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

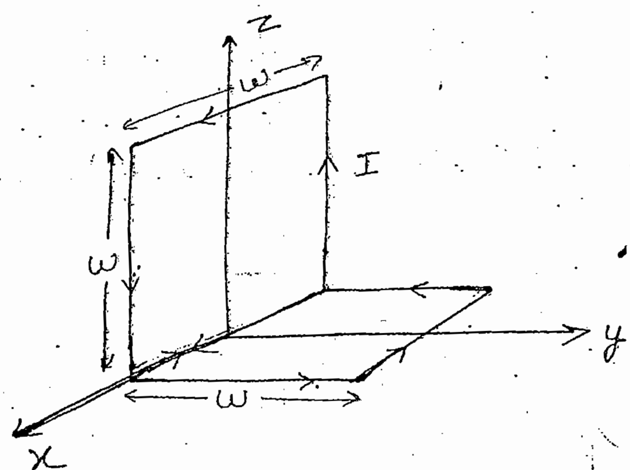
$$\vec{B} = \frac{1}{r^2 \sin\theta}\left[\hat{r}\left\{\frac{\partial}{\partial\theta} r\sin\theta\, A_\phi - \frac{\partial}{\partial\phi}\underbrace{rA_\theta}_{0}\right\} - r\hat{\theta}\left\{\frac{\partial}{\partial r} r\sin\theta A_\phi - \underbrace{\frac{\partial}{\partial\phi} A_r}_{0}\right\}\right]$$

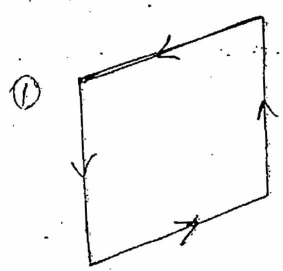
$$\boxed{\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]}$$

Similar as $\vec{E}$, $\quad \vec{E} = \frac{p}{4\pi\epsilon_0 r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$

Q. Find the mag. dipole moment of a bookend-shape loop. All sides have length w and current I.

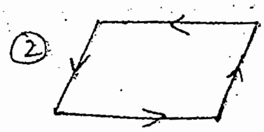
Seperate both loop. & find the mag mom. of each loop.





This loop is in $x-z$ plane
so dir^n of mag. mom. is $\hat{y}$.

$$\vec{m_1} = Iw^2\hat{y}$$



Again for IInd loop, loop is in $x-y$ plane
so dir^n of mag. mom. is $\hat{z}$

$$\vec{m_2} = Iw^2\hat{z}$$

So total mag mom., vector sum of $\vec{m_1}$ & $\vec{m_2}$

$\vec{m} = \vec{m_1} + \vec{m_2} \Rightarrow \boxed{\vec{m} = Iw^2(\hat{y}+\hat{z})}$

$$\boxed{|\vec{m}| = \sqrt{2}\, Iw^2}$$

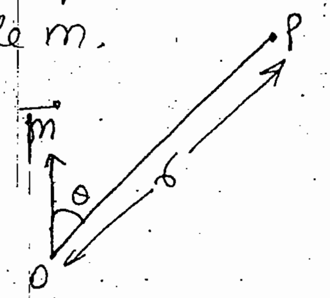
* Magnetic dipole mom. is always independent on choice of origin bcoz mag. monopole mom. is always 0.

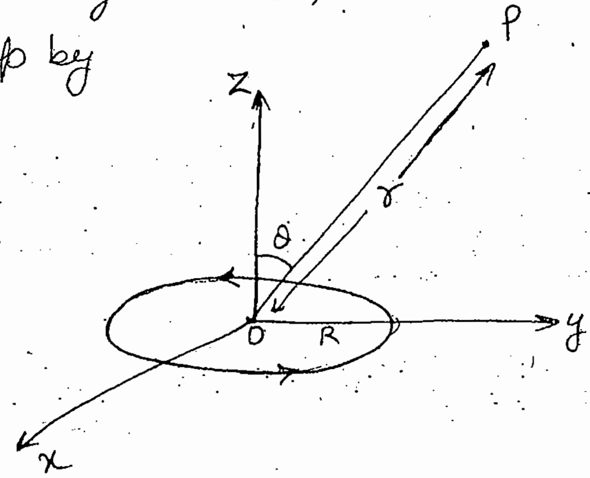
Q. A circular loop of wire with radius R lies in the x-y plane centred at the origin & carries a current I running counter clockwise as viewed from z-axis.

(a) find mag. dipole moment.

(b) Approximate mag. field at large distances

We can replace the whole loop by a single m.





Mag. dipole mom.

$$\vec{m} = I\vec{a}$$

$$\boxed{\vec{m} = I\pi R^2 \hat{z}}$$

Mag. field at point P,

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

$$\boxed{\vec{B} = \frac{\mu_0 I \pi R^2}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]}$$

• Magnetic Vector potn at the axis of loop, $\theta = 0$

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi}$$

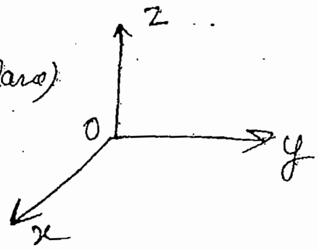
$$\boxed{\vec{A}_{axis} = 0}$$

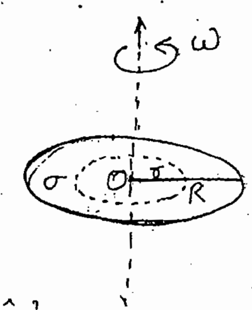
Q. A phonograph record of radius R carrying a uniform surface charge σ is rotating at a constant angular velocity ω. Find its magnetic dipole moment.

$$\vec{v} = \vec{\omega} \times \vec{r}$$
$$= \omega r \sin\theta\,\hat{\phi} \qquad \{\hat{z}\times\hat{r} = \sin\theta\,\hat{\phi}\}$$

$\vec{v} = \omega r \hat{\phi}$

$(\theta = 90°)$ in x-y plane)





$\vec{K} = \sigma \vec{v}$

$\boxed{\vec{K} = \sigma \omega r \hat{\phi}}$

Surface current

Total current $I = \int_0^R K \cdot dr$ $\{$No change in $\hat{\theta}\}$ $\hat{r}, \hat{\theta}, \hat{\phi}$

Magnetic mom. $\vec{m} = I \vec{a}$

Area vector, ⊥ to the flow of current (θ is not changing)

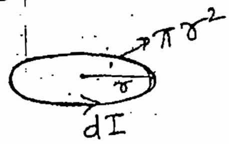
$$\begin{aligned} da &= r \sin\theta \, d\phi \, dr \\ &= r \, d\phi \, dr \quad (\theta = 90°) \\ &= 2\pi r \, dr \end{aligned}$$

$I = \int_0^R \sigma \omega r \, dr$ $\left\{ \vec{m} = \frac{\sigma \omega R^2}{2} \times \pi R^2 \text{ is wrong} \right\}$

I is func^n of r. At the differt point of disc, the value of current will be different & corresponding mag mom. will be different.

Total current in the ring of radius r,

$$dI = \sigma \omega r \, dr$$



Mag. Mom. $\vec{m} = I \vec{a}$

$$\vec{m} = \int a \cdot dI$$

$$\vec{m} = \int_0^R \sigma \omega r \, dr \cdot \pi r^2 \, \hat{z}$$

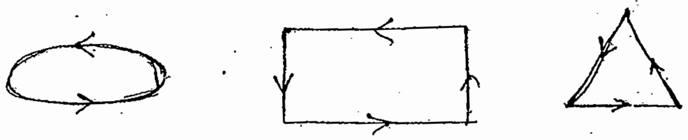
$$\boxed{\vec{m} = \frac{\sigma \omega \pi R^4}{4} \hat{z}}$$

The disc is made up of small rings then mag. mom. in ring $= dI \cdot a$

When disc is form then Mag. mom. $= \int dI \cdot a$

<u>Note</u>- dir^n of length vector is always along the line while the " " area " " perpendicular to the area.

Mag. dipole is a current loop, It may be of any shape - circular, square, rectangular --- etc.



$\oint dI =$ Net displacement $= 0$

So closed loop behaves like Mag. ~~Monopole~~ dipole.

- In 8 (eight shape), there are 2 loops. So we have to seperate these 2 loops & then find $\vec{m}$ of each.

- Definet wire carrying current I not form a loop so not behave like magnetic dipole.

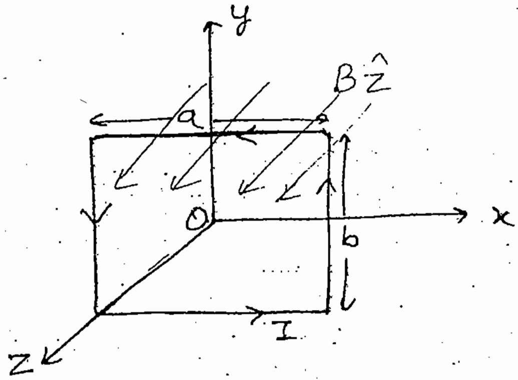
Force & Torque on a magnetic dipole :-

Magnetic dipole experiences no force (0 force) in a uniform magnetic field but it experiences Non-zero torque in uniform magnetic field.

Let us consider a mag. dipole of rectangular shape. Its mag. mom,

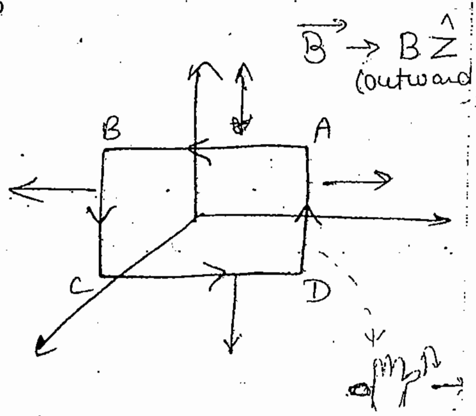
$\vec{m} = I \cdot ab \, \vec{a}$

$$\boxed{\vec{m} = Iab\,\hat{z}}$$



If we apply uniform mag. field $\vec{B}$ along $\hat{z}$, then force on this current loop

$$\vec{F} = I \oint (d\vec{l} \times \vec{B})$$



Sum of total forces for AB, BC, CD, DA will cancel out. So

$$\boxed{\vec{F} = 0}$$

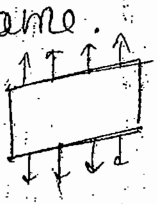
Thus there is No force on the mag. dipole in Uniform mag. field

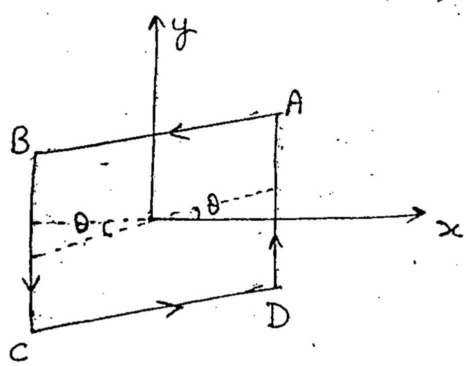
Now Torque :- Rotate the loop s.t. angle b/w $\vec{m}$ & $\vec{B}$ is θ.

(Earlier angle b/w $\vec{m}$ & $\vec{B}$ was zero as $\vec{m} = Iab\hat{z}$ & $\vec{B} = B\hat{z}$)

Now line AD & BC are not at the same value of z.

forces of line AB & CD will cancel out bcoz their line of action is same.





But forces along BC & AD will not cancell out bcoz their line of action is not same, it is different.

$$\boxed{\vec{F}_{AB} + \vec{F}_{CD} = 0}$$

If line of action is different (not same) than they produce Torque.

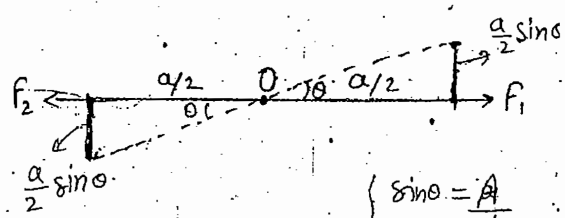
For line BC & ADA, the angle b/w current & mag. field is always 90°.

$$|\vec{F}_{BC}| = |\vec{F}_{DA}| = IbB$$

$$\left\{\begin{aligned} F &= I\int dl \times B \\ &= IB\int dl \\ &= IBb \end{aligned}\right.$$

Now Torque $\vec{N} = \vec{r} \times \vec{F}$

$$N = IbB\frac{a}{2}\sin\theta + IbB\frac{a}{2}\sin\theta$$



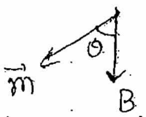
$$N = IabB\sin\theta = mB\sin\theta$$

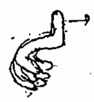
$$\left\{\begin{aligned} \sin\theta &= \frac{A}{a/2} \\ A &= \frac{a}{2}\sin\theta \end{aligned}\right.$$

$$\boxed{\vec{N} = \vec{m} \times \vec{B}}\ \hat{y}$$

$$N = \vec{p} \times \vec{E}$$

* $\vec{m}$ & $\vec{B}$ are in x-z plane so dir^n of $\vec{N}$ will be $\hat{y}$.





→ dir^n of torque → $\hat{y}$

Torque will try to rotate the plane s.t. $\vec{B}$ & $\vec{m}$ are ~~parallel~~ parallel. & when $\vec{m}$ & $\vec{B}$ ~~are~~ become parallel then

$\vec{m}\uparrow \quad \uparrow\vec{B}$ Torque $= 0$.

Force & Torque in Non-Uniform mag. field :-

$$\boxed{\vec{F}_{\text{Non-Uniform}} = \vec{\nabla}(\vec{m}\cdot\vec{B})} \neq (\vec{m}\cdot\vec{\nabla})\vec{B}$$

$$\left\{ \vec{F}_{\text{Non-unifm}} = (\vec{p}\cdot\vec{\nabla})\vec{E} = \nabla(\vec{p}\cdot\vec{E}) \right.$$

conditionally true cond^n if p is not a func^n of position x

bcoz in magnetostatic $\vec{\nabla}\times\vec{B} \neq 0$

electrostatic $\vec{\nabla}\times\vec{E} = 0$

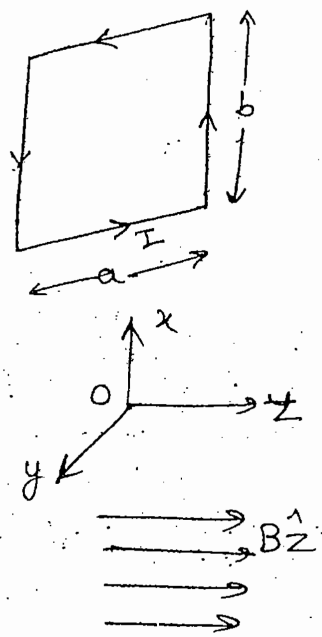
Torque :- $\boxed{\vec{N} = \vec{m} \times \vec{B} + \vec{r} \times \vec{F}_{Non-uniform}}$

Potential Energy of a Magnetic Dipole :-

In Electrostatic relation, Replace P by m & E by B ;

$U = -\vec{P} \cdot \vec{E}$

i.e. $\boxed{\vec{U} = -\vec{m} \cdot \vec{B}}$

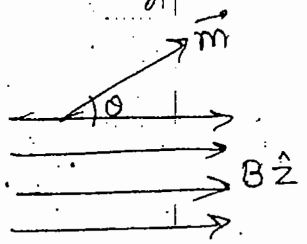


Suppose, we have a loop & current is flowing in this loop is I.

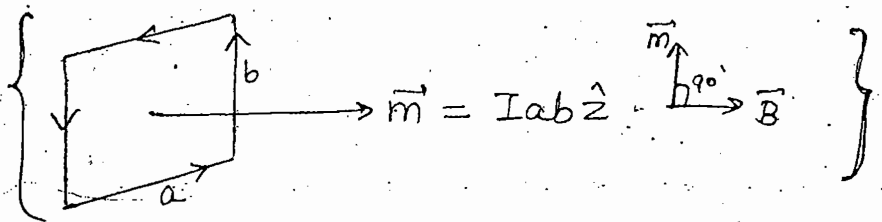
Mag. field is along z-dir^n.

If loop is straight then work done to take the loop from ∞ to in the mag field is zero, as $\vec{m}$ & $\vec{B}$ are in same dir^n.

But Now loop is tilted by the angle θ. So work done is Non-zero as angle b/w m & B is θ.



$$dW = N\, d\theta$$



Now angle changes from 90° to θ, So Work done

$$U = W = \int_{\pi/2}^{\theta} mB\sin\theta\, d\theta = -mB[\cos\theta]_{\pi/2}^{\theta}$$

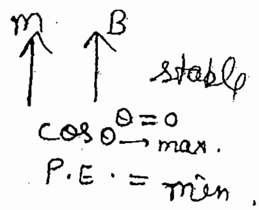
This work will store as P.E.

$$= -mB\cos\theta$$

$$\boxed{U = -\vec{m} \cdot \vec{B}}$$

When ∠ b/w $\vec{m}$ & $\vec{B}$ is 0 i.e. $U = -mB$ then P.E will be minimum.

This will be the stable configuration.



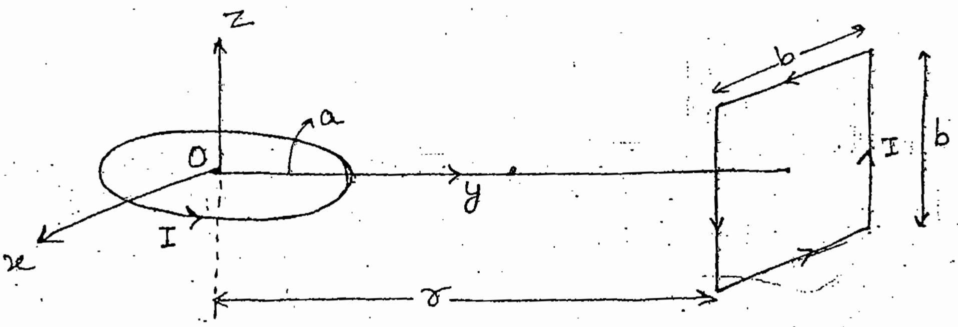
As torque rotate the plane then flux changes & it will induce electromagnetic field will do work.

(Mag. force does not do work)

Magnetic Dipole Interaction Energy :-

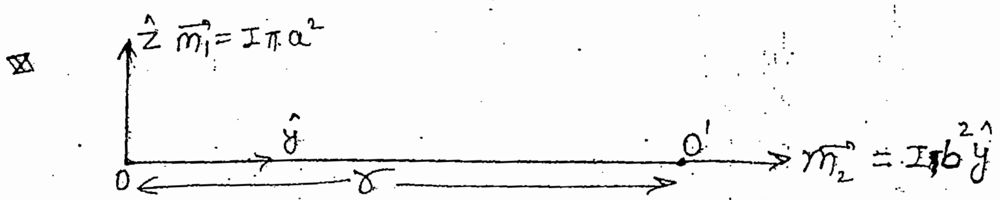
$$\boxed{U = \frac{\mu_0}{4\pi r^3}\left[\vec{m}_1\cdot\vec{m}_2 - 3(\vec{m}_1\cdot\hat{r})(\vec{m}_2\cdot\hat{r})\right]}$$

Q. Calculate the torque & the interaction energy for 2 loops as shown in the figure.



When $r > a, b$ then these 2 loops can be replaced by their mag. moment.

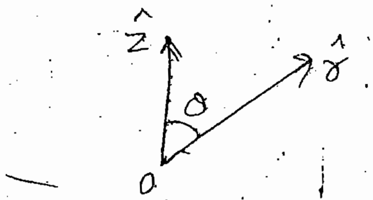
These 2 loops are equilant to 2 dipoles.



Mag. field of loop 2 applys a torque on loop 1.

$$\vec{N}_1 = \vec{m}_1 \times \vec{B}_2$$

Magnetic field on loop (2) at a distance r

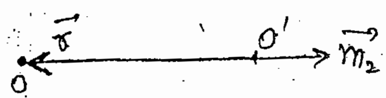


$$\vec{B}_2 = \frac{\mu_0 m_2}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right].$$

$$\vec{B}_2 = \frac{\mu_0 m_2}{4\pi r^3}\left[-2\hat{r}(-\hat{y})\right]$$

$\theta = 180°$
$\hat{r} = -\hat{y}$

$$\vec{B}_2 = \frac{2\mu_0 m_2}{4\pi r^3}\hat{y}$$



$\hat{r}$ is always away from the origin.

$$\boxed{\vec{B}_2 = \frac{2\mu_0 I b^2}{4\pi r^3}\hat{y}}$$

On loop ~~2~~, $\cancel{\vec{N}_2} = \cancel{\vec{m}_2 \times \vec{B}_1}$

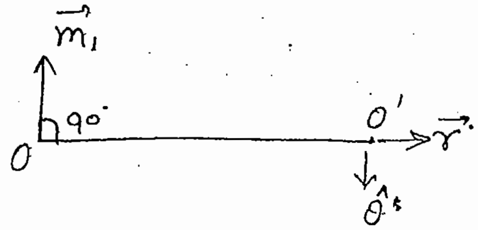
Torque $\vec{N_1} = \vec{m_1} \times \vec{B_2}$

$$\vec{N_1} = I\pi a^2 \frac{2\mu_0 I b^2}{4\pi r^3} (\hat{z} \times \hat{y})$$

$$\boxed{\vec{N_1} = \frac{\mu_0 I^2 a^2 b^2}{2r^3} (-\hat{x})}$$

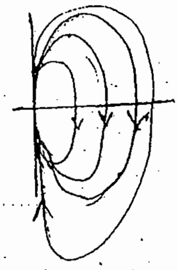
Torque on loop 2, $\vec{N_2} = \vec{m_2} \times \vec{B_1}$

$m_2 = I b^2 \hat{y}$



$$\vec{B_1} = \frac{\mu_0 m_1}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

$$\vec{B_1} = \frac{\mu_0 I \cdot \pi a^2}{4\pi r^3}\,\hat{\theta}$$



$$\hat{\theta} = -\hat{z}$$

$$\vec{B_1} = \frac{\mu_0 I \pi a^2}{4\pi r^3}(-\hat{z})$$

$$\vec{N_2} = \vec{m_2} \times \vec{B_1}$$

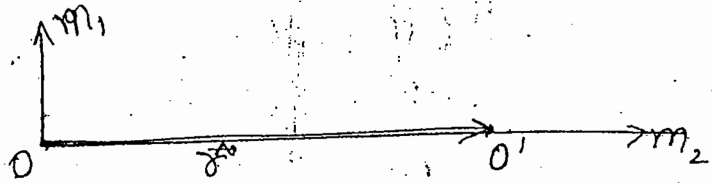
$$\boxed{\vec{N_2} = \frac{\mu_0 I^2 a^2 b^2}{4r^3}(-\hat{x})}$$

$\{\hat{y} \times (-\hat{z}) = -\hat{x}\}$

Interaction Energy :-

$$U = \frac{\mu_0}{4\pi r^3}\left[\vec{m_1}\cdot\vec{m_2} - 3(\vec{m_1}\cdot\hat{r})(\vec{m_2}\cdot\hat{r})\right]$$

Mutual angle b/w $\vec{m_1}$ & $\vec{m_2}$ is 90°.



$\vec{r}$ can be from O to O' or can be from O' to O

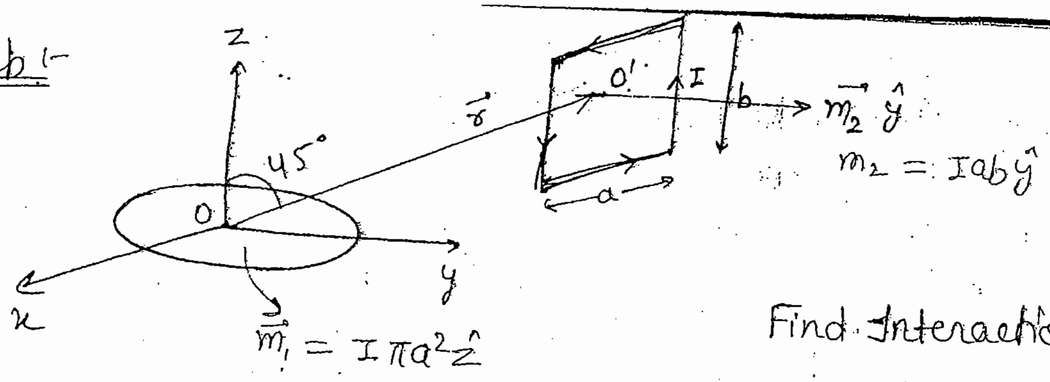
$(\vec{m_1}\cdot\hat{r}) = 0$

$(\vec{m_2}\cdot\hat{r}) \neq 0$

so $U = \frac{\mu_0}{4\pi r^3}\left[m_1 m_2 \cos 90^\circ - 3\cancel{(0)}\, 0\right]$

$$\boxed{U = 0}$$

Prob:-



Find Interaction Energy?

$$U = \frac{\mu_0}{4\pi r^3}\left[\vec{m}_1\cdot\vec{m}_2 - 3(\vec{m}_1\cdot\hat{r})(\vec{m}_2\cdot\hat{r})\right]$$

$$\vec{m}_1\cdot\vec{m}_2 = m_1 m_2 \cos 90^\circ = 0 \qquad (\angle \text{ b/w } m_1 \,\&\, m_2 = 90^\circ)$$

$$\vec{m}_1\cdot\hat{r} = m_1 \cos 45^\circ = \frac{m_1}{\sqrt{2}} = \frac{I\pi a^2}{\sqrt{2}}$$

$$\vec{m}_2\cdot\hat{r} = m_2 \cos 45^\circ = \frac{m_2}{\sqrt{2}} = \frac{Iab}{\sqrt{2}}$$

$$\therefore U = \frac{\mu_0}{4\pi r^3}\left[0 - \frac{3\pi I^2 a^3 b}{2}\right]$$

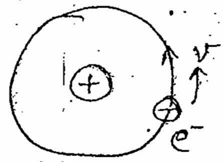
$$U = -\frac{3\mu_0 I^2 a^3 \pi b}{8\pi r^3} \Rightarrow \boxed{U = -\frac{3\mu_0 I^2 a^3 b}{8 r^3}}$$

Bound Currents:- There are 2 types of currents :-

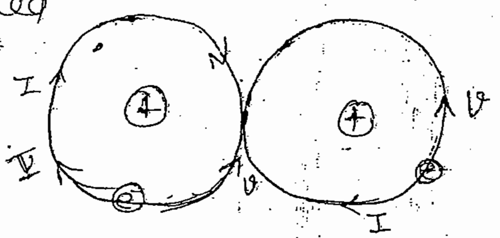
free currents - are due to motion of free charges
If inside the metal free charges are moving under pot^n then current arries.

Bound currents - arriess due to the motion of bound charges.

e^- perform a orbital motion & this motion is equalent to magnetic dipole. If a e^- is moving with velocity v around nuclei & correspond to a current, Now it 'll act as a current dipole. It is the bound current.



If another atom is placed near this atom. The current b/w them cancel out & Net current will be on surface.



Then it is called Surface bound Current &
If any atom is missing in b/w, there will be current Non-zero then net current will exist in volume. It is called Volume bound current.

$\vec{K_b} \rightarrow$ Surface bound current
$\vec{J_b} \rightarrow$ Volume bound current

As $\vec{P}$ is polarisation i.e. dipole mom. per unit volume.
Similarly in magnetostatic,
Magnetisation (M):- Magnetic mom. $\vec{m}$ per unit volume.

As $\sigma_b = \vec{P}\cdot\hat{n}$, ρ_b =

$\rho_b = -\vec{\nabla}\cdot\vec{P}$

if P is uniform then $\rho_b = 0$
P is Non uniform then $\rho_b \neq 0$

In Magnetostatics, Total current will be surface current if magnetisation is uniform.

$$\boxed{\vec{K_b} = \vec{M}\times\hat{n}}$$

If Magnetisation is Non-uniform then K_b & J_b both contribute, volume bound current

$K_b \rightarrow$ surface bound current

$$\boxed{\vec{J_b} = \vec{\nabla}\times\vec{M}}$$

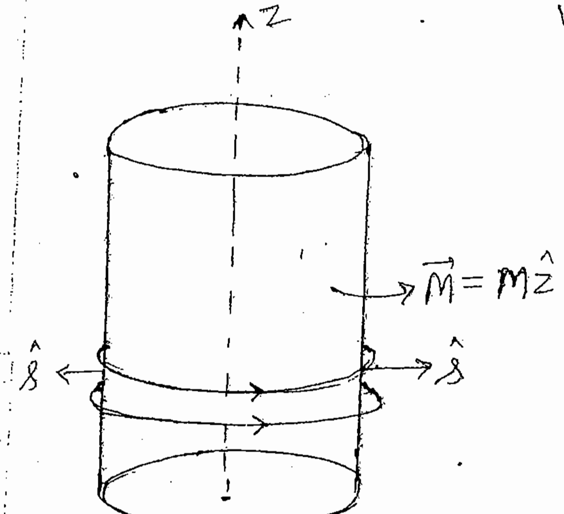
Div of Curl is zero so $\boxed{\vec{\nabla}\cdot\vec{J_b} = 0}$

Que: An infinitely long circular cylinder carries a uniform magnetization M parallel to its axis. Find surface & volume bounds current & also find mag. field inside & outside the cylinder.

$\vec{M}$ is uniform, not depanding on space co-ordinate.
No free current is present.
First find bound current, Using Ampere's law

$$\boxed{I_{enc} = I_b + I_f}$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$$



Vol. bound current, $J_b = 0$ bcoz $\vec{M}$ is uniform.

Surface bound Current,

$$\vec{K_b} = \vec{M} \times \hat{n}$$

$\perp$ vector to $\vec{M}$ will be $\hat{s}$..

$$\vec{K_b} = M(\hat{z} \times \hat{s})$$

$$\vec{K_b} = M\hat{\phi}$$

Magnetization is like mag. field.
↓ (arises from bound current). ↓ arises from current (bound as well as free)

$B \xrightarrow{\text{determines from}} I_b + I_f$

$M \rightarrow I_b$

$H \rightarrow I_f$ (free current)
↓
Mag. field Intensity

$$\boxed{\text{Note :- Curl the finger in the dir}^n \text{ of bound current, thumb will tell the dir}^n \text{ of Magnetisation.}}$$

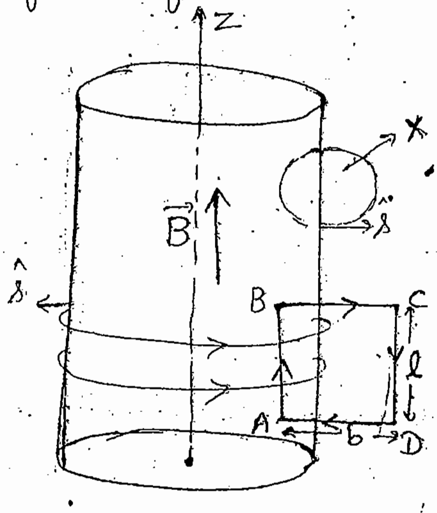
There is no free current so total current will be bound current.

It is like a solanoid. So to find the current of this infinitely long cylender we must draw a Ampirical loop. Ampirical loop should be s.t. current passes through the loop. So loop should be $\perp$ to the flow of current.

Take a <u>ractangle as amperical loop</u>.

(If we take a ~~cy~~ circular loop then dl will be different at each point of loop so B.dl will be variable. So we can not take a circle as ampirical loop).

Dir^n of loop will be according to the dir^n of mag. field $\vec{B}$.



$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{enc}$$

$$\oint \vec{B}\cdot\vec{dl} = \int_{AB} \vec{B}\cdot\vec{dl} + \int_{BC} \vec{B}\cdot\vec{dl} + \int_{CD} \vec{B}\cdot\vec{dl} + \int_{DA} \vec{B}\cdot\vec{dl}$$

$\int_{BC}$: ↓ $\angle$ b/w them is $90°$ $= 0$

$\int_{CD}$: ↓ out of cylender No current $= 0$

$\int_{DA}$: ↓ $\angle$ b/w then $= 90°$ $= 0$

<u>Note</u> :- Bound Current can never be in the dir^n of Magnetisation.

$$\oint \vec{B}\cdot d\vec{l} = \int_{AB} \vec{B}\cdot d\vec{l} = Bl$$

Surface current $= \dfrac{\text{Total current}}{\text{length} \perp \text{ to the flow}} \Rightarrow \dfrac{I}{l_\perp} = M$

$$I = M \cdot l$$

$$B\cancel{l} = \mu_0 M \cancel{l}$$

$$\boxed{\vec{B} = \mu_0 \vec{M}\ \hat{z}}$$

Mag. field through bound current.

in terms of surface current, (K) is

$$\boxed{B = \mu_0 K}$$

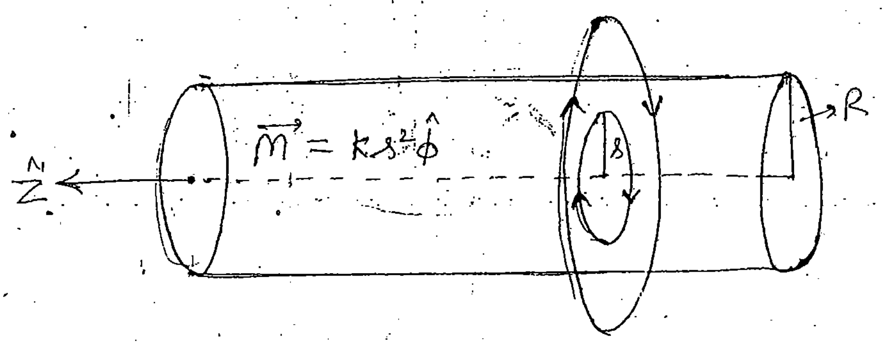
for n no. of loops ; $\vec{B} = \mu_0 n I$

$B = \mu_0 \underset{\downarrow}{n I}$

Total current / unit length

i.e. surface current

Ques:- A long circular cylender of radius R carries a magnetisation $\vec{M} = K s^2 \hat{\phi}$ where K is the constant & s is the distance from the axis, $\hat{\phi}$ is the azimuthal vector. Find the mag. field due to $\vec{M}$ inside & outside the cylender.



If $\vec{M}$ is in the $\hat{\phi}$ then bound current will be in $\hat{z}$ &

" $\vec{M}$ " " $\hat{z}$ " " " " " $\hat{\phi}$.

Vol. bound current, $\vec{J}_b = \vec{\nabla} \times \vec{M}$

$$= \frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ M_s & sM_\phi & M_z \end{vmatrix}$$

$$= \frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & sKs^2 & 0 \end{vmatrix} \qquad (M_\phi = Ks^2)$$

$$= \frac{1}{s}\hat{z}\left[\frac{\partial}{\partial s}(Ks^3)\right] = \frac{1}{s}K3s^2\hat{z}$$

$$\boxed{\vec{J_b} = 3Ks\,\hat{z}}$$

Surface bound current,

$$\vec{K_b} = \vec{M}\times\hat{n}$$

(unit vector normal to surface $\Rightarrow \hat{n} = \hat{s}$)

$$= Ks^2(\hat{\phi}\times\hat{s})\big|_{s=R}$$

$$= KR^2(-\hat{z})$$

$(\hat{\phi}\times\hat{s} = -\hat{z})$

$$\boxed{\vec{K_b} = -KR^2\hat{z}}$$

Surface bound Current & Vol. bound current are opositelly directed.

Magnetic field Inside :-

Only volume bound curent will contribute. Take a circlla Ampirical loop

Current Enclose by the circular loop,

$$I_b(\vec{J_b}) = \int \vec{J_b}\cdot d\vec{S_\perp}$$

$dS_\perp = s\,ds\,d\phi$ (z is not changing)

$$= \int_0^s\int_0^{2\pi} 3Ks\,s\,ds\,d\phi$$

$$= 3K\frac{s^3}{3}\cdot 2\pi$$

$\boxed{I_b(\vec{J_b}) = 2\pi Ks^3}$ The Ampirical loop will enclose this current.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$B\cancel{2\pi}s = \cancel{2\pi}Ks^3\mu_0$$

$$\boxed{\vec{B}_{in} = \mu_0 Ks^2\,\hat{\phi}}$$

Magnetic field Outside :- both volume & surface bound currents will contribute.

$$\rightarrow I_b(\vec{J_b}) = \int_0^R\int_0^{2\pi} 3Ks\,s\,ds\,d\phi$$

$$I_b(\vec{J_b}) = 2\pi KR^3$$

$$\rightarrow I_b(K_b) = K_b \times 1\,\text{length}$$

$$= -KR^2 2\pi R\,\hat{z} \Rightarrow I_b(K_b) = -2\pi KR^3$$

Total bound current $= 2\pi K R^3 - 2\pi K R^3$

$$\boxed{I_b(\text{Total}) = 0}$$

So $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$$\Rightarrow \boxed{B_{out} = 0}$$

So Mag field outside

Ampere's law in Magnetised Material :- The division of materials is based on how they respond to external ~~elect~~ magnetic field.

If a material is placed in mag field & there is no change in its property then this material is called Non-magnetic material.

Because Every magnetic material respond to mag field i.e. On placing it in mag field its properties changes.

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$$

Total $\vec{J} = \vec{J_b} + \vec{J_f}$

$$\therefore \nabla\times\vec{B} = \mu_0(\vec{J_b}+\vec{J_f})$$

$$\Rightarrow \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\times\vec{M} + \vec{J_f}$$

$$\nabla\times\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) = \vec{J_f}$$

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J_f}}$$

$\left\{ \frac{B}{\mu_0} - M = H \right.$

$B = \mu_0(M+H)$

This is the differential form of Ampere's law in magnetisation material,

H → Magnetic field intensity

Its M.K.S. Unit → A/m {ampere/meter}

$\vec{J_f}$ → free current density

Now, Take Surface integral of both the sides,

$$\int_s (\nabla \times H) \cdot ds = \int_s J \cdot ds$$

$$\boxed{\oint \vec{H} \cdot \vec{dl} = I_{f\,enc.}} \rightarrow \text{Integral form}$$

In Magnetostatics → H, determine only by the free current. It includes bound current in itself. Similar as D in electrostatics.

Que :- In previous Question, $M = K s^2 \hat{\phi}$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$\vec{H}_{in} = 0$
$\vec{H}_{out} = 0$ } bcoz there is no free current inside & outside. (only bound current)

So $\vec{B}_{in} = \mu_0 \vec{M}_{in}$

$$\boxed{\vec{B}_{in} = \mu_0 K s^2 \hat{\phi}}$$

$$\vec{B}_{out} = \mu_0 \vec{M}_{out} = \mu_0 \times 0$$

$$\boxed{\vec{B}_{out} = 0}$$

$$\vec{B} = \mu \vec{H}$$

$\mu \rightarrow$ permiability of the medium.

Permiability :- It is defined as

Relative permiability $\mu_r = \frac{\mu}{\mu_0}$ as $\epsilon_r = \frac{\epsilon}{\epsilon_0}$

μ_r decides the type of material.

$\mu_0 \rightarrow$ permiablity in free space.

For Diamagnetic Material, $\mu_r < 1$

Paramag. " $\mu_r > 1$

ferromag. " $\mu_r >> 1$.

Physical Meaning - If we have a hollow cylinder, wrape conducting wires over it. Current flow is constant. It'll become a solanoid. Curent flowing in conductor is free current. Mag. field intensity H inside will be Non-zero.

It is hollow before. Now if we place a iron rod inside it then, (Iron is ferromag. material)

$\vec{H}$ → No change bcoz it totally determine by free current

& $\vec{B}$ increases. bcoz of bound current

In free space $\vec{B} = \mu_0 \vec{H}$

If we place a rod then The magnetisation will be in the dirn of mag field then

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\vec{B} = \mu \vec{H} \Rightarrow B = \mu_r \mu_0 \vec{H}$$

⇒ If $\mu_r < 1$ then $B \uparrow$

$\mu_r > 1$ then $\underset{\text{can be}}{B} \uparrow$ or $\underset{\text{can be}}{B} \downarrow$ depanding upon material

while $\vec{E} \downarrow$.

⇒ Polarisation is always in the dirn of external E-field

$E_{ext} \uparrow$ - $P \uparrow$

But for a Mag. Material

We have a dia mag. matrial & apply H then

$H_{ext} \uparrow$ $M \downarrow$

for para mag. $H_{ext} \uparrow$ $M \uparrow$

Magnetic Succeptability :-

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\chi_m = \frac{M}{H} \Rightarrow \mu H = \mu_0 (H + \chi_m H)$$

$$\frac{\mu}{\mu_0} = 1 + \chi_m$$

$$\boxed{\mu_r = 1 + \chi_m}$$

$$\epsilon_r = 1 + \chi_e$$

$\epsilon_r > 1$ always but $\mu_r > 1$ or may be $\mu_r < 1$

$\mu_r < 1$ i.e. $\chi_m < 0$ then Diamag. material.

$\mu_r > 1$ i.e. $\chi_m > 0$ " Para " "

If Magnetisation is proportional to first power of H then these types of materials are called Linear Magnetic Materials. i.e. $M \propto H$

defination of susceptability $\chi_m = \frac{M}{H}$ is true only for linear mag. material

Dia, para are linear mag. material but ferro mag. material is not linear.

$$\chi_m \neq \frac{M}{H} \quad (\text{for ferro})$$

for ferro mag. material $\boxed{\chi_m = \frac{\partial M}{\partial H}}$

for ex :- $M = \tanh\left(\frac{\mu H}{KT}\right)$

M is not linearly depanding on H. bcoz $\tan x \simeq x$ only for small value of x. [i.e. $x<1$]

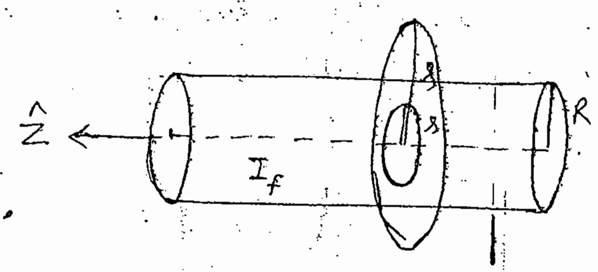
It is ferro mag. material.

Que :- A long Copper rod of radius R carries a uniformly distributed free current I. find the H inside & outside the rod.

Current $I = J \times (\text{Area})$

$$I = J \pi R^2$$

$$J = \frac{I}{\pi R^2}$$



Inside :-

$I_{enc} = \int \vec{J} \cdot d\vec{s} = J \cdot \text{area}$ (J is uniform)

$$= \frac{I}{\cancel{\pi} R^2} \cancel{\pi} s^2$$

$$\boxed{I_{enc} = \frac{I s^2}{R^2}}$$

$$\oint \vec{H} \cdot d\vec{l} = I_{f\,enc}$$

$$H \cdot 2\pi s = \frac{I s^2}{R^2}$$

$$\Rightarrow \boxed{\vec{H} = \frac{I s}{2\pi R^2} \hat{\phi}}$$

disⁿ of H is same as B.

So $\boxed{H_{in} \propto s}$

Outside :-

$$I_{enc} = I$$

$$H \cdot 2\pi s = I$$

$$\boxed{\vec{H} = \frac{I}{2\pi s}\hat{\phi}}$$

If permiablity of that medium is μ then mag field will be $B = \mu H \Rightarrow \boxed{\vec{B}_{in} = \frac{\mu I s}{2\pi R^2}\hat{\phi}}$

(but can't determine Bin Here bcoz μ is not given)

We can determine Bout bcoz outside permiability is μ_0

So $\boxed{B_{out} = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$

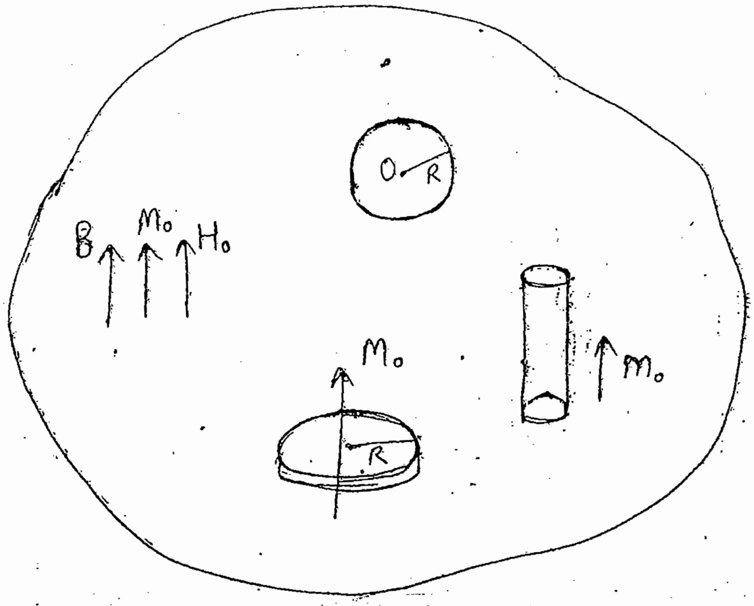
Ques* :- A infinitely long cylender of radius R carries a magnetisation $\vec{M} = ks\hat{z}$ parallel to its axis where s is the distance from the axis of the cylender. There is no free charge anywhere. Find surface & volume bound currents. Also find mag-field inside & outside the cylender by using 2 different methods → (i) by bound currents & (ii) by H.

Ques :- Suppose the field inside a large piece of magnetic material is B_0 such that $H_0 = \frac{B_0}{\mu_0} - M$.

(a) Now a small spherical cavity is hollowed out of the material, find the mag field at the centre of the cavity.

(b) Do the same for a long niddle shaped cavity parallel to M.

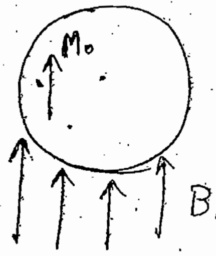
(c) Do the same for a ~~waffour~~ wafer shaped cavity $\perp$ to $\vec{M}$.



When B & M are in same dir^n then mag. field inside the material will decrease – (Para & ferro mag. material B & M are in same dir^n but in dia dir^n of B & M are opposite)

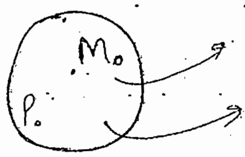
~~sphere~~ In free space of ~~Mate~~ sphere

$$\vec{B} = \vec{B_0} - \frac{2}{3}\mu_0 \vec{M_0} \quad \text{i.e. } B_{in} = \frac{-2}{3}\mu_0 \vec{M_0}$$



~~This is~~ the mag. field of uniformly magnetised sphere,

$$B_{in} = \frac{2}{3}\mu_0 \vec{M_0}$$ (vol. current in + = surface current in)



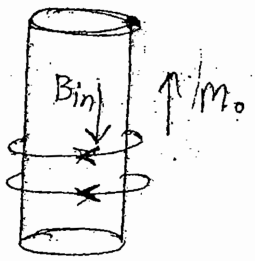
$$E_{in} = -\frac{1}{3}\frac{P_0}{\epsilon_0}$$

~~So~~ Mag. field inside the spherical cavity

$$\vec{B_0} - \frac{2}{3}\mu_0 \vec{M_0}$$

Niddle shape :-

$$\vec{B} = \vec{B_0} + \vec{B_{in}}$$



In free space, Magnetisation = 0

Outside Magnetisation M_0 then we have to place $-M$ mag. inside the cavity to make the total mag. zero. (Magnetisation → by superposition)

Mag. field inside the cavity :

$$\boxed{\vec{B_{in}} = \vec{B_0} - \mu_0 \vec{M_0}}$$

{ If Thumb → $\vec{B}$, fingers ~~tell the~~ dir^n of current

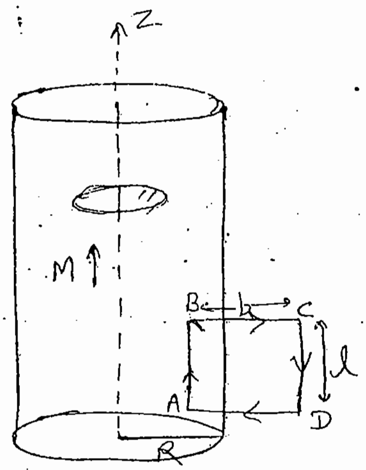
* If the length of rod is long → more effect of surface current (in magnetostatics)
If length of rod is long (in Electrostatic) → surface charge are far apart & effect will be less.

Disc (Waffer shape) :- Current will flow on the surface of disc → Total current = surface current × ⊥ length

In disc, amount of bound ~~charge~~ current is very less bcoz ⊥ length → 0
so bound current → 0, Mag. field $\vec{B} = \vec{B_0}$ $\left\{ E = E_0 + \frac{P}{\epsilon_0} \right\}$

In case of disc, induce mag. field = 0 bcoz length is very small, $\vec{B}_{in} = 0$
so total mag. field $\boxed{\vec{B} = \vec{B_0}}$

Ques * :- $\vec{M} = Ks\,\hat{z}$



Volume bound current

$$\vec{J_b} = \vec{\nabla} \times \vec{M}$$

$$= \frac{1}{s} \begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & Ks \end{vmatrix}$$

$$= -\frac{1}{s}\, s\hat{\phi}\, \frac{\partial}{\partial s}(Ks)$$

$$\boxed{\vec{J_b} = -K\hat{\phi}}$$

Surface bound Current

$$\vec{K_b} = \vec{M} \times \hat{n}$$

$$= Ks(\hat{z} \times \hat{s}) = Ks\hat{\phi}$$

$$\boxed{\vec{K_b} = Ks\hat{\phi}} \Rightarrow \boxed{\vec{K_b} = KR\hat{\phi}}$$

$\int ds \cdot \int dz$

• Magnetic field Inside by bound current :-

$$I_b(\vec{J_b}) = \int \vec{J_b} \cdot d\vec{S_\perp} = \int_0^s \int_0^l -K\, ds\, dz = -K\, s\, l$$

$$\oint B\cdot dl = \int_{AB} B\cdot dl + \int_{BC} B\cdot dl + \int_{CA} B\cdot dl + \int_{DA} B\cdot dl$$

$$= Bl + 0 + 0 + 0 = Bl$$

So $\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{enc}$

$$B\cdot l = \mu_0 (+Ksl)$$

$$\boxed{B = +\mu_0 K s \hat{z}}$$

By H, There is No free charge so $\vec{H_{in}} = 0$

$$\vec{B_{in}} = \mu_0(\vec{H_{in}} + \vec{M_{in}})$$

$$\vec{B_{in}} = \mu_0(0 + Ks\hat{z})$$

$$\boxed{\vec{B_{in}} = \mu_0 K s \hat{z}}$$

Magnetic field Outside by bound current;

$$I_b(\vec{J_b}) = \int_0^R\int_0^l -K\,ds\,dz = -KRl$$

$$I_b(\vec{K_b}) = K_b \times l_\perp = KR \times l = KRl$$

$$I_{enc} = 0$$

So $\boxed{B_{out} = 0}$

By H, $H_{out} = 0$

so $B_{out} = \mu_0 M_{out}$

$B_{out} = \mu_0 \times 0 \Rightarrow \boxed{B_{out} = 0}$

Boundry Conditions on $\vec{H}$:-

$$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \boxed{B^{\perp}_{above} = B^{\perp}_{below}}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \Rightarrow \boxed{B^{\parallel}_{above} - B^{\parallel}_{below} = \mu_0 K}$$

Tangential comp. of mag. field is discontinous by the amount $\mu_0 K$.

$$\vec{\nabla}\cdot\vec{H} = -\vec{\nabla}\cdot\vec{M}$$

$$\boxed{H^{\perp}_{above} - H^{\perp}_{below} = M^{\perp}_{below} - M^{\perp}_{above}}$$

$B = \mu_0(\vec{H}+\vec{M})$
$\nabla\cdot B = 0$
$\Rightarrow \mu_0\nabla\cdot(H+M) = 0$
$\nabla\cdot H = -\nabla\cdot M$

$\nabla\cdot H$ can be 0 only if $\vec{H}$ is not uniform.

$$\vec{\nabla}\times\vec{H} = \vec{J}_f \qquad J_f \to \text{free vol. charge density}$$

$$\boxed{H^{\parallel}_{above} - H^{\parallel}_{below} = K_f}$$

Tangential comp. of H is discontinous by the amount K_f (free current)

If there exist a boundries whose permiability are different (i.e. mag. properties are different) for such a boundries

① μ_1
dielectric
② μ_2

$$K_f = 0$$

then tangential comp. of H i.e. $H^{\parallel}_{above} = H^{\parallel}_{below} = 0$

Que :- At the Interface b/w two linear magnetic materials the mag. field lines bend. Show that

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{\mu_2}{\mu_1}$$

Assuming that, there is No free current at the boundry.

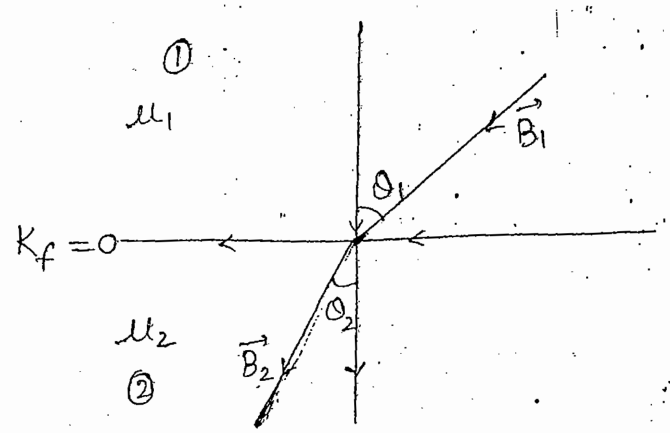
If no free current then boundry cond^{n}s are $H^{\parallel}$ & $B^{\perp}$ are continous.

$B^{\perp} \to$ always continous
$H^{\parallel} \to$ conditionally "

i.e. B.Cs are
$$H_1^{\parallel} = H_2^{\parallel}$$
$$B_1^{\perp} = B_2^{\perp}$$

$B_1^{\parallel} = \mu_1 H_1^{\parallel}$

$B_2^{\parallel} = \mu_2 H_2^{\parallel}$



Now $B_1^{\perp} = B_2^{\perp}$

$\Rightarrow \boxed{B_1 \cos\theta_1 = B_2 \cos\theta_2}$ —— (1)

& $H_1^{\parallel} = H_2^{\parallel}$

$\Rightarrow H_1 \sin\theta_1 = H_2 \sin\theta_2$

$\Rightarrow \boxed{\frac{B_1}{\mu_1} \sin\theta_1 = \frac{B_2}{\mu_2} \sin\theta_2}$ —— (2)

Deviding (2) by (1) $\Rightarrow$

$$\frac{B_1}{\mu_1} \Big/ B_1 \tan\theta_1 = \frac{B_2}{\mu_2} \Big/ B_2 \tan\theta_2$$

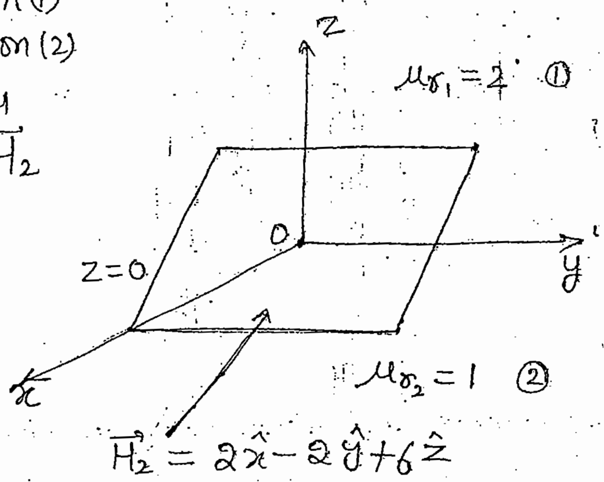
$$\frac{1}{\mu_1} \tan\theta_1 = \frac{1}{\mu_2} \tan\theta_2$$

$$\Rightarrow \boxed{\frac{\tan\theta_2}{\tan\theta_1} = \frac{\mu_2}{\mu_1}} = \frac{\epsilon_2}{\epsilon_1} \text{ (for electric lines)}$$

for magnetic lines.

Ques :- Given $\mu_r = 2$ in region (1)
$\mu_r = 1$ in region (2)
Apply a mag field which is tilted by some angle & $\vec{H_2}$ exist in region (2)

$\vec{H_2} = 2\hat{x} - 2\hat{y} + 6\hat{z}$



Find (i) $\vec{B_1}$ (ii) $\vec{H_1}$

No free current at the surface i.e. $K_f = 0$

If $K_f = 0$ then B.C. are
$H_1^{\parallel} = H_2^{\parallel}$ & $B_1^{\perp} = B_2^{\perp}$

$H_2 = 2\hat{x} - 2\hat{y} + 6\hat{z}$

$\Rightarrow H_{2n} = 6\hat{z}$ & $H_{2t} = 2\hat{x} - 2\hat{y}$

$B_1^{\perp} = B_2^{\perp} \Rightarrow B_{1n} = B_{2n} \Rightarrow B_{1n}$

$$① \boxed{H_1'' = H_2''} \Rightarrow H_{1t} = H_{2t} = 2\hat{x} - 2\hat{y} = H_1''$$

$$B_1^{\perp} = B_2^{\perp} \Rightarrow B_{1n} = B_{2n}$$

$$\mu_1 H_{1n} = \mu_2 H_{2n} \text{ ——— } ①$$

$$B_2^{\perp} = \mu_2 H_2^{\perp}$$

$$= \mu_0 \mu_{r_2} H_2^{\perp}$$

$$B_2^{\perp} = \boxed{6\mu_0 \hat{z} = B_1^{\perp}}$$

$$B_1^{\perp} = \mu_1 H_1^{\perp} = \mu_0 2 H_1^{\perp} \Rightarrow H_1^{\perp} = 3\hat{z}$$

$$\text{so } \vec{H}_1 = H_1'' + H_1^{\perp}$$

$$\boxed{\vec{H}_1 = 2\hat{x} - 2\hat{y} + 3\hat{z}}$$

$$① \Rightarrow \frac{B_1''}{\mu_1} = H_2'' = H_1''$$

$$\frac{B_1''}{\mu_0 \mu_{r_1}} = H_2''$$

$$B_1'' = 2\mu_0 H_2''$$

$$B_1'' = \mu_0(4\hat{x} - 4\hat{y})$$

$$\boxed{\vec{B}_1 = \mu_0(4\hat{x} - 4\hat{y} + 6\hat{z})}$$

Note :-

$$\vec{B}_1 = \mu_1 \vec{H}_1$$

$$= \mu_0 \mu_{r_1} \vec{H}_1$$

$$\vec{B}_1 = \mu_0(4\hat{x} - 4\hat{y} + 6\hat{z})$$

Ques ① A long wire has a circular cross-section with radius a the current density in the wire is $J(r) = J_0\left(\frac{a^2 - r^2}{a^2}\right)$ where r is the distance from the axis. Calculate :-

(i) Total current in the wire

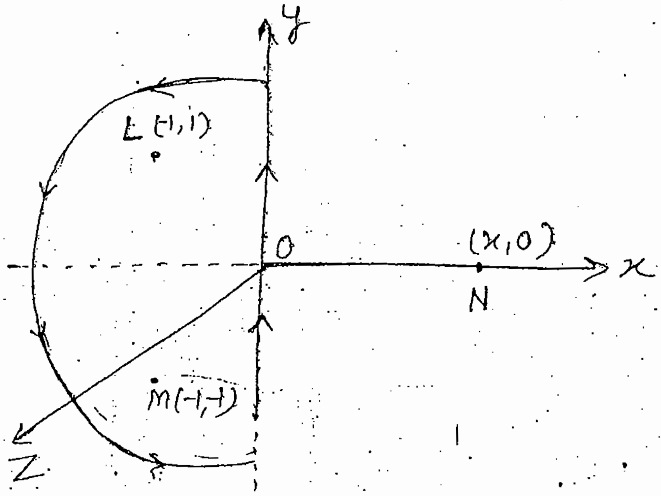
(ii) Mag. field inside & outside the wire

#

Q.2 :- Consider 2 infinitely long wires parallel to z-axis carrying the same current I. One of the wire passes through the point L with co-ordinates (−1, 1) and other through point M with co-ordinates (−1, −1) in the x-y plane as shown in the figure. The dirⁿ of the current in both the wires is in +ve z-dirⁿ.

Find :

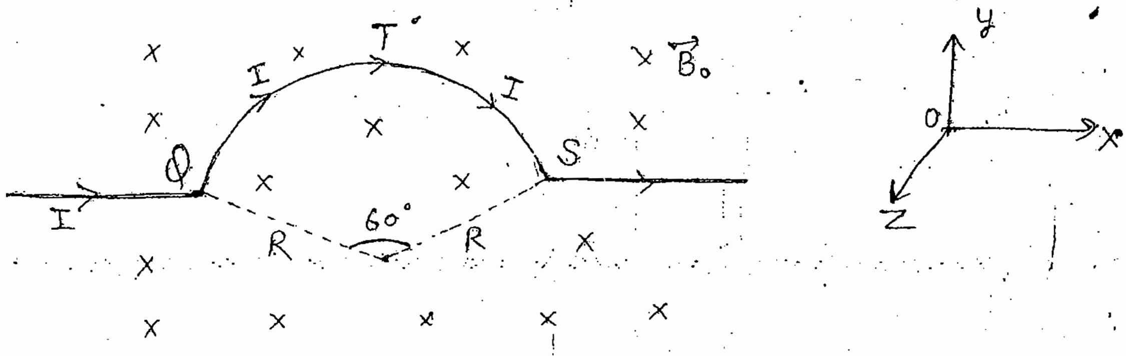
(i) the value of $\oint \vec{B}\cdot\vec{dl}$ for a loop shown in the figure.

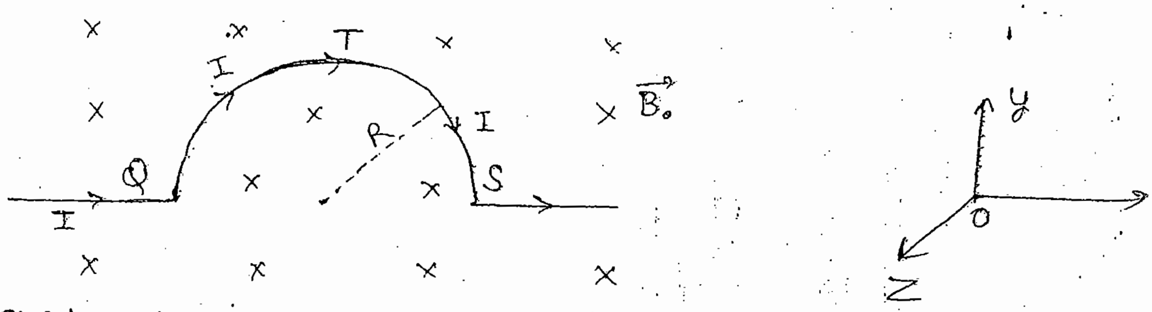


(ii) A IIIrd long wire carrying current I also perpendicular to the x-y plane is placed at the point N with co-ordinate (x, 0) s.t. mag. field at the origin is doubled.

Find the value of x & dirⁿ of the current in IIIrd wire.

Q.3 :- A circular Arc QTS is kept in an external magnetic field $\vec{B_0}$ as shown in the figure. The arc carries a current I. Find the force on the ~~earth~~ arc.



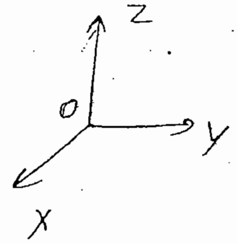


mag. field → into the page

Q.3 :- (i) $\vec{dl_1} = dy\,\hat{y} + dz\,\hat{z}$

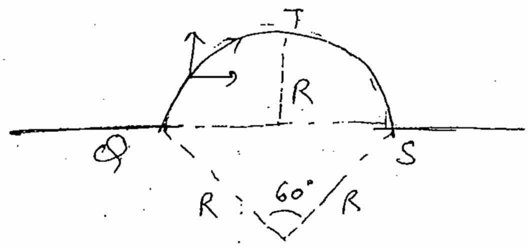
$$\vec{dl_2} = dy\,\hat{y} + (-dz\,\hat{z})$$

$$\vec{dl_1} + \vec{dl_2} = 2dy\,\hat{y}$$



$$\vec{F} = I\int \vec{dl} \times \vec{B}$$

$$= I\,2B\int_0^{R/2} dy\,\hat{z}$$



$$\boxed{\vec{F} = IBR\,\hat{z}}$$

(ii) $\vec{F} = I\int \vec{dl} \times \vec{B}$

$$= I\,2B\int_0^{R} dy\,\hat{z}$$

$$\boxed{\vec{F} = 2IBR\,\hat{z}}$$

Electrodynamics

In electrodynamics,
Changing electric & magnetic field with time.
Now, the fields are time variable.
Static mag. field is produced by steady current.
" electric " " " charge at rest.

Steady Current – charges are not accumulating anywhere.
If current I is the fun^n of time then Magnetic field will also be the fun^n of time.

$$I(t) \longrightarrow B(t)$$

& If Mag. field is time dependent then Mag. field & elec. field are dependent on each other.
Time dependent mag. field can produce time dependent Elec. field
" " Elec. " " " " Mag. "

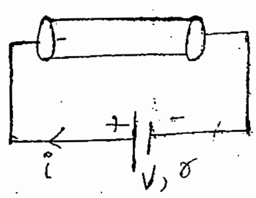
$$I(t) \rightarrow B(t) \rightarrow E(t)$$

ElectroMotive force (emf) :-

Electromotive force is Work done per unit charge.
emf is denoted by $\mathcal{E}$,

$$\mathcal{E} = -\int \vec{E}\cdot d\vec{l}$$

$\vec{F}\cdot d\vec{l}$ is Work done ~~/unit charge~~
$\vec{E}\cdot d\vec{l}$ Work done per unit charge
If we have a wire connected to battery then it is conducting & battery supply the force then free charge in wire are in motion & produce work. This work is done by battery. This work done is emf.
If battery pot^n is $V - iR$
Internal resistance is r

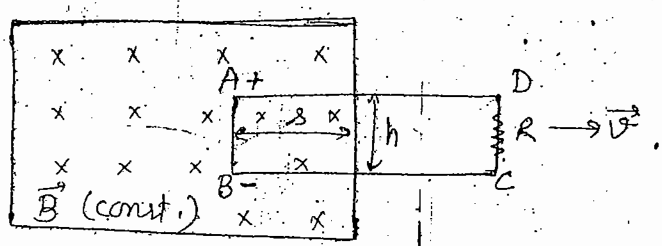


then By Bettery do work by this pot^n $V-iR$. So emf or work done per unit charge is $\Rightarrow$ emf $\mathcal{E} = V - ir$

Motional Electromotive force (emf) :-

Generators uses motional emf concept to generate power. This type of emf arrises when a conducting loop moves in a constant mag. field i.e. $\vec{B}$ remains constant with time.

Let us consider a wooden block. In this we have a closed loop. A resistance is connected with loop.



If someone is pulling the loop then Mag. flux changes.

∵ Mag. flux passes through a closed surface is always zero.

$$\phi_m = \int_S \vec{B} \cdot d\vec{s} \qquad \{ \phi_m = \oint \vec{B} \cdot d\vec{s} = 0$$

$$\mathcal{E} = -\frac{d\phi_m}{dt}$$

This is closed loop but open surface bcoz closed surface bound some volume.

It works on Faraday's law.

Acc. to Faraday law,

Changing flux always induces the emf & dir^n of emf is s.t. it oppose the change.

If flux is ↑ then dir^n of emf is s.t. try to ↓ it. i.e. we take −ve sign.

i.e. $$\boxed{\mathcal{E} = -\frac{d\phi_m}{dt}} \longrightarrow \text{Faraday's law}$$

$\Rightarrow$ Change in flux can be of 3 types.

(i) Either change $\vec{B}$, or (ii) change area ds or (iii) change the angle b/w $\vec{B}$ & $d\vec{s}$. $\quad \phi_m = \int_S B\, ds \cos\theta$

In Case of Motional emf,
B is always constant.
So flux can be changed due to either by changing area or by change angle b/w them.
If we pull the loop out then s will varry so area will change.
To find motional emf, we use formula,

$$\boxed{\mathcal{E} = \oint \vec{f}_{mag} \cdot d\vec{l}}$$

$\vec{f}_{mag}$ → Magnetic force per unit charge.
⇒ In the present Case, when loop is pulled with vel. $\vec{v}$ then amount of emf = ? Current in resistance & dir = ?
If a charge is moving with vel. $\vec{v}$ in mag. field $\vec{B}$ then force on charge

$\vec{f}_{mag} = \vec{v} \times \vec{B}$ if charge is +ve

$f_{mag} = \vec{B} \times \vec{v}$ if " " -ve

Here Conside charge is +ve. for +ve charge the dir^n of f_{mag} will be upward.
& force on e^- will be downward,

A → +ve
B → -ve

Current will flow due to motion of charges.
Now part AB will behave like a battery. & Current will flow from +ve to -ve terminal. i.e. A → B.
If AB is perfect conducting wire then its resistance = 0.
Hence AB part of loop is only contributing to the emf,

$$\mathcal{E} = \oint \vec{f}_{mag}\, d\vec{l}$$

$$= vB \int dl$$

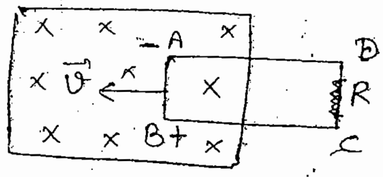
$$\boxed{V = \mathcal{E} = vBh}$$

& current $V = iR \Rightarrow i = \frac{V}{R}$

$$\boxed{i = \frac{vBh}{R}}$$

& dir^n of flow is from D → C

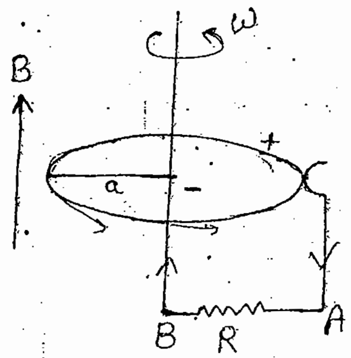
- If loop is pulling in opposit dir^n



force on +ve charge downward
" " e^- upward.
then A → −ve & B → +ve
& Now current will flow from C → D.

Que:- A metal disc of radius 'a' rotates with angular velocity ω about vertical axis through a uniform mag field $\vec{B}$ pointing in the upward dir^n. A circuit is made by connecting one end of the resistor to the axle & other ~~two~~ to the sliding contact which touches the outer edge of the disc. Find the current in the resistor with dir^n.

If charges are not in motion then no change in flux & there will be No emf.
Emf induces due to motion of charges.
In present case due to the motion of charges, their area is changing bcoz disc is rotating & charges are displacing from their position.



$$\vec{\omega} = \omega\hat{z}$$

$$\vec{v} = \vec{\omega}\times\vec{r}$$

$$= \omega r(\hat{z}\times\hat{r})$$

$$\boxed{\vec{v} = \omega r\hat{\phi}}$$

$\vec{r}$ → radius vector of the disc having dir^n $\hat{r}$.

for charges ~~of~~ on the circumference the speed v is constant (same) but for all other charges speed is different.

force per unit charge,

$$\vec{f}_{mag} = \vec{v} \times \vec{B}$$
$$= \omega r \hat{\phi} \times B \hat{z}$$
$$= \omega r B (\hat{\phi} \times \hat{z})$$
$$\boxed{\vec{f}_{mag} = \omega r B \hat{r}}$$

B'coz of this force, charges move towards outer edge so outer edge become +ve terminal of battery & centre " " −ve " "

$$\mathcal{E} = \int_0^a \omega B r \, dr$$
$$\boxed{\mathcal{E} = \frac{\omega B a^2}{2}}$$

The Current flow, $I = \frac{\mathcal{E}}{R}$

$$I = \frac{\omega B a^2}{2R}$$

Dir^n of the current will be $A \to B$

- If disc rotate oppositely then dir^n of v will be opposite & dir^n of force opposite & then +ve charge on centre & −ve on surface so terminal changes.
- The Current flow on External Circuit is from +ve to −ve terminal.

Ques 1- A metal bar of mass m slides frictionlessly on two parallel conducting rails. A distance l apart a resistor R is conected across the rails & a uniform mag. field $\vec{B}$ is pointing into the page.

(A) find dir^n & magnitude of the current in the resistor,

(B) Magnetic force on the metal bar with dir^n.

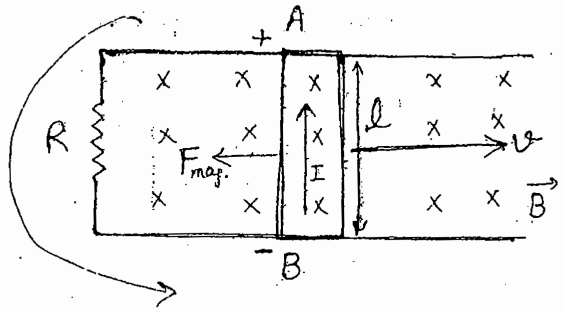
(C) If V_0 is the speed at $t=0$ time. find the speed at later time t of the conducting bar.

(A)

For +ve charges push upward
−ve " " down

So A → +ve
B → −ve

Current will flow towards resistance as this is open circuit from one side.



emf, $\varepsilon = \int \vec{f}_{mag} \cdot \vec{dl}$

$$\boxed{\varepsilon = vBl}$$

Current $I = \frac{\varepsilon}{R} \Rightarrow \boxed{I = \frac{vBl}{R}}$

dir^n → downwards

Inside the rod, current from B → A

Metal Bar does not have any resistance. So total resistance is R on whole circuit.

(B) If a wire of length l & current flowing in it is I then Magnetic force will be

$$\vec{F}_{mag} = I\int \vec{dl} \times \vec{B}$$

$$= \frac{vBl}{R} \cdot lB$$

$$\boxed{\vec{F}_{mag} = \frac{vB^2l^2}{R}}$$

dir^n → to the left

(C) Bar is moving in dir $\longrightarrow$
& $\vec{F}_{mag}$ is in dir^n $\longleftarrow$

Both will try to stop the bar.

If m be the mass of bar & v be the vel. then mechanical force

$$m\frac{dV}{dt} = -\frac{VB^2l^2}{R} \Rightarrow \frac{dV}{dt} = -\frac{VB^2l^2}{mR}$$

$$\frac{dV}{V} = -\frac{B^2l^2}{mR}dt$$

$$\ln v = -\frac{B^2 l^2 t}{Rm} + C$$

At $t = 0$, $v = v_0$

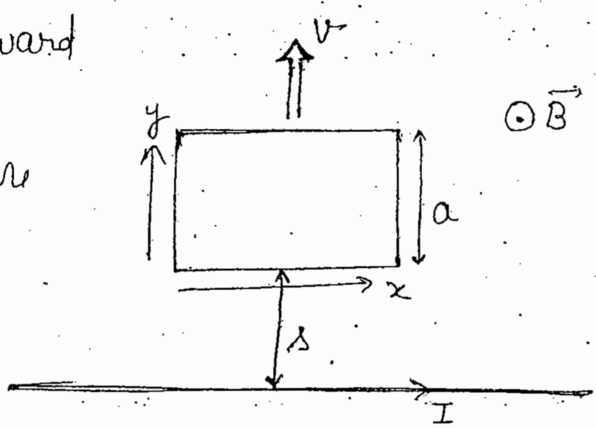
$$\ln\left(\frac{v}{v_0}\right) = -\frac{B^2 l^2}{Rm} t$$

$$\boxed{v = v_0 e^{-\frac{B^2 l^2 t}{Rm}}}$$

velocity is exponentially ↓ with time.
As B ↑, the decrease in v will be more.

Que:- A square loop of wire of side 'a' lies on a table at a distance s from a very long straight wire which carries a current I. If someone pulls the loop away from the wire at speed v. What is the induced emf in the loop. Also find the dirⁿ of induced current.

Square loop is pulled upward dirⁿ.



Flux passes through the square loop

$$\phi_m = \int B \cdot ds$$

B → mag. field of wire
ds → area of loop

Mag. field of a wire is

$$B = \frac{\mu_0 I}{2\pi s} \hat{\phi}$$

dirⁿ of Mag. field → It is coming out.

Area, $ds = dx\,dy$

$$\phi_m = \int_0^a \int_s^{s+a} \frac{\mu_0 I}{2\pi y} dx\,dy$$

{limits of y → s to $s+a$

Here variation of mag field is along y dirⁿ

$$\boxed{\phi_m = \frac{\mu_0 I}{2\pi} a \ln\left(\frac{s+a}{s}\right)}$$

If flux → const then No emf.

s is variable. If loop is away from wire then $s \uparrow$ & then flux $\downarrow$. (area of loop & angle $\rightarrow$ constant & mag. field changes) depending on s

So Here flux is not Constant.

Now, Emf,

$$\mathcal{E} = -\frac{d\Phi_m}{dt}$$

$$= -\frac{\mu_0 I a}{2\pi}\left(\frac{s}{s+a}\right)\left(\frac{-a}{s^2}\right)\frac{ds}{dt}$$

$\frac{ds}{dt} \rightarrow$ Rate of change of distance i.e. velocity v.

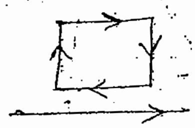
$$\boxed{\mathcal{E} = \frac{\mu_0 I a^2}{2\pi s(s+a)} v}$$

This much emf will enduce in the loop.

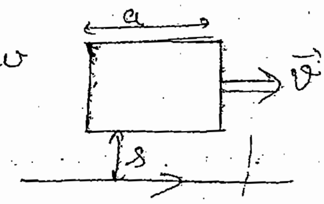
Dir^n of the induce current :- As loop is moving away from wire $\rightarrow s \uparrow$ - Here flux is decreasing so induce current will try to increase it, & flux is decreasing due to dec. in mag. field so Induce current will try to inc. the mag. field. To do so, it will develop a mag. field opposite to original.

So Dir^n of Current – Anticlockwise

- If we are pulling the loop towards the wire, then flex $\uparrow$, $\vec{B} \uparrow$, so current try to dec. the $\vec{B}$. Dir^n of Current - Clockwise



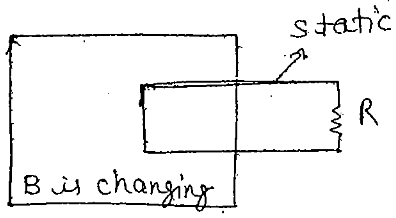
- If we pull the loop parallel to wire then there will be No change in flex. Now flex will pass through the loop but change in flux $= 0$ so emf $= 0$



* If wire is finite then on the ends of wire, mag. field will be complicated.
* If loop pulling away from wire, dir^n of I $\rightarrow$ Anti
  " " towards " " " $\rightarrow$ clockwise
* Induced emf try to dec. the flex

Magnetic Flux with changing Magnetic field :- (II case)

Loop is static but B is changing.



emf,

$$\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_m}{dt}$$

This is the integral form of Faraday's law.

$\vec{E} \rightarrow$ This is not constant electrostatic field. (i.e. Not static)

so $\oint \vec{E}\cdot d\vec{l} \neq 0$ Here.

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{S} \qquad \phi_m = \int \vec{B}\cdot d\vec{S}$$

Loop is static i.e. area is not changing

so $\frac{\partial S}{\partial t} = 0$

$$\oint E\cdot dl = -\int \frac{d\vec{B}}{dt}\cdot d\vec{S}$$

$$\Rightarrow \int_S (\nabla \times E)\cdot d\vec{S} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

$$\Rightarrow \int_S \left(\vec{\nabla}\times\vec{E} + \frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{S} = 0$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

So we have induced electric field only if $\vec{B}$ is changing with time.

& E is not electrostatic here bcoz for electrostatic E, $(\nabla \times E = 0)$

So $\vec{E}$ is not conservative as well,

$\vec{E} \rightarrow$ Non conservative & Non electrostatic

No scalar potn is defined for that electric field. This ele. field is Induced electric field, & The properties of induced elec. field are same as prop of mag. field as it arises from mag. field.

E - field lines $\rightarrow$ Open Curve
B - field lines $\rightarrow$ Close Curve

Field lines of Induced electric field - Closed Curve

& $\boxed{\vec{\nabla}\cdot\vec{E} = 0}$

bcoz there is no charge correspond to it otherwise $\nabla\cdot E = \frac{\rho}{\epsilon}$

~~E~~ Induced Elec. field is $\rightarrow$

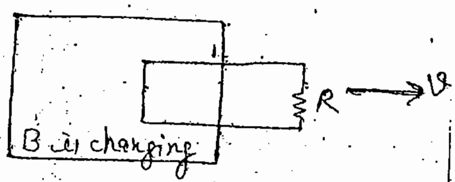
$\rightarrow$ Non Conservative
$\rightarrow$ Non electrostatic
$\rightarrow$ Closed Curve
$\rightarrow$ Div $\vec{E} = 0$

emf, $\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_m}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{S}$

$$= -\int \frac{d\vec{B}}{dt}\cdot d\vec{S}$$ (if B is not changing with t)

Case III :- If loop & ~~coil~~ mag. field both are changing then

$$\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{S}$$



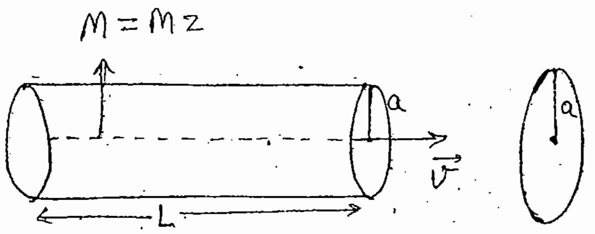
$$= -\int \frac{d\vec{B}}{dt}\cdot d\vec{S} + \oint f_{mag}\cdot d\vec{l}$$

$$= -\int \frac{\partial\vec{B}}{\partial t}\cdot d\vec{S} + \oint(\vec{v}\times\vec{B})\cdot d\vec{l}$$

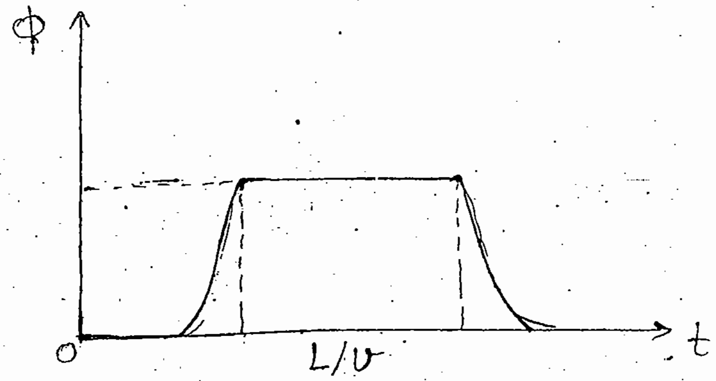
Ques :- A long cylindrical magnet of length 'L' & radius 'a' carries a uniform magnetization $\vec{M}$ parallel to its axis. It passes at a constant velocity v through a circular ring of slightly larger diameter. Graph the induce emf in the ring as a func^n of time.

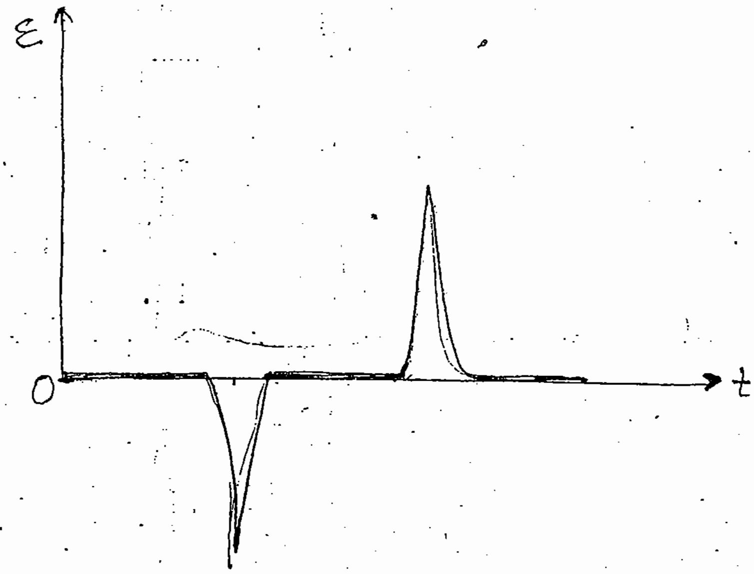
If Magnetization of ~~coil~~ magnet is M then mag. field of a magnet is

$$\boxed{\vec{B} = \mu_0 M\hat{z}}$$



When magnet come close to ring then flux ↑. When it is in the ring then flux is constant through the passing of the magnet & then it will dec. bcoz it leave the ring.
So emf occur when it enters & leaves the ring.



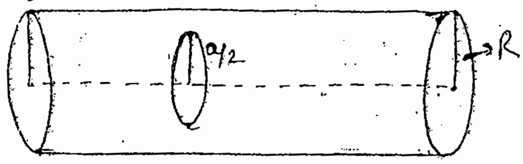


Qu :- A long solenoid of radius a is driven by an alternating current s.t. field inside the solenoid is

$$\vec{B}(t) = B_0 \cos\omega t \, \hat{z}$$

A circular loop of wire of radius $a/2$ & resistance R is placed inside the solenoid & coaxial with it. Find the current induce in the loop as a funⁿ of time.

$\vec{B}(t) = B_0 \cos\omega t \, \hat{z}$



B is not a func^n of distance i.e.
B uniform. So flux

$$\phi = B \cdot \pi \left(\frac{a}{2}\right)^2$$

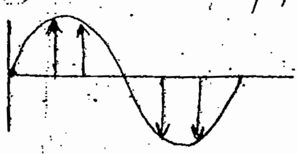
$$= B \frac{\pi a^2}{4}$$

$$\boxed{\phi = \frac{B_0 \pi a^2}{4} \cos\omega t}$$

$$\mathcal{E} = -\frac{d\phi}{dt} = \frac{\omega B_0 \pi a^2}{4} \sin\omega t \Rightarrow \boxed{\mathcal{E} = \frac{B_0 \pi a^2 \omega}{4} \sin\omega t}$$

$$\text{current } I = \frac{\mathcal{E}}{R} \Rightarrow \boxed{I = \frac{B_0 \omega \pi a^2}{4R} \sin\omega t.}$$

Dir^n of current → changing accordingly sin ωt. For upper half circle & lower half circle dir^n will be different,

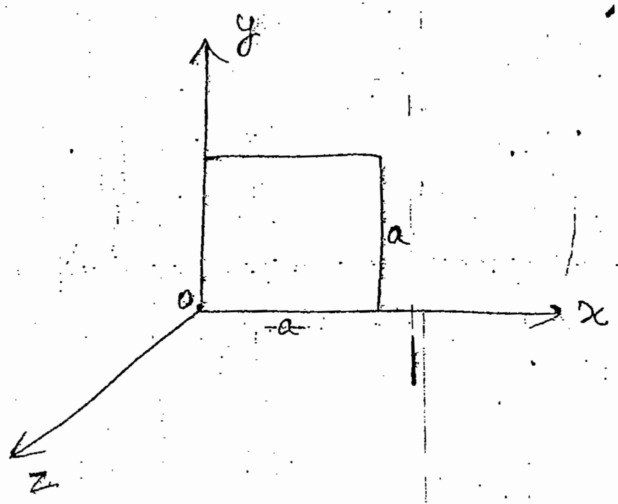


Que 1 – A square loop of wire with side 'a' lies in the Ist quadrant of x–y plane with one corner at the origin. In this region there is Non-uniform time dependent magnetic field $\vec{B}(y,t) = Ky^3 t^2 \hat{z}$ is coming out of the page. where K is some constant; Find the induce emf in the loop.

$$\vec{B}(y,t) = K y^3 t^2 \hat{z}$$

↓

Non uniform & time dependent.



$$\text{flux } \phi = \int B \cdot ds$$

$$= \int_0^a\int_0^a K y^3 t^2 \, dx\, dy$$

$$= Kt^2 [x]_0^a \left[\frac{y^4}{4}\right]_0^a$$

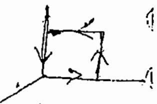
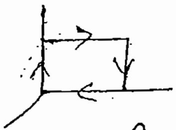
$$= Kt^2 a \cdot \frac{a^4}{4}$$

$$\boxed{\phi = \frac{Kt^2 a^5}{4}}$$

flux is changing with time.

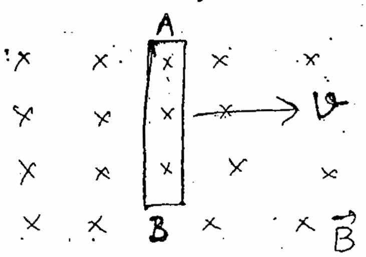
emf $\varepsilon = \frac{-d\phi}{dt} = -\frac{d}{dt}\left[\frac{Kt^2a^5}{4}\right] = \frac{-Ka^5}{4}\cdot 2t$

$$\boxed{\varepsilon = -\frac{Kta^5}{2}}$$

Current producing such mag. field will be Anticlockwise 
B is ↑ with time, so induce current will try to ↓ it.
So Dirⁿ of induced current for induced ~~ele.~~ mag. field will be clockwise. 

Note :- Induced current always less than the original current except the case of Superconductors ($I_{in} = I$).

Ques.1 - A metallic rod AB moves with a uniform velocity v in a uniform mag. field B as shown in the figure.

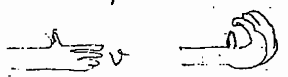


(a) The rod becomes electrically charged.
(b) The end A becomes +vely charged.
(c) " B " " "
(d) The rod becomes hot because of joule heating.

Ques.2 - A rod of length l rotates with a uniform angular velocity ω about its ⊥r bisector. A uniform mag. field $\vec{B}$ exist parallel to the axis of rotation. The potⁿ difference b/w the centre of the rod & an end is
(a) 0 (b) $\frac{1}{8}\omega Bl^2$ (c) $\frac{1}{2}\omega Bl^2$ (d) $B\omega l^2$

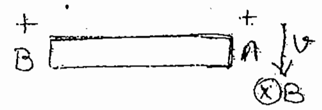
Q.3 - The potⁿ diff. b/w two ends of the rod is
(a) 0 (b) $\frac{1}{2}Bl^2\omega$ (c) $Bl^2\omega$ (d) $2Bl^2\omega$

① (b) A → +ve , B → −ve

B → ⊗ 

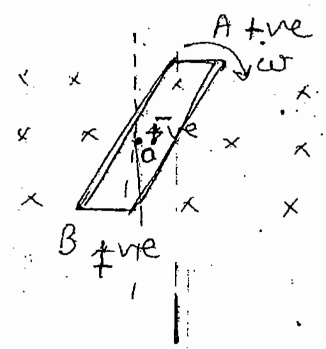
so upward → +ve

(2) Rod is rotating about mid point.



End A & B → +ve
Centre O → −ve
It will become a battery of length l.

$$\vec{v} = \vec{\omega} \times \vec{r} = \omega r$$



The charges in the rod are different. The linear velocity is 0 at the centre & increased away from origin & max. at A.

$$\varepsilon = \int_0^{l/2} \omega r B\, dr = \frac{\omega B}{2}\left(\frac{l}{2}\right)^2 = \frac{1}{8}\omega B l^2$$

$$\boxed{\varepsilon = \frac{1}{8}\omega B l^2}$$

(3) A & B both have +ve charge so Potn diff. b/w them is 0.

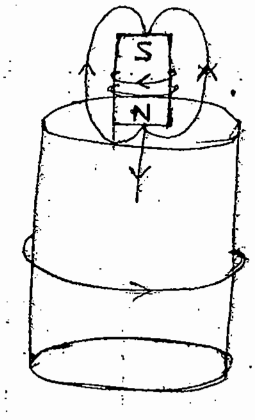
$$\varepsilon = \int_{-l/2}^{l/2} \omega r B\, dr = \frac{\omega B}{2}\left(l^2\right)_{-l/2}^{l/2} = \frac{\omega B}{2}\left[\frac{l^2}{4} - \frac{l^2}{4}\right] = 0$$

$$\boxed{\varepsilon = 0}$$

Q. :- A bar magnet is realeased from the rest along the axis of a very long vertical copper tube. After some time the magnet

(i) will stop in the tube.
(ii) will move with almost constant speed
(iii) will move with an acceleration g.
(iv) will oscillate.

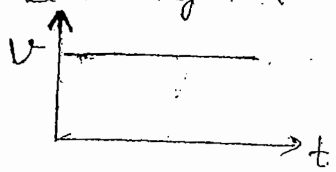
[(ii) is correct.]



If it is in the copper tube then it will move with 0 acceleration & constant speed. (almost constant). It depends on how strong the magnet is.

If ~~copper rod~~ magnet move downword inside the tube. As it pass through the rings (again Layer cylendr is made by rings) → change in the flux, & induces mag. field which oppose its motion.

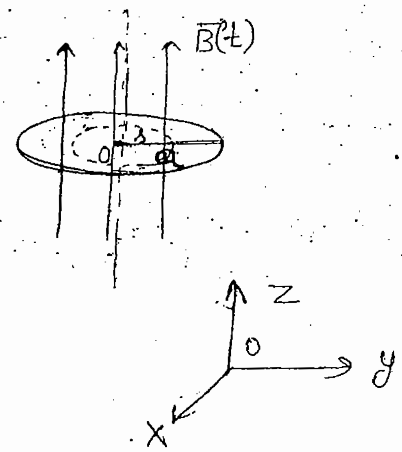
As magnet enter in one ring then flux constant & again it enter in the next new ring so flux will be change, & & give rise to induce mag. field.



When magnet enter int the tube then current will be opposite to the actual current,
& when leaving the tube then dir^n of current will be same as actual current.

<u>Que</u> :- A uniform mag. field B(t) pointing in the up dir^n fills the circular region as shown in the figure. If B is changing with time. What is the induce electric field.

Induce Electric field;



$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi}{dt}$$

$$= -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s} \quad \text{———(1)}$$

sketch a Ampirical loop of radius s.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = \mu_0 \int_S \vec{J}\cdot d\vec{s}$$

flux passing through the loop,

$$(1) \Rightarrow \quad \phi_m = B\pi s^2$$

Now, $\oint E\cdot dl = -\frac{d\phi}{dt} \Rightarrow E\cdot 2\pi s = -\pi s^2 \frac{\partial B(t)}{\partial t}$

$$\boxed{E_{in} = -\frac{s}{2}\frac{\partial B(t)}{\partial t}\hat{\phi}}$$

If mag. field is in z dir^n then E_{in} will be in ϕ dir^n.

- If $\vec{B} = at^2\hat{z}$

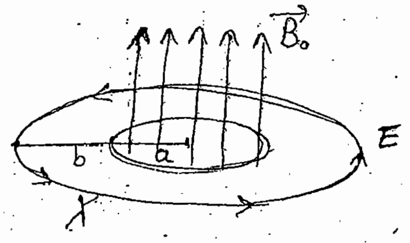
  Here B is ↑ with t so Flux ↑.

  So induce current or induce ele. field will try to ↓ the flux. [so its dir^n will be opposite to the actual curnt (antic.)]

  so dir^n of Ein will be Clockwise by watching from up.

- If B is ↓ with t then dir^n of Ein will be Anticlockwise
- If we switch off the mag. field, i.e. B = 0 then flux ↓ & induce current will try to ↑ the flux then dir^n of Ein will be Anticlockwise.

Q. A line charge λ is uniformly distributed on the rim of a wheel of radius b which is then suspended horizontally as shown in the figure, & it is free to rotate. In the central region upto radius a there is a uniform mag-field $\vec{B}$ pointing in the up dir^n. Now if someone turn off the magnetic field then calculate the angular momentum acquired by the wheel.



$B_0 \rightarrow$ const. Not depending on time

Right now flux pass

$$\phi_m = B_0 \pi a^2$$

Now switch off $\vec{B}$ i.e. $B = 0$

$$\phi_m = 0$$

Flux changes then there is a induce electric field.
dir^n of induce electric field is s.t. it will try to restore the flux.

Each charge is fix in ring. On applying force, charges do motion & due to this whole rim will do motion so this rim have angular mom.

$$\oint E \cdot dl = -\frac{d\phi_m}{dt}$$

mag. field is lying in the region of radius a.

$$= -\pi a^2 \frac{dB_0}{dt}$$

So Torque $N = \vec{r} \times \vec{F}$

$\vec{F} \rightarrow$ force required to rotate the charge, electric field will do work for rotation of charges – Electric force.

force/unit length $= f = \lambda E$

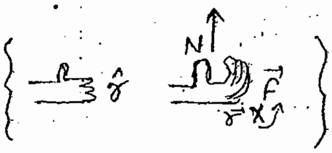
$\{q = \oint \lambda\, dl\}$

So total force $= \lambda \oint \vec{E} \cdot \vec{dl}$

$$\left\{\begin{aligned} F &= qE \\ &= \int \lambda\, dl\, E \\ &= \lambda \int E\, dl \end{aligned}\right.$$

$$\therefore N = b \lambda \oint \vec{E} \cdot \vec{dl}$$

dir^n of torque $\rightarrow$ upward



$$N = \frac{dL}{dt} = -\pi a^2 b \lambda \frac{dB_0}{dt}$$

$L \rightarrow$ angular Mom.

$$dL = -\pi a^2 b \lambda \, dB_0$$

when $B = B_0$; $L = 0$
$B = 0$, $L = L$

so $\int_0^L dL = -\pi a^2 b \lambda \int_{B_0}^{0} dB_0$

$$\Rightarrow \boxed{L = \pi a^2 \lambda b B_0}$$

This much angular mom. will accuired by the rim.

We know $L = mvr$ If mass of the rim is m.

$r = b$, $L = mvb$

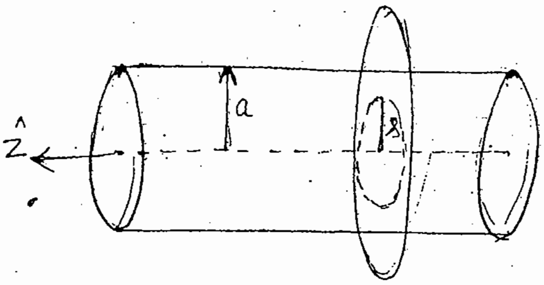
$L = m\omega b^2$ $(v = \omega r = \omega b)$

so $m\omega b^2 = \pi a^2 b \lambda B_0$

$$\boxed{\omega = \frac{\pi a^2 \lambda B_0}{mb}}$$

Q. :- A long solenoid with radius a & n turns per unit length carries a time dependent current $I(t)$ in the $\hat{\phi}$ dirn. Find the electric field both dirn & magnitude at a distance s from the axis both inside & outside the solanoid.

$$\oint E \cdot dl = -\frac{d\phi_m}{dt}$$



Mag. field inside the solanoid $\Rightarrow B = \mu_0 n I(t)$

$$E \cdot 2\pi s = -\mu_0 n \cdot \pi s^2 \frac{dI}{dt}$$

$$\boxed{\vec{E}_{in} = -\frac{\mu n s}{2} \frac{dI}{dt} \hat{\phi}}$$

If B is in $\hat{z}$ dirn & $\vec{E}_{in}$ (induced) will be in $\hat{\phi}$ dirn

Outside :- Flux $= \mu_0 n I \pi a^2$

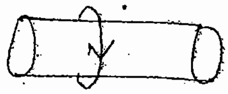
$$E \cdot 2\cancel{\pi} s = -\mu_0 n \cancel{\pi} a^2 \frac{dI}{dt}$$

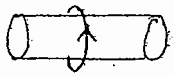
$$\boxed{\vec{E}_{out} = -\frac{\mu_0 n a^2}{2s}\frac{dI}{dt}\hat{\phi}}$$

Although mag. field outside the solenoid is 0 but induce electric field is not $\vec{E}_{out} \neq 0$

$$(E_{induce})_{inside} \propto s$$

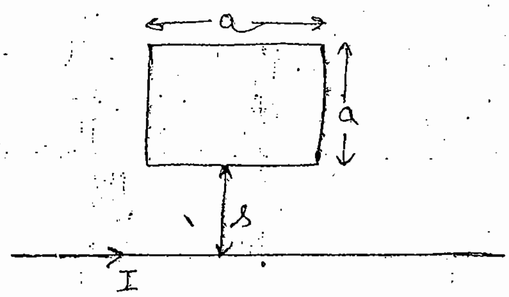
$$(E_{induce})_{outside} \propto \frac{1}{s}$$

If I (current) is ↑ with time then  induce current oppose the flux

If I ↓ with time  create the flux.

Q. A square loop of side a & resistance R lies at a distance s from an infinite straight wire that carries a current I. Now someone cut the wire s.t. I drops to zero. In what dir^n induce current in loop will flow. And calculate the total charge passes a given point in the loop during this time of current flow.

emf $\mathcal{E} = \frac{-d\Phi}{dt}$



$$\mathcal{E} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi s} \text{ (out of page)}$$

$$ds = dx\,dy$$

$$\mathcal{E} = -\frac{d}{dt}\int_s^{s+a} \frac{\mu_0 I}{2\pi s}\cdot a\,ds$$

$$= -\frac{d}{dt}\left[\frac{\mu_0 I a}{2\pi}(\ln s)_s^{s+a}\right] = -\frac{\mu_0 a}{2\pi}\ln\left(\frac{s+a}{s}\right)\frac{dI}{dt}$$

$$emf = I_{loop}\cdot R$$

$$I_{loop}\cdot R = -\frac{\mu_0 a}{2\pi}\ln\left(\frac{s+a}{s}\right)\frac{dI}{dt}$$

$$\Rightarrow \boxed{I_{loop} = -\frac{\mu_0 a}{2\pi R}\ln\left(\frac{s+a}{s}\right)\frac{dI}{dt}}$$

We know $I = dQ/dt$

$$\frac{d\phi_{loop}}{dt} = -\frac{\mu_0 a}{2\pi R}\ln\left(\frac{s+a}{s}\right)\frac{dI}{dt}$$

$$\int_0^{\phi} d\phi_{loop} = -\int_I^0 \frac{\mu_0 a}{2\pi R}\ln\left(\frac{s+a}{s}\right) dI$$

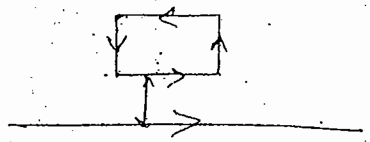
$$\Rightarrow \phi_{loop} = \frac{\mu_0 I a}{2\pi R}\ln\left(\frac{s+a}{s}\right)$$

$$\Rightarrow \phi_{loop} = \frac{1}{R}\left[\frac{\mu_0 I a}{2\pi}\ln\left(\frac{s+a}{s}\right)\right]$$

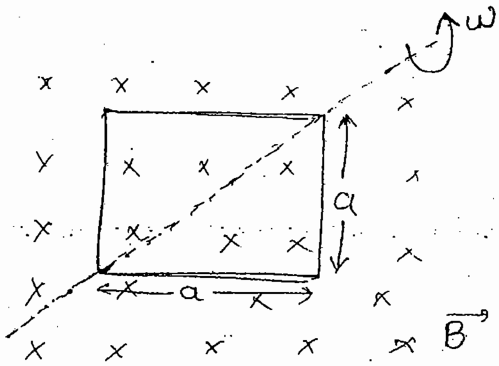
$$\Rightarrow \boxed{\phi_{loop} = \frac{\phi}{R}} \checkmark$$

Dirn of Induce current → Anticlockwise

$I\downarrow$, $B\downarrow$, $\phi\downarrow$ So induce currt will try to ↑ the flux.



Q. A square loop of edge 'a' having 'n' turns is rotated with a uniform angular velocity ω about one of its digonals which is kept fixed in horizontal position as shown in the figure. A uniform mag. field exist in the vertical dirn. Find the induce emf in the coil. Also plot the graph b/w induce current & phase ωt.



Area is fixed. B is fixed but angle is changing.
So flux changes due to angle change.

flux $\phi = B a^2 \cos\theta$

If loop is rotating with ang. velocity ω then in time t, loop rotate by angle $\theta = \omega t$

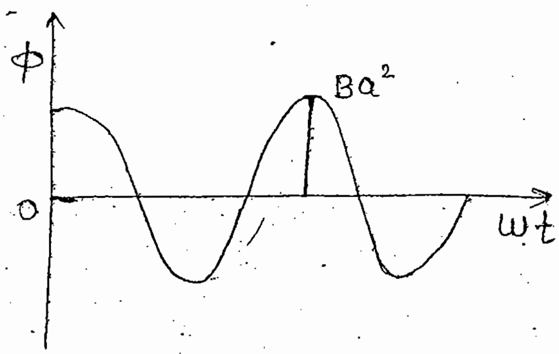
So $\phi = Ba^2 \cos\omega t$

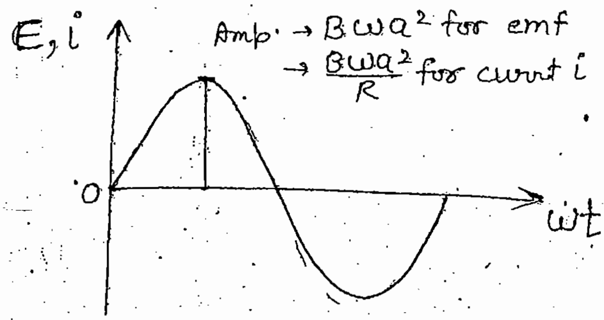
emf $\quad \varepsilon = \dfrac{-d\phi}{dt} = B\omega a^2 \sin\omega t$

If Resistance of the loop is R then current flow through the loop will be

$$i = \frac{\varepsilon}{R} \Rightarrow \boxed{i = \frac{B\omega a^2 \sin\omega t}{R}}$$

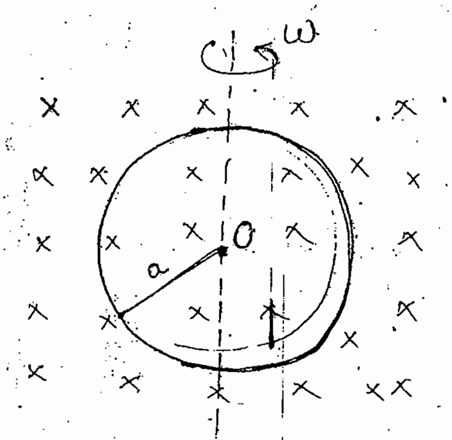
<u>Plot</u>:-





Plot of emf & current will be same. Only difference is of amplitude.

Q. A conducting circular loop of radius a is rotated about its diameter at ~~co~~ a constant angular velocity ω. In a mag. field B, ⊥r to the axis of rotation, in what position of the loop, induced emf will be zero.



flux passes through the loop

$$\phi = B\pi a^2 \cos\theta$$

$\because \theta = \omega t$

$$\boxed{\phi = B\pi a^2 \cos\omega t}$$

emf $\boxed{\varepsilon = \dfrac{-d\phi}{dt} = B\pi a^2 \omega \sin\omega t}$

When $\theta = \omega t = \frac{\pi}{2}$ then emf will be maximum & flux $= 0$

(phase diff. b/w emf & flux is of $90°$)

flux is leading in phase than emf.

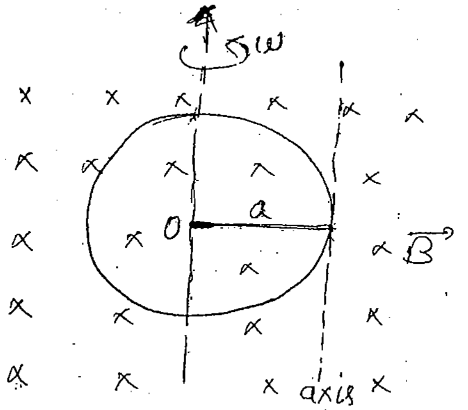
as $\sin(\omega t + \frac{\pi}{2}) = \cos\omega t$

As compare to cos & sine, we get ~~sin is π/2 ahed~~ flux is ahead in phase

- In this case

$$\text{flux } \phi = B\pi a^2 \cos\omega t$$

$$\text{emf } \varepsilon = B\pi a^2 \omega \sin\omega t$$



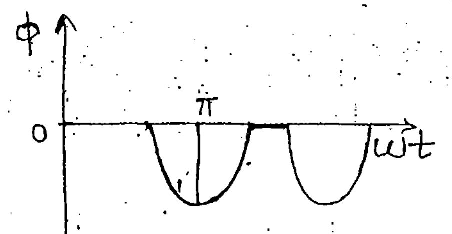
Now the whole loop is rotating about the axis. There will be No differenc in ϕ & ε.

- But if there is no mag field on the R.H.S. of axis then:

flux will exist only from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$

i.e. $-\frac{\pi}{2} < \omega t < \frac{\pi}{2}$ $\quad (\theta = \omega t)$

then wave of flux will be Half wave rectifier



If flux wave is half cos wave then emf wave will be half sine wave.

$\varepsilon \rightarrow$ half wave Sinusoidal.

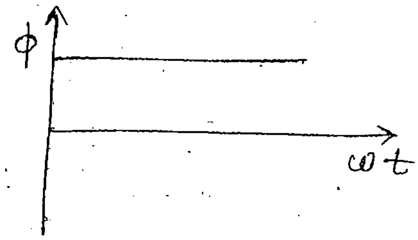
~~It~~ wave can be on +ve or -ve axis depands upon the rotation.

The Axis of rotation is parallel to the axis of loop.

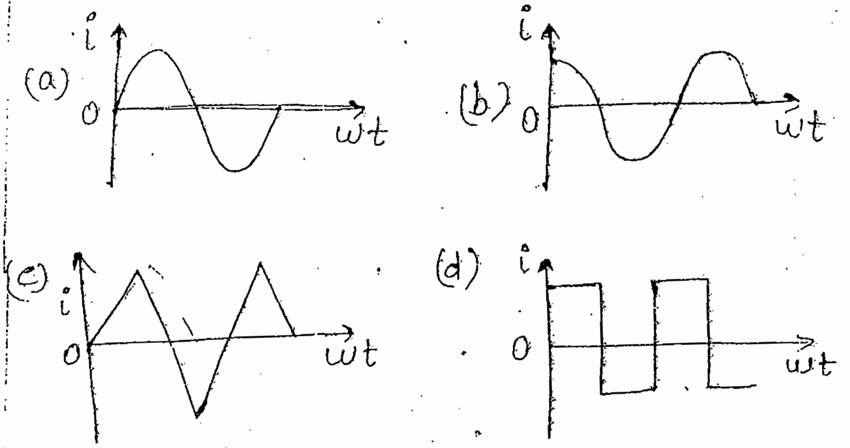
flux pass through loop $\phi = B\pi a^2$

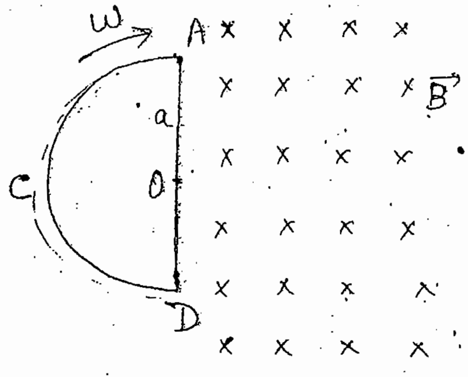
change in flux $= 0$

So Induced emf $= 0$.



Q. The mag. field B shown in the figure is directed into the plane of the paper; ACDA is a semicircular loop of radius 'a' with the centre at O. The loop is now made to rotate clockwise with a constant angular velocity ω about axis passing through O & $\perp^r$ to the plane of the paper. The resistance of the loop is R. Obtain an expression for the magnitude of the induce current of the loop. Plot a graph b/w induced current & time.



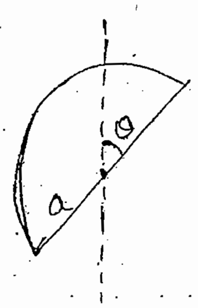


axis of rotation is O.

Axis of Rotation is O i.e. loop is rotating about O. So flux passing through the loop is variable.

Maximum area limit $= \frac{\pi a^2}{2}$

It is rotating so after some time position of loop will be



→ Angle b/w A (area vector) & B is Not changing

→ B is Not changing

→ Area is changing.

If loop rotate by 180° then it comes in mag. field i.e. area A is completely inside the B.

$\theta = \omega t$, {If $\theta = \pi$}

then area $A = \left(\frac{\pi a^2}{2}\right)\left(\frac{\theta}{\pi}\right) = \frac{a^2\theta}{2}$

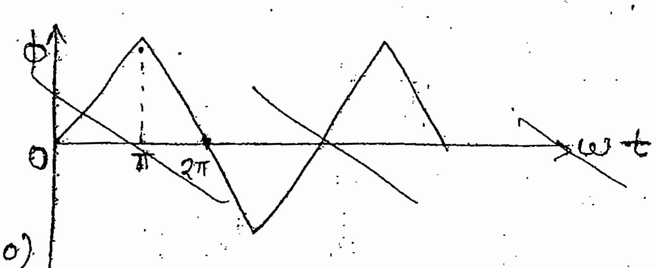
If $0 < \theta < \pi$ then flux ↑ (area linearly ↑ with θ)

If $\pi < \theta < 2\pi$ then flux ↓

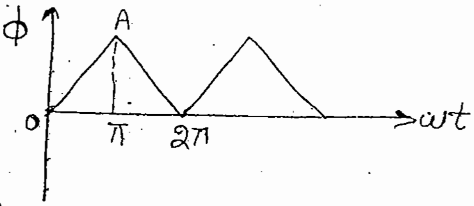
So flux $(0 < \theta < \pi)$

$$\phi = \frac{Ba^2\theta}{2} \Rightarrow \phi = \frac{Ba^2(\omega t)}{2}$$

Flux ↑ linearly with θ (ωt) up to 0 to π. then from π to 2π flux linearly ↓ with θ bcoz loop is out of B now. At θ = 2π, φ again zero bcoz (2π same 0)



1 wave complete $0 \rightarrow 2\pi$



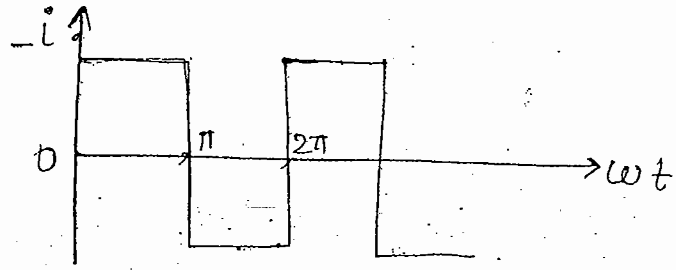
Now emf, $\varepsilon = \frac{-d\phi}{dt}$

flux linear with t so on differentiating we get constant. So we get squar wave.

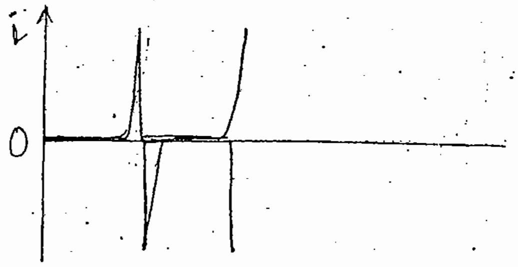
Slop of OA is +ve so current or emf will be -ve.

$\varepsilon = -\frac{d\phi}{dt}$ (here -ve sigh present so take $-i$ on axis)

* Differntiation of linear wave is always a Square wave.



If we apply differntiator of square wave - then we get 'Spikes.'



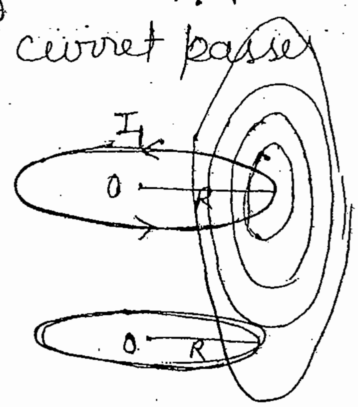
If Sharp change; then spikes are narrow.
If broad change, then spikes are spread.

If we apply Integrator on spikes - then we get Square wave again.

Wave of $\phi \longrightarrow$ triangular
wave of $i \longrightarrow$ Square Wave

Mutual Inductance :-

If we have 2 current carrying coils of radius R.
Put these coils close to each other. If current passes through coil (1) is I_1 & coil (2) is zero.
Due to current I_1 of loop (1),
Mag. field $\vec{B}$ lines of loop (1) will pass through the loop (2), So There will be a mag. flux through loop (2).



~~So there will be a induced emf~~

If we change the current I_1 then mag. field B_1 will change

as $B \propto I_1$

So flux changed which is linked to loop (2).

So there will be induced emf.

The dir^n of

If flux ϕ $\downarrow$, dir^n of current in loop (1) & (2) are same

" $\uparrow$ " " " " opposite

So flux pass through loop (2) is due to the mag. field of loop (1) & area of loop (2)

$\phi_2 \propto I_1$ { as ~~B~~ flux $\phi \propto B$ & $B \propto I$ so $\phi \propto I$

And constant of proportionallity is called Mutual Inductance. (M)

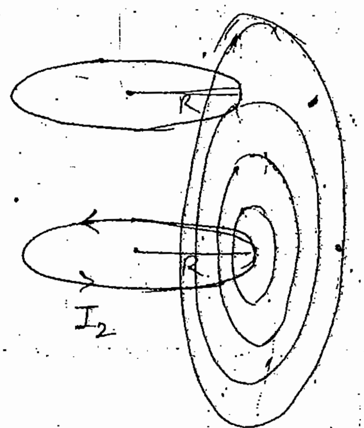
i.e. $\boxed{\phi_2 = M_{21} I_1}$

$M_{21} \rightarrow$ Mutual Inducance b/w 2 & 1.

$\Rightarrow$ Now inter change the current, (I_1 & I_2 $\rightarrow$ same magnitude)

Now current is flowing through loop (2)

~~So~~ And Mag. flux pass though loop (1) will depand upon due to the mag. field B of loop (2).



then $\phi_1 \propto I_2$

$I_1 = I_2 = I$ (same mag.)

So both flux ϕ_1 & ϕ_2 will be same.

$\boxed{\phi_1 = M_{12} I_2}$ $\boxed{\phi_1 = \phi_2}$

flux $\propto$ current

On interchanging current, flux will be intechange irrespective of their areas.

Mutual Inducer is purely geometrical quantity. This depands on size, shape & positions of the loop.

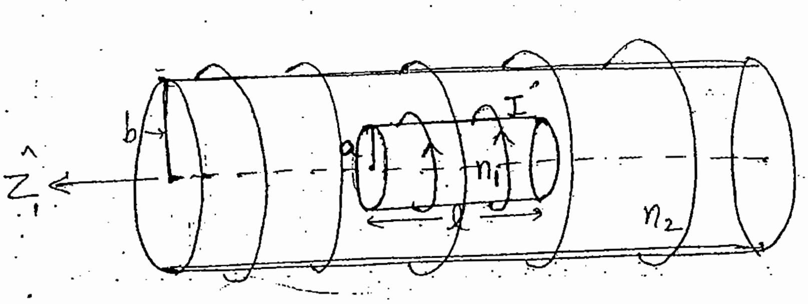
⇒ If Radius of both loops are different still flux is same.

$$\left.\begin{array}{l} \text{loop}(1) \rightarrow 2R \\ \text{loop}(2) \rightarrow R \end{array}\right\} \text{then } \phi_1 = \phi_2$$

⇒ On changing size of loop, M will change.

⇒ To calculate flux of any loop, we have to fix the value of M (i.e. In the process, size, shape & distance b/w both loops can not be changed)

Q.:- A short solenoid of length L & radius a with n_1 turns per unit length lies on the axis of a very long solenoil of radius b & n_2 turns per unit length. Current I flows in the short solenoid. What is the flux through the long solenoid.



No current on large solenoid.

Mag. field of small solenoid passing through the large one

Mag. field of a short solenoid is complicated.

but Mag. field of a short solenoid

Inside → $B = \mu_0 n I$ but outside, Not exatly zero at the ends of short loop.

Beoz Mag. field of short solenoid is complicated.

Hence interchange the current.

Now assume, current I is flowing in ~~sho~~ the long solenoid & we have to find the ~~mag~~ flux through short solenoid

Mag. field of large solenoid

$$\vec{B} = \mu_0 n_2 I \hat{z}$$

By single ring of a ~~short~~ solanoid, flux passout

$$\phi_{single} = \mu_0 n_2 I \cdot \pi a^2$$

And in short solanoid, n_1 are the no. of turns & l is the length.

So $n_1 l \rightarrow$ turns per unit length.

then flux

$$\phi_{Total} = n_1 l \, \phi_{single}$$

$$\phi_{Total} = \mu_0 n_1 n_2 \, l \cdot \pi a^2 I$$

So same flux will pass through the short & long 's.

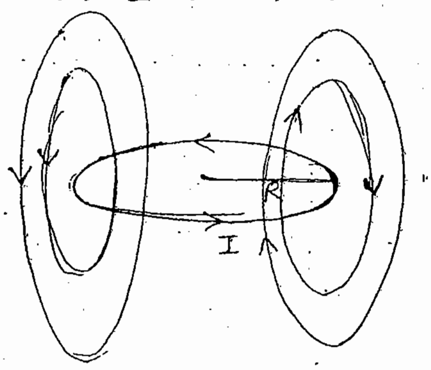
Now Mutual Inductance of this configuration

$\phi = MI \Rightarrow M = \mu_0 n_1 n_2 l \cdot \pi a^2$

M depends on medium b/w both solanoids.

Self Inductance (L) :-

Suppose we have a circular coil. Current flowing in this is I & its radius is R. Then there will be flux passing through the coil due to its own magnetic field.



flux $\phi \propto B$. & $B \propto I$

So flux passing through the coil is proportional to the current

$$\phi \propto I$$

And constant of proportionality is Self Inductance.

Self $\rightarrow$ bcoz it is due to self current.

$$\boxed{\phi = LI}$$

If we are changing current with time then B is changing with time & hence induce ϕ is changingin with time Area is const. L depends on size & shape of loop.

So there will be an induced emf

$$\text{emf} \quad \varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$$

If $I \uparrow$ with t then $\frac{dI}{dt} \to +ve$ but $\varepsilon = -ve$

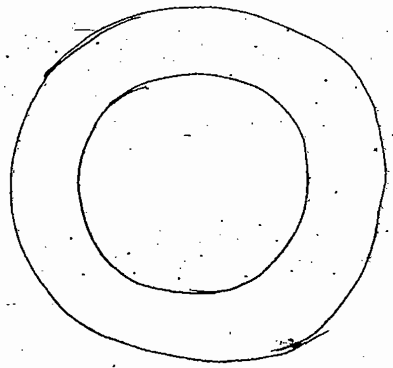
So induce current will oppose the change in current.

This is called <u>Self Inductance</u>.

Q. Find the self inductance of a toroidal coil with rectangular cross-section inner radius a & outer radius b & height h which carries a total N turns.

Toroid $\to$ Endless solenoid.

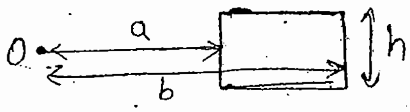
Mag. field of toroidal coil is



$$\vec{B} = \frac{\mu_0 N I}{2\pi s}\hat{\phi} \quad \text{[Non-uniform]}$$

(Inside the toroidal coil) where $a<s<b$

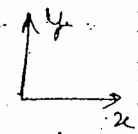
There are N no. of turns.



Flux passing through a single turn,

area $= (b-a)h$

$\int_a^b \int_0^h dx\,dy$



$$\phi_{single} = \frac{\mu_0 N I}{2\pi}\int_a^b \frac{1}{s} h\, ds$$

$$\phi_{single} = \frac{\mu_0 N I h}{2\pi}\ln(b/a)$$

Mag. field varying with b

$ds = dx\,dy$. but B is not changing with y so $dy = h$

Total flux passing through N turns of toroid, will be

$$\phi_{Total} = N\phi_{single}$$

$$\boxed{\phi_{Total} = \frac{\mu_0 N^2 h}{2\pi}\ln(b/a) I}$$

We know $\phi = LI$

So self Inductance will be

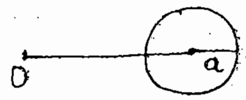
$$\boxed{L = \frac{\mu_0 N^2 h}{2\pi}\ln(b/a)}$$

This is the ~~seflf~~ self inductance of toroidal coil with rectangular cross-section.

If Cross-section is circular instead of Rectangular :-

Area element $= s\,d\phi\,ds$

$= 2\pi\, s\,ds$



(no variation with ϕ, only with s)

Now, $\phi_{single} = \dfrac{\mu_0 N I}{2\pi}\displaystyle\int_0^a \frac{1}{\cancel{s}}\,\cancel{s}\,ds\cdot 2\pi$

$\displaystyle\int_0^a\int_0^{2\pi} s\,ds\,d\phi$

$\displaystyle\int_0^a s\,ds\; 2\pi$

$\Rightarrow \pi a^2$

$\phi_{single} = \mu_0 N I a$

Total flux $\phi_{Total} = N\cdot\phi_{single}$

$$\boxed{\phi_{Total} = \mu_0 N^2 I a}$$

So Self Inductance $\boxed{L = \mu_0 N^2 a}$

Analogy b/w Mechanics & Electromagnetism :-

In Mechanics, K.E. is $K = \frac{1}{2} m v^2$

& P.E. is $U = \frac{1}{2} K x^2$

If we have a coil & current ~~is~~ flowing in it is I & ~~seft~~ self inductance of that coil is L then magnetic energy stored by that coil is $U_m = \frac{1}{2} L I^2$

Capacitor stores the electrical energy & Inductor stores magnetic energy

↳ in the form of charge q or field inself

↳ in the form of current.

current → dynamic

charge → static

Inductor → L → I

Capacitor → c → q

$U_e = \dfrac{q^2}{2C}$ & Also $U_e = \dfrac{\epsilon_0}{2}\displaystyle\int E^2 d\tau$

$K = \frac{1}{2} m v^2 \longleftrightarrow U_m = \frac{1}{2} L I^2$

$U = \frac{1}{2} K x^2 \longleftrightarrow U_e = \dfrac{q^2}{2C}$

Electromag. Waves carry energy but not carry charge or current.
With E.M. Wave, both energies are associated in electrical & magnetic. So magnetic energy resides in the magnetic field & electric energy resides in Ele. field. bcoz there is No charge (or current)

In Magnetostatics, $U_m = \cancel{\frac{1}{2}\mu_0} \frac{1}{2\mu_0} \int_{\text{all space}} B^2 \, d\tau$

Mechanics ~~Magnetostatic~~	Electromag.
$m \longrightarrow$	L
$v \longrightarrow$	I
$K \longrightarrow$	$\frac{1}{C}$
$x \longrightarrow$	q

⇒ If we have 2 bodies of mass m & $2m$ then body of mass m will accelerate more.
Mass → less, More → acceleration
If M is More it will be difficult to ↑ the velocity.

⇒ If we have 2 coils of ~~seff~~ self inducane L & $2L$ then it is easy to ↑ the current of coil of S.I. L.
To ↑ current, have to fight against its back emf.
If L is more, it will be difficult to ↑ the current.

26/8/2012

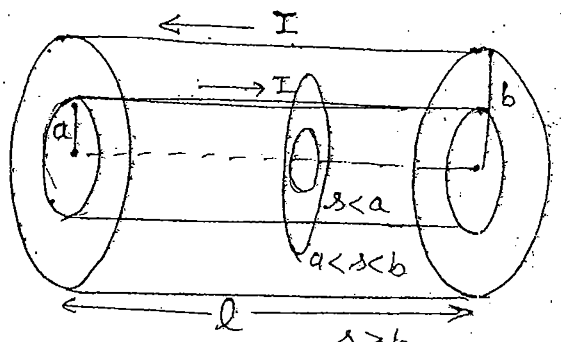
Q. 1- A long co-axial cable carries a current I, the current flows down the surface of inner cylender of radius a & back along the cylender of radius b. Find the magnetic energy stored in the section of length l.

Magnetic energy stored

$$U_m = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 \, d\tau$$

$$U_e = \frac{1}{2} \int_{\text{all space}} \vec{D} \cdot \vec{E} \, d\tau$$

Also $U_m = \frac{1}{2} \int_{\text{all space}} \vec{H} \cdot \vec{B} \, d\tau$



current is flowing on the surface of both inner & outer cylinder

for $s<a$, $I_{enc} = 0$

so $\boxed{B = 0}$

for $a<s<b$, $I_{enc} = I$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$$

dirn of Ienc is $\hat{z}$ Hence dirn of mag. field is $\hat{\phi}$.

for $s>b$, $I_{enc} = 0$

bcoz current on both cylinders are equal & opposite so cancel out $I_{enc} = +I - I = 0$

So $\boxed{B=0}$

So $U_m = \frac{1}{2\mu_0}\int_{all\,space} B^2 d\tau$

$$U = \frac{\mu_0^2 I^2}{2\mu_0 4\pi^2}\int_a^b \frac{1}{s^2}\, s\,ds\,d\phi\,dz$$

{limits on $s \to 0$ to ∞ but from $\int_0^a = 0 \Rightarrow (B=0)$ & $\int_b^\infty = 0$ $(B=0)$}

$$U = \frac{\mu_0 I^2}{8\pi^2}\, 2\pi\, l \ln(b/a)$$

$$\boxed{U = \frac{\mu_0 l}{4\pi}\ln(b/a)\, I^2}$$

This much energy will be stored in this configuration.

<u>Self Inductance of this confi. :-</u>

$$U = \frac{1}{2}LI^2$$

$$\boxed{L = \frac{\mu_0 l}{2\pi}\ln(b/a)}$$

Q. Find the mag. energy stored in a toroidal coil of rectangular cross-section in a radius a, outer radius b & height h. Total No. of turns in the coil are N and current is I.

Electrodynamics before Maxwell :-

(Gauss Law in electrostatics) (i) $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$ (ii) $\vec{\nabla}\cdot\vec{B} = 0$ (Gauss law in magnetostatics)

(iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ (Faraday's law)

(iv) $\vec{\nabla}\times\vec{B} = +\mu_0\vec{J}$ (Ampere's law)

These 4 eqn deriving the electrodynmics before Maxwell.

Are these consistent in Electrodynamics :-

Physical significance of $\vec{\nabla}\cdot\vec{B} = 0$ (sometimes No name)

→ Non existance of magnetic monopols.

⇒ Taking ~~Curl~~ div of eqn (iii),

L.H.S: $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{E}) = 0$ (div of curl is always zero)

R.H.S. $-\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{B}) = 0$

$\vec{\nabla}\cdot\vec{B} = 0$ always, No restriction.

So Faraday's law i.e. this eqn is valid in electrodynamics.

⇒ Taking div. of eqn (iv),

$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0$

$\vec{\nabla}\cdot\vec{J} = 0$

but $\nabla\cdot J = 0$ only in magnetostatics.

So this Ampere's law is valid only in Magnetostatics i.e. where there is No changing fields & current must be steady.

If accumulation of charge is present then this is not valid.

So this law must be modified.

Modification of Ampere's law by Maxwell :-

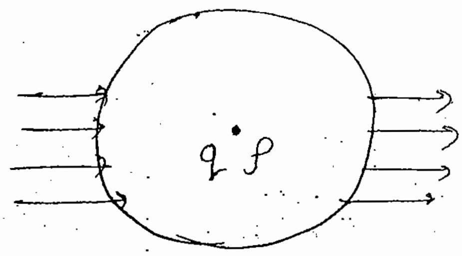
After modification, eqn (iv) becomes Modified Ampere's law. All 4 eqn together called Maxwell's eqns.

In Magnetostatics $\vec{\nabla}\cdot\vec{J}=0$

But if charge is changing with time i.e.
ρ " " " " then $\vec{\nabla}\cdot\vec{J}\neq 0$

If we have a close surface & charge q is placed in this close surface & its charge density is ρ. This ρ is Not changing with time i.e. charge is conserved. i.e. No charge is flowing in or out.

OR IInd possibility is that
- charge coming in = charge coming out.



Flux of the current passing through the boundary of sphere $= 0$

then $\vec{\nabla}\cdot\vec{J}=0$

In IInd case ∵, still flux $=0$
then $\vec{\nabla}\cdot\vec{J}=0$

i.e. $\boxed{\vec{\nabla}\cdot\vec{J}=0 \text{ if } \rho \text{ is not depending on time.}}$

But If ρ or q is a func^n of time.
- then flux pass through boundary $\neq 0$

→ If +ve ~~flux~~ charge is dec. with time then current will flow outward (in the dir^n of flow of +ve charge)

→ If charge ↑ with time then current will flow into the sphere.

→ If charge is not change with time then current flow $=0$

If Electrodynamics,
$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$$

If ρ dec. with time then $\rho = -ve$ then $\vec{\nabla}\cdot\vec{J} = \frac{\partial \rho}{\partial t}$
current flow outward.

If ρ ↑ with time $\rho = +ve$

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \vec{\rho}}{\partial t}$$ then current flow into

This eqⁿ is called eqⁿ of Continuity $\boxed{\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}}$

& its physical significace is local conservation of charge.

→ charge is conserved globally, neither be created nor be destroyed.

Here charge is conserved on a local body that's why it is called Local conservation of charge.

eqⁿ (i) ⇒ $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$

$$\Rightarrow \rho = \epsilon_0(\vec{\nabla}\cdot\vec{E})$$

Put it into Continuity eqⁿ,

$$\vec{\nabla}\cdot\vec{J} = -\epsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E})$$

Change the order then (bcoz both operator are indipendent on each other)

$$\vec{\nabla}\cdot\vec{J} = -\epsilon_0\vec{\nabla}\left(\frac{\partial \vec{E}}{\partial t}\right)$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot\left[\vec{J} + \epsilon_0\frac{\partial \vec{E}}{\partial t}\right] = 0}$$

In electrodynamics, We need to replace J by $J + \epsilon_0\frac{\partial E}{\partial t}$. then we get Modified Ampere's law.

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \Rightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0\left[\vec{J} + \epsilon_0\frac{\partial \vec{E}}{\partial t}\right]} \quad \text{(A)}$$

Now L.H.S. = 0 & R.H.S. = 0

So this eqⁿ is consistent with electrodynamics.

This eqⁿ is called Modified Ampere's law.

Also called 4th Maxwell eqⁿ.

electro

In Magnetostatic, B & E are independent of time

$$\frac{\partial E}{\partial t} = 0$$

so $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

In eqn (A) ⇒ $\epsilon_0 \frac{\partial \vec{E}}{\partial t}$ → called displacement Current density in vaccum.

Displacement Current - follow no other property of current. This is being called current only bcoz it is producing mag. field. There are no charges corresponding to this displacement current. Corresponding to $\vec{J}$ there are charges.

4 eqn's together →

3rd eqn say → change in Mag-field producing Ele. field.
4rth " " " " Ele. " " Mag. " .

$$\nabla \cdot E = \rho/\epsilon_0$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$\boxed{\text{Differential forms}}$

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{s} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_s \vec{B} \cdot d\vec{s}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d$$

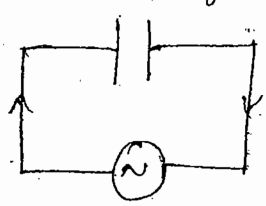
$\boxed{\text{Integral forms}}$

Practical Example of Displacement Current :- is found in air filled capacitor.

If we connect it with source then current flow.

In Air gape, No current.

↳ current is flowing — displacement Current.

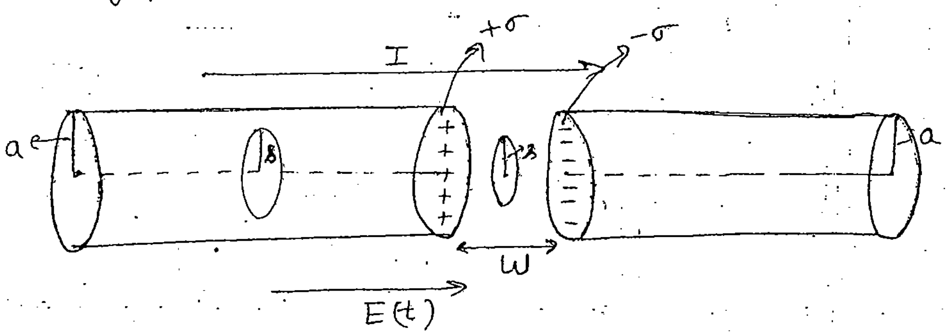


If source is time dependent → charge accumulate on plate of capacitor → E-field produce. Here $\sigma = \sigma(t)$ bcoz charge is built up with time. So E-field will also be the fxn of time. So a current produces due to the change in Ele. field called Displacement Current.

Current produce due to motion of free electron called Conduction Current.

{ bound current → due to motion of bound charge }.

Q. A thick wire of radius a carries a constant current I uniformly distributed over its cross-section. A narrow gap in the wire of width $w \ll a$ forms a parallel plate capacitor as shown in the figure. Find the mag. field in the gap at a distance $s < a$ from the axis.



$$E(t) = \frac{\sigma(t)}{\epsilon_0}$$

Current density $\vec{J}_d = \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \cancel{\epsilon_0} \frac{1}{\cancel{\epsilon_0}} \frac{\partial \sigma}{\partial t} = \frac{\partial \sigma}{\partial t}$

dirn of current is the dirn of ele. field.

Current Enclose by the s loop,

$$I_{enc} = \int \vec{J_d} \cdot d\vec{s}$$

$$= \cancel{\epsilon_0} \frac{\partial \sigma(t)}{\partial t} \cdot \frac{\pi s^2}{\cancel{\epsilon_0}}$$

$$= \frac{\partial \sigma(t)}{\partial t} \pi s^2 \times \frac{\pi a^2}{\pi a^2}$$

$$I_{enc} = \frac{1}{\pi a^2} \underbrace{\frac{\partial q(t)}{\partial t}}_{\text{Current}} \pi s^2$$ {charge / unit time - current}

$$\boxed{I_{enc} = \frac{I s^2}{a^2}}$$

Ampere's law, $\oint \vec{B} \cdot d\vec{l} = \mu_0 \underbrace{I_c}_{\text{Conduction current}} + \mu_0 \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{s}$

$I_c = 0$ bcoz no free e^- in the gap

so Total current will be due to IInd term.

$$B \cdot 2\pi s = \mu_0 \frac{I s^2}{a^2}$$

$$\left\{ I_{enc} = \int J_d \cdot ds = \epsilon_0 \int_s \frac{\partial E}{\partial t} ds \right\}$$

$$\boxed{\vec{B} = \frac{\mu_0 I s}{2\pi a^2} \hat{\phi}}$$

Displacement current is necessary to make the current continuous across the capacitor.

This mag. field is same as → In case of uniform mag. field inside the wire.

Maxwell's Eqn in Matter :-

(i) $\vec{\nabla} \cdot \vec{D} = \rho_f$

(ii) $\vec{\nabla} \cdot \vec{B} = 0$ $(\vec{\nabla} \cdot \vec{H} \neq 0$ as Magnetization is non-zero)
may or may not (depending on condn)

{ If M not depend on t then $\nabla \cdot H = 0$ }

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(iv) $\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$

Before Maxwell → 4th eqn → $\vec{\nabla} \times \vec{H} = \vec{J}_f$

$$\nabla \times B = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Find in terms of H = ?

Inside the matter, there will be polarisation & Magnetisation.

$$\vec{J} = \vec{J_b} + \vec{J_f}$$

$$\vec{J} = \vec{J_f} + \underbrace{\vec{\nabla} \times \vec{M}} \longrightarrow \text{Magnetisation Current.}$$

$$\frac{1}{\mu_0}(\vec{\nabla} \times \vec{B}) = \underbrace{\vec{J_f}} + \vec{\nabla} \times \vec{M} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \underbrace{\frac{\partial \vec{P}}{\partial t}}_{\text{Polarisation current}}$$

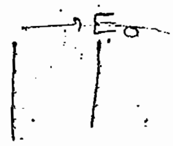
(i) Due to the motion of free charges

(ii) Due to the bound charges – bound or Magnetisation current.

(iii) Due to the change in Elec. field – Displacement current.

(iv) If E is changing with time then P also changing with time & current produce – Polarisation current.

In free space E. field is E_0



but if space is not free fill the matter in it then elec field will be changed.

$$\epsilon E = \epsilon_0 E_0 + P$$

$$E = \frac{E_0}{\epsilon_r} \pm \frac{P}{\epsilon}$$

~~On~~ Due to E. field, charges become polarised & began to oscillate so polarisation current produce.

$$\therefore \frac{1}{\mu_0}(\vec{\nabla} \times \vec{B}) = \vec{J_f} + \vec{\nabla} \times \vec{M} + \frac{\partial}{\partial t}(\epsilon_0 \vec{E} + \vec{P})$$

$$= J_f + \vec{\nabla} \times \vec{M} + \underbrace{\frac{\partial D}{\partial t}}$$

displacement current density in matter

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J_f} + \frac{\partial \vec{D}}{\partial t}} \rightarrow \text{called Maxwell's eq}^n \text{ in Matter.}$$

Maxwell's Eqn. in Free Space :-

In free space – there is no charge & no current.

(i) $\nabla \cdot D = 0 \Rightarrow \boxed{\nabla \cdot E = 0}$ $(D = \epsilon E)$

(ii) $\nabla \cdot B = 0$

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(iv) $\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

$\left\{ \nabla \times H = \underset{\substack{\| \\ 0}}{J_f} + \frac{\partial D}{\partial t} \right\}$ (free space)

Maxwell's Eqn for Static Fields :-

(i) $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$

(ii) $\vec{\nabla} \cdot \vec{B} = 0$

(iii) $\vec{\nabla} \times \vec{E} = 0$

(iv) $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

{static} $\left\{ \frac{\partial D}{\partial t} = 0 \right\}$

Maxwell's eqn for Isotropic Linear Dielectric :-

In dielectric → free charges $= 0$

$\rho_f = 0$

for Linear dielectric, $D = \epsilon E$

Isotropic means ϵ is not depend on space co-ordinate

i.e. ϵ is not a funcn of position.

(i) $\vec{\nabla} \cdot \vec{D} = 0 \Rightarrow \boxed{\vec{\nabla} \cdot \vec{E} = 0}$

$\left\{ \begin{array}{l} \nabla \cdot (\epsilon E) = 0 \\ \Rightarrow \epsilon (\nabla \cdot E) = 0 \end{array} \right.$

(for Anisotropic $\nabla \cdot D = 0$ but $\nabla \cdot E \neq 0$ bcoz ϵ is a funcn of position)

(ii) $\vec{\nabla} \cdot \vec{B} = 0$

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(iv) $\vec{\nabla} \times \frac{\vec{B}}{\mu} = \underset{\substack{\| \\ 0}}{\vec{J_f}} + \frac{\partial \epsilon \vec{E}}{\partial t}$

$\Rightarrow \boxed{\vec{\nabla} \times \vec{B} = \mu \epsilon \frac{\partial \vec{E}}{\partial t}}$

$J_f = 0$ (No free current)

for linear → $B = \mu H$

$\nabla \times H = \epsilon (\nabla \times B)$

Solution of Maxwell's eqn in free space :-
OR
Electromagnetic Waves in Vaccum :-

Maxwell's eqn in free space,

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \text{——— (1)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{——— (2)}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{——— (3)}$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \quad \text{——— (4)}$$

Maxwell's eqn are four first order differential eqn. We need to convert these first " " " into 2 IInd order diff. eqn.

Take the curl of eqn (iii),

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

Put the value of $\vec{\nabla}\times\vec{B}$ from eqn (4) into this eqn,

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$\nabla\cdot E = 0$ from eqn (1),

$$\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0} \quad \text{——— (5)}$$

Now, Take the curl of eqn (4),

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \mu_0 \epsilon_0 \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E}) \qquad \text{(from (3))}$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \qquad (\nabla\cdot B) = 0$$

$$\boxed{\nabla^2 B - \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} = 0} \quad \text{——— (6)}$$

eqn (5) & (6) are the IInd order diff. eqn of electric & mag. field.

All the information of 4 Maxwell eq^n contained in two eq's (5 & 6)
eq^n (5) & (6) are the Wave eq^{n}s of electric & mag. field.
General Wave eq^n,

$$\boxed{\nabla^2 f - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = 0} \quad (7)$$

Compare (5) & (6) with (7) ⇒ We get

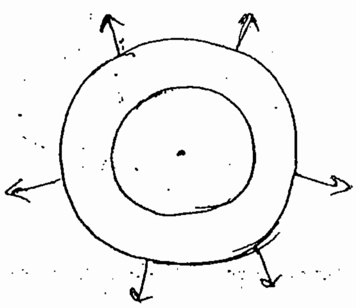
$$\boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c} \quad (8)$$

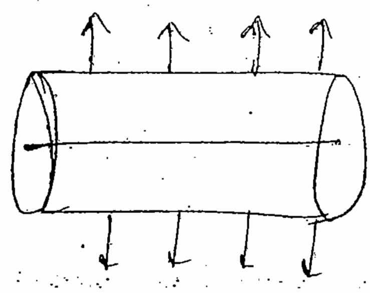
$c \rightarrow$ speed of light.

Hence, Light Waves are Electromagnetic Waves.
They don't require any medium to propagate.
[This can't be proved if Maxwell did'nt give Displacement current term]

- Solution of IInd order diff. eq^n gives → one give Ele. field & another give Mag. field. And both field depend on each other. They can not exist without each other.

Solution may be of many king.
In Cartisian co-ordinate → we get plane waves
" sperical-polar " → spherical waves
" cylendrical " → cylendrical wavy





Small portion of spherical wave frount ~~& cylendri~~ is a plane. (large radius sphere)
We are interested in plane wave soln.

Plane wave Sol^n of $\vec{E}$ & $\vec{B}$

Suppose $\vec{E}$ & $\vec{B}$ are propogate in x, y, z dir^n

$$\vec{E} = E_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)}$$

$$\vec{B} = B_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)}$$

These sol^ns must satisfy the maxwell's eq^n as they are the solution of Maxwell's eq^n.

$$\vec{\nabla}\cdot\vec{E} = 0 \Rightarrow \boxed{\vec{K}\cdot\vec{E} = 0} \quad \text{——(A)}$$

$i(K\cdot E) = 0$

∇-operator apply on plane wave. (it operates on space part) After operation it gives iK.

& time operator gives $-i\omega$.

$$\boxed{\begin{array}{l} \nabla \longrightarrow iK \\ \dfrac{\partial}{\partial t} \longrightarrow -i\omega \end{array}}$$

$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow i(K\cdot B) = 0 \Rightarrow \boxed{\vec{K}\cdot\vec{B} = 0}$ ——(B) from (A) & (B)

Vibration of $\vec{E}$ & $\vec{B}$ are the $\perp^r$ to the wave propagation.

Wave vector tells the dir^n of propogation.

So $\boxed{\vec{E} \text{ \& } \vec{B} \text{ are Transvers in Nature}}$

Put the value of $\vec{E}$ & $\vec{B}$ in eq^n (3) & (4),

$\mathrm{eq}^n(3) \Rightarrow$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$i(\vec{K}\times\vec{E}) = -(-i\omega)\vec{B}$$

$$= +i\omega\vec{B}$$

$$\vec{K}\times\vec{E} = \omega\vec{B}$$

$$\boxed{\vec{B} = \left(\frac{\vec{K}\times\vec{E}}{\omega}\right)} \quad \text{——(11)}$$

As $E \perp^r K$ & $B \perp K$ so

$\vec{E}, \vec{B}$ & $\vec{K}$ are Mutually perpendicular.

dir^n of $\vec{B} = \vec{K}\times\vec{E}$

$eq^n (4) \Rightarrow \quad \vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$i(\vec{K} \times \vec{B}) = \mu_0 \epsilon_0 (-i\omega) \vec{E}$$

$$\vec{K} \times \vec{B} = -\mu_0 \epsilon_0 \omega \vec{E}$$

$$\boxed{\vec{E} = \frac{-c^2(\vec{K} \times \vec{B})}{\omega}} \quad \text{---} (12)$$

When wave vector K & mag. field B are given & find e. field $\vec{E}$ then use above relation directely.

dir^n of $\vec{E} \rightarrow -(\vec{K} \times \vec{B})$

from (12) $\rightarrow$ $\vec{E}, \vec{B}, \vec{K}$ are mutually $\perp^r$.

In free space, Electromag. waves are plane waves.

Let $\vec{E} = E\hat{x}$

$\vec{B} = B\hat{y}$

Hence $\vec{K} = k\hat{z}$

Now, $$\boxed{\vec{E} = E_0 e^{i(kz - \omega t)} \hat{x}} \qquad (\vec{K} \cdot \vec{r} = kz)$$

& $$\boxed{\vec{B} = B_0 e^{i(kz - \omega t)} \hat{y}}$$

- $E = E_0 e^{i(kz - \omega t)} \hat{x}$ — $\hat{x} \longrightarrow$ dir^n of E. field; $\omega t \longrightarrow$ freq. of Vibration of field is ω; $E_0 \downarrow$ amplitud of wave; $kz \downarrow$ dir of propogation is $+z$

- If $E = E_0 e^{i(\omega t - kz)} \hat{x}$

  still dir^n of ~~field~~ propogation is $+\hat{z}$

- Either $(\omega t - kz)$ or $(kz - \omega t)$ then dir^n of prop. is $+z$

  i.e. b/w ωt & kz, one plus, one minus then $+z$.

- If sign of ωt & kz is same then dir^n of propogation $-\hat{z}$

  i.e. $\vec{E} = E_0 e^{i(-\omega t - kz)} \hat{z} \longrightarrow -\hat{z}$

  $= E_0 e^{i(\omega t + kz)} \hat{z} \longrightarrow -\hat{z}$

* $\boxed{\begin{array}{l}\text{Among } \omega t \text{ \& } kx \\ \quad \text{same sign} \longrightarrow -\hat{z} \\ \quad \text{opposite sign} \longrightarrow +\hat{z}\end{array}}$

Energy Density :- (u) Energy per unit volume.

Electric field vector gives the Electric energy density u_e & magnetic " " " " magnetic " " u_m

$$u_e = \frac{\varepsilon_0}{2}\int_{\text{all space}} E^2 \, d\tau \qquad E \rightarrow \text{Total elec. field}$$

$$\boxed{u_e = \frac{\varepsilon_0}{2} E^2}$$

& $u_m = \frac{1}{2\mu_0}\int_{\text{all space}} B^2 \, d\tau$

$$\boxed{u_m = \frac{B^2}{2\mu_0}}$$

We have, $\vec{B} = \frac{\vec{K} \times \vec{E}}{\omega}$ $\qquad \vec{K} \rightarrow K\hat{z}$, $\vec{E} \rightarrow E\hat{x}$ $\qquad \{\hat{z} \times \hat{x} = \hat{y}\}$

$$\vec{B} = \frac{KE}{\omega}\hat{y}$$

$\boxed{v = \frac{\omega}{K}}$ = wave velocity (in free space)

$$\text{so} \quad \vec{B} = \frac{E}{c}\hat{y}$$

$$\therefore \quad u_m = \frac{B^2}{2\mu_0} = \frac{E^2}{2c^2\mu_0} = \frac{E^2 \mu_0 \varepsilon_0}{2\mu_0} = \frac{\varepsilon_0 E^2}{2}$$

$$\boxed{u_m = u_e}$$

Mag. field energy = Electric field energy.

In free space, Mag. field & Ele. field carry equal energy. This energy remain in field, i.e. mag. energy remain in mag. field & E. energy remain in Elec. field.

Total Energy density $\boxed{u = u_e + u_m = \varepsilon_0 E^2}$

Energy Flux or Poynting Vector ($\vec{S}$) :-

Poynting Vector is defined as Energy per unit area per unit time carried by the electromagnetic wave.

OR Power per unit area is called Poynting Vector.

Unit :- $J/m^2\text{-sec}$ or W/m^2 (watt/m²)

$$\vec{S} = \vec{E} \times \vec{H}$$

$$= \frac{\vec{E} \times \vec{B}}{\mu_0}$$

dir^n of $E \rightarrow \hat{x}$
$B \rightarrow \hat{y}$
$(\hat{x} \times \hat{y} \rightarrow \hat{z})$

$$\vec{S} = \frac{EB}{\mu_0}\hat{z}$$

$$B = \frac{E}{c} \Rightarrow \vec{S} = \frac{E^2}{\mu_0 c}\hat{z}$$

$$\vec{S} = \frac{E^2}{\mu_0 c} \times \frac{c}{c}\hat{z} = \frac{E^2 c}{\mu_0 c^2} = \frac{E^2 \cancel{\mu_0} \epsilon_0 c}{\cancel{\mu_0}}$$

$$\vec{S} = c\,\epsilon_0 E^2 \hat{z}$$

$$\boxed{\vec{S} = cu\hat{z}} \qquad (u = \epsilon_0 E^2)$$

This is Relation b/w Poynting vector & energy density.

$\vec{S} \rightarrow$ tells the dir^n of Energy propogation.

$\vec{K} \rightarrow$ " " " wave "

Generally, dir^n of $\vec{S}$ & $\vec{K}$ matches but not every time.

In present case, dir^n of $\vec{K} = \hat{z}$
dir^n of $\vec{S} = \hat{z}$

In free space, dir^n of Wave propogation is same as dir^n of energy flow.

Electromag. wave not only carry the energy but also carry the momentum.

Momentum Density of EM Wave :-

Total mom. per unit volume called Mom. density.
denoted by $\vec{p}$

In free Space, $\vec{p} = \mu_0 \varepsilon_0 \vec{S}$

$$\boxed{\vec{p} = \frac{\vec{S}}{c^2} = \frac{u}{c}\hat{z}} \qquad (\vec{S} = cu)$$

This is called Electromag. mom. density.

Average Value of these quantities :-

We have, Energy density $u = \varepsilon_0 E^2$

Poynting vector $\vec{S} = c\,\varepsilon_0 E^2 \hat{z}$

Momentum density $p = \frac{\varepsilon_0 E^2}{c}\hat{z}$

Now, we calculate average energy density, Average poynting vector & average mom. density over a cycle.

We know, the solutions are

$$\vec{E} = E_0 e^{i(kz-\omega t)}\hat{x}$$

$$\vec{B} = B_0 e^{i(kz-\omega t)}\hat{y}$$

We can write,

$$\vec{E} = E_0\Big[\underbrace{\cos(kz-\omega t)}_{\text{real Part}} + i\underbrace{\sin(kz-\omega t)}_{\text{imaginary Part}}\Big]\hat{x}$$

Only Real part carries the energy.

$$E^2 = \vec{E}\cdot\vec{E} = E_0^2 \cos^2(kz-\omega t)$$

Average Value of energy density

$$\langle u \rangle = \varepsilon_0 E_0^2 \langle \cos^2(kz-\omega t)\rangle$$

Average value of $\cos^2$ over a cycle of 2π gives $\frac{1}{2}$.

so $\boxed{\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2}$ → we can also write it in terms of mag. field (B_0) → amp of mag. field

E_0 → amplitude

This is the combination of electric & magnetic energy density

So seperately,

$$\langle u_e \rangle = \frac{1}{4}\varepsilon_0 E_0^2$$

$$\langle u_m \rangle = \frac{1}{4}\varepsilon_0 E_0^2$$

Relation b/w amplitude of elec & mag. field, $B_0 = \frac{E_0}{c}$

$$\Rightarrow E_0 = B_0 c$$

$$\Rightarrow E_0^2 = B_0^2 c^2$$

$$\Rightarrow E_0^2 = \frac{B_0^2}{\mu_0 \varepsilon_0}$$

So $\boxed{\langle u \rangle = \frac{1}{\cancel{2}4}\varepsilon_0 E_0^2 = \frac{B_0^2}{4\mu_0}}$

<u>Average value of poynting vector</u> also called <u>Intensity</u>

it is the average energy per unit area per unit time associated with the EM wave.

$$\langle \vec{S} \rangle = I$$

e.g. An EM wave of given intensity, incident on $1\,m^2$ area for 10 sec then calculate energy transfered to the surface.

$I = 2\ J/m^2\text{-sec}$

energy / unit area / unit time

So Energy $= 2 \times 10 = 20$ Joule

<u>Now</u>,

$$\langle \vec{S} \rangle \equiv I \text{ (intensity)}$$

$$= c \langle u \rangle \hat{z}$$

$\boxed{\langle \vec{S} \rangle = \frac{1}{2}\varepsilon_0 c E_0^2 \hat{z}}$ → This much intensity is transfe to the surface.

This can be in another form (in terms of mag. field)

<u>Average value of Mom. density</u>

Mom. density → mom. / unit volume.

If Mom. density of EM wave is given ~~for~~ a unit volume & calculate mom. transfer to the given volume then

$$\langle \vec{p} \rangle = \frac{1}{2}\frac{1}{c}\,\varepsilon_0 \langle E_0^2 \rangle$$

$$\boxed{\langle \vec{p} \rangle = \frac{\varepsilon_0}{2}\frac{E_0^2}{c}}$$

This much avg. mom./unit volume will transfer to the volume.

Radiation pressure (P)

Pressure → force per unit area

$$P = \frac{F}{A}$$

If a ~~surface~~ is made of atoms ~~contain~~ charges & e⁻ ~~selove~~ revolve in the orbites. If EM wave incident on the surface of atom. Then the force applied per unit area is called the Radiation pressure.

force → Rate of change of momentum
i.e. mom. per unit time

We know the mom. density which is mom./unit volume.
i.e. if Mom. density × volume = ~~force~~ mom.
& ~~force~~ mom per unit ~~area~~ time gives force & pressure.
force & force/unit area gives pressure.

$$\langle \vec{p} \rangle = \frac{1}{2}\frac{\varepsilon_0 E_0^2}{c}$$

Let Mom. transfered in time Δt is Δp

$$\Delta p = \langle p \rangle A c \Delta t$$

{ If EM wave travell with speed of light then in time Δt it will travell distance $c\Delta t$. }

time = distance / speed = $\frac{c\Delta t}{c}$

Now force

$$F = \frac{\Delta p}{\Delta t} = \frac{1}{2}\frac{\varepsilon_0 E_0^2}{\cancel{c}} \times \cancel{c} A$$

Pressure $P = \frac{F}{A} \Rightarrow \boxed{P = \frac{c}{2c}\varepsilon_0 E_0^2 = \frac{I}{c}}$

This much pressure will be on the surface due to electromag. wave.
So Radiation pressure is Intensity devided by speed of light of EM wave.

→ This is the radiation pressure if the ~~master~~ medium is perfectely Absorbing ($P = \frac{I}{c}$)

→ If surface is perfeectely reflecting then pressure will be doubled.

$$\boxed{P = \frac{2I}{c}}$$

Note:-

- Metal reflects EM waves (Mirrors are reflectors) bcoz metal contains large no. of free e^-s. so free e^-s are responsible for this reflection.

→ If we incident EM wave on a mirror then free e^- starts to vibrate with the freq. of EM wave then it will oscillate & emit radiation. (i.e. e^- Reradiate the EM wave) in this phenomenon No time lag.

→ When EM wave incident on surface then (one) pressure produce. & when it reradiate then again there will be pressure so pressure will be doubled.

→ If for perfeetelly Absorbing medium, when wave re-radiate then there will be no pressure. So in this case there is single pressure.

Microwaves & EM wave they can travell with Insulators.

Wave Impedence of free Space :-

It is denoted by Z_0. & defined as

$$Z_0 = \left|\frac{E_0}{H_0}\right|$$

E_0 → Amp. of E-field

H_0 → Amp. of mag. field intensity.

$$B_0 = \mu H_0 \Rightarrow H_0 = \frac{B_0}{\mu}$$

$$\text{So } Z_0 = \left|\frac{E_0}{H_0}\right| = \frac{\mu_0 E_0}{B_0} = \mu_0 c = \mu_0 \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\boxed{Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi}$$ This is the impedence of free space.

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi = 377\ \Omega$$

In free space, Z_0 is a real quantity.

Hence $\vec{E}$ & $\vec{B}$ vibrates in <u>same phase</u>. (in free space)

$$\vec{E} = E_0 \cos(kz - \omega t)\ \hat{x}$$

$$\vec{B} = B_0 \cos(kz - \omega t)\ \hat{y}$$

There is No phase diff.

<u>Important points of free space</u> :-

→ EM waves are transvers in free space.

i.e. $E \perp K$ & $B \perp K$

$E \perp B \perp K$

They are moving with speed of light c.

→ $\vec{E}$ & $\vec{B}$ are vibrating in same phase

→ Same energy lies in $\vec{E}$ + $\vec{B}$ field.

Electromagnetic waves in Isotropic Linear Dielectric Medium :-

for <u>linear Isotropic</u> (dielectric medium), $\vec{D} = \varepsilon \vec{E}$, $\vec{B} = \mu \vec{H}$

~~$D = \varepsilon E$~~

Maxwell's Eqn

$$\left.\begin{aligned} \vec{\nabla}\cdot\vec{D} &= \rho_f \\ \vec{\nabla}\cdot\vec{B} &= 0 \\ \vec{\nabla}\times\vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla}\times\vec{H} &= \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \end{aligned}\right\}$$

for dielectric medium, No free charge, No free charge density.

$$\rho_f = 0$$

$$J_f = 0$$

$$\Rightarrow \quad \nabla\cdot(\varepsilon E) = 0 \Rightarrow \boxed{\vec{\nabla}\cdot\vec{E} = 0} \quad \text{——— (1)}$$

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0} \quad \text{——— (2)}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad \text{——— (3)}$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu\varepsilon\frac{\partial \vec{E}}{\partial t}} \quad \text{——— (4)}$$

Take curl of eqⁿ (3),

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

$$\Rightarrow \nabla^2 \vec{E} - \mu \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \text{——— (5)}$$

Take curl of eqⁿ (4),

$$\nabla^2 \vec{B} - \mu \varepsilon \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \quad \text{——— (6)}$$

If we compare these (2) IInd order eqⁿ with original wave eqⁿ, ~~we get~~

$$\nabla^2 f - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = 0 \quad \text{——— (7)}$$

We get $v = \frac{1}{\sqrt{\mu \varepsilon}}$ ——— (8)

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

$$v < c$$

So In linear isotropic dielectric, speed of EM wave in that medium is less than speed of light in free space for Any Transparent Medium,

$$v = \frac{c}{n} \quad \text{——— (9)}$$

$n \to$ refractive index of medium,
$v \to$ wave speed

Comparing (8) & (9), we get

$$\boxed{n = \sqrt{\mu_r \varepsilon_r}}$$

So Refractive index of the medium can be calculated by Relative permitivity & Relative permiability.

If medium is Non magnetic $\to \mu_r = 1$

then $\boxed{n = \sqrt{\varepsilon_r}}$

Now, Maxwell's eqn implies the solutions,

$$\left.\begin{aligned}\vec{K}\cdot\vec{E} &= 0 \\ \vec{K}\cdot\vec{B} &= 0\end{aligned}\right\}$$ These eqns implies that →

→ Inside isotropic dielectric, EM waves are transverse wave.

If E & K is given & we have to find B then

$$\boxed{\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}}$$

$$\Rightarrow \cancel{i}(\vec{K}\times\vec{B}) = -\cancel{i}\omega\mu\varepsilon\vec{E}$$

$$\vec{E} = -\frac{1}{\mu\varepsilon\omega}(\vec{K}\times\vec{B})$$

$$\boxed{\vec{E} = -\frac{v^2}{\omega}(\vec{K}\times\vec{B})} \qquad \left\{ v = \frac{1}{\sqrt{\mu\varepsilon}} \right.$$

Now, Magnitude of $\vec{B}$,

$$\boxed{|\vec{B}| = \frac{|\vec{E}|}{v}}$$

where $\frac{\omega}{K} = v$

If wave is travelling in isotropic dielectric medium then $\left[\frac{\omega}{K} = v\right]$

Conclusion :- $\vec{E} \perp \vec{B} \perp \vec{K}$ i.e. mutually $\perp$ to each other.

→ Energy density :-

electric energy density $u_e = \frac{\varepsilon}{2}E^2$

Magnetic " " $u_m = \frac{B^2}{2\mu} = \frac{E^2\mu\varepsilon}{2\mu} = \frac{E^2\varepsilon}{2}$

$$u_m = \frac{1}{2}\varepsilon E^2$$

$$\Rightarrow \boxed{u_e = u_m},$$

Total energy density $u = u_e + u_m$

$$\boxed{u = \varepsilon E^2}$$

→ Poynting Vector :- $\vec{K}\cdot\vec{E} = 0$, $\vec{K}\cdot\vec{B} = 0$

$$\vec{E} = E_0\, e^{i(kz-\omega t)}\,\hat{x}$$

$$\vec{B} = B_0\, e^{i(kz-\omega t)}\,\hat{y}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$= \frac{\vec{E} \times \vec{B}}{\mu} = \frac{1}{\mu v} E^2 = \frac{v}{\mu v^2} E^2 \qquad (B = E\sqrt{\mu\varepsilon} = E/v$$

$$\boxed{\vec{S} = \varepsilon v E^2 \hat{z} = v u \hat{z}} \qquad v^2 = \frac{1}{\mu\varepsilon}$$

So Energy flow is in the dirn of wave propagation.
(dirn of wave propagation is $\hat{z}$)

On Comparing with free space, We get if we replace

$\mu_0 \longrightarrow \mu$

$\varepsilon_0 \longrightarrow \varepsilon$

$c \longrightarrow v$

then we get, expressions for isotropic dielectric

<u>Momentum density</u> $\quad \vec{p} = \mu\varepsilon\vec{S} = \frac{u}{v}\hat{z}$

<u>Wave Impedence</u> $\quad Z = \left|\frac{E}{H}\right| = \sqrt{\frac{\mu}{\varepsilon}}$

$$Z = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

If $\left.\begin{matrix}\mu_r \geq 1 \\ \varepsilon_r \geq 1\end{matrix}\right\}$ $\quad Z = 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}}$ = Real Value

If Z is Real that means $\vec{E}$ & $\vec{B}$ are vibrating in same phase.

<u>Conclusion</u> :-

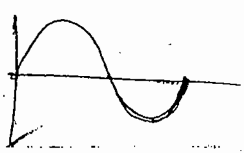
→ speed of medium v is less than c. & → $n = \sqrt{\mu_r \varepsilon_r}$

→ for a linear isotropic dielectric medium

$\mu_r \geq 1$ & $\varepsilon_r \geq 1$

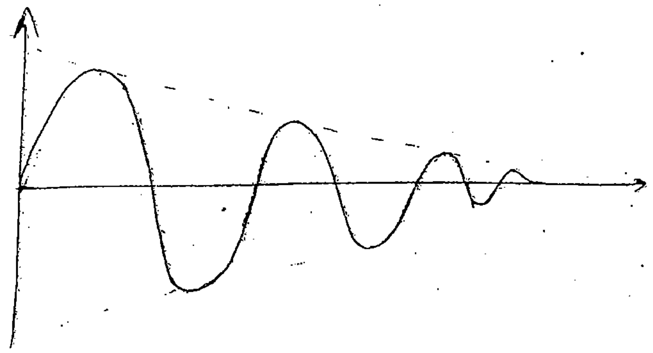
(i) If $\varepsilon_r = 1$, $\mu_r = 1$ that means free space
free space is also a dielectric medium of dielectric constant one.

(ii) If $\varepsilon_r > 1$, $\mu_r > 1$ then n is real
then soln of EM will be Oscillatory



(iii) If $\varepsilon_r < 0$ (-ve), $\mu_r \geq 1$ $\Rightarrow n \rightarrow$ imaginary

If n is imaginary or complex, then solution of EM wave will be damped.



If $\varepsilon_r \geq 0$, $\mu_r < 0$, Again sol$^n \rightarrow$ damped

(Inside the conducting medium)

- In all these cases, dir^n of $\vec{S}$ & $\vec{K}$ will be same.

Energy flow wave prop.

Here If, ε_r & μ_r are not the fun^n of position then medium will be isotropic.

<u>Note</u> :- If ε_r & μ_r are fun^n of position then dir^n of $\vec{S}$ & $\vec{K}$ are different.

<u>Case (iv)</u> :- If ε_r & μ_r are simultanously negative. then $\vec{S}$ & $\vec{K}$ are antiparallel.

$$\left\{\begin{array}{l}\mu = \mu_0 \mu_r \\ \varepsilon = \varepsilon_0 \varepsilon_r\end{array}\right.$$

No medium exist in nature, in which both ε_r & μ_r are simultanously -ve.

For Left Handed Material, $\vec{S}$ & $\vec{K}$ are antiparallel.

i.e. This type of medium is called L.H. material.

Electromagnetic Waves inside an Anisotropic Linear Dielectric Medium :-

Generally, for dielectrics, permiability

$\mu \simeq \mu_0$

This meadium is Anisotropic w.r. to permitivity ε only.

Maxwell's eqn,   Linear $\rightarrow$ $\vec{D} = \varepsilon \vec{E}$

$\vec{\nabla} \cdot \vec{D} = \rho_f \Rightarrow \boxed{\vec{\nabla} \cdot \vec{D} = 0}$ — (1)   Dieletric $\rightarrow$ $\rho_f = 0$

$\Rightarrow \nabla \cdot (\varepsilon \vec{E}) = 0$ Here ε is not const.

but $\nabla \cdot \vec{E} \neq 0$

$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$ — (2)

$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$ — (3)

$\vec{\nabla} \times \vec{H} = J_f + \frac{\partial \vec{D}}{\partial t}$ for dielectric $J_f = 0$

$\Rightarrow \boxed{\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}}$ — (4)

$\vec{\nabla} \cdot \vec{D} = 0 \Rightarrow \vec{K} \cdot \vec{D} = 0$

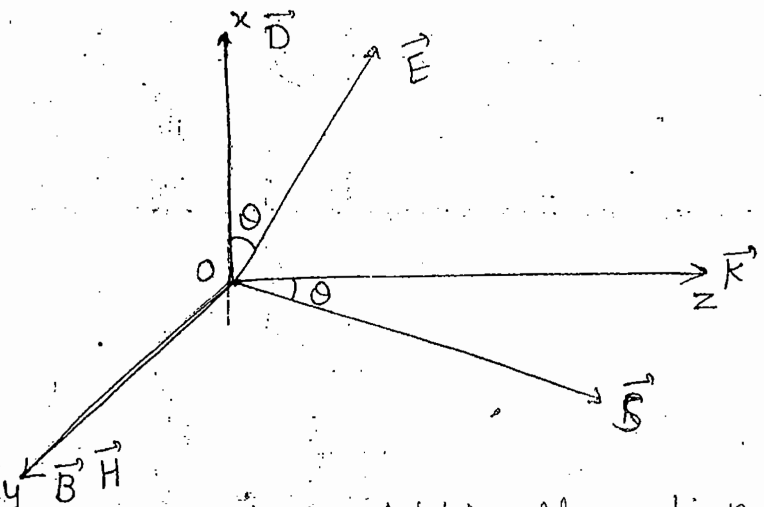
$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \vec{K} \cdot \vec{B} = 0$

i.e. $\vec{B}$ & $\vec{D}$ are $\perp^r$ to wave propogation i.e.

$\vec{B} \perp \vec{D} \perp \vec{K}$

$\vec{S} = \vec{E} \times \vec{H}$

& $\vec{B} = \mu_0 \vec{H}$

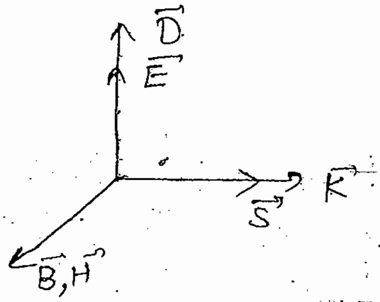


i.e. If medium is Anisotropic then dirn of energy flow is not same as wave prop.

Main Points :-

- EM waves in Anisotropic medium are transverse w.r.to $\vec{B}$ & $\vec{H}$. Not w.r.to $\vec{E}$ & $\vec{D}$.
- Dirn of Energy flow is not same as the dirn of wave propogation.
- The Electric field is making angle θ with D. Hence $\vec{S}$ is also making angle θ with wave propogation K.

<u>for Isotropic</u> :-



→ <u>Doubly refecting Systems</u> ($n_x \neq n_y$) [ref. index is not same in all dirn.] are the examples of Anisotropic medium.

<u>Electromag. Waves in Conducting Medium</u> :-

Maxwell's eqn,

$$\vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \vec{J}_f \times \frac{\partial \vec{D}}{\partial t}$$

for a conducting Medium, $\vec{D} = \varepsilon\vec{E}$ & $\vec{B} = \mu\vec{H}$

volume free charge $\rho_f = 0$ & $\boxed{\vec{J}_f = \sigma\vec{E}}$

If we put any free charge in conducting medium then free charge will be on surface. No free charge can reside inside the conducting medium.

$$\vec{\nabla}\cdot\vec{D} = 0 \Rightarrow \boxed{\vec{\nabla}\cdot\vec{E} = 0} \quad \text{——— (1)}$$

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0} \quad \text{——— (2)}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}} \quad \text{——— (3)}$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu\sigma\vec{E} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}} \quad \text{——— (4)}$$

Take curl of eqⁿ (3),

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}\left[\mu\sigma\vec{E} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}\right]$$

$$\nabla^2 E = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 E - \mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0} \quad \text{——— (5)}$$

Take curl of eqⁿ (4),

$$\vec{\nabla}\times\vec{\nabla}\times\vec{B} = \mu\sigma(\vec{\nabla}\times\vec{E}) + \mu\varepsilon\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B} = \mu\sigma\left(-\frac{\partial\vec{B}}{\partial t}\right) + \mu\varepsilon\left(-\frac{\partial^2\vec{B}}{\partial t^2}\right)$$

$$\boxed{\nabla^2 B - \mu\sigma\frac{\partial\vec{B}}{\partial t} - \mu\varepsilon\frac{\partial^2\vec{B}}{\partial t^2} = 0} \quad \text{——— (6)}$$

eqⁿ (5) & (6) are IInd order differential eqⁿs.

IInd term, of eqⁿ (5) & (6) are damping terms, coms from damped harmonic oscillator. $\frac{d^2x}{dt^2} - \omega^2 x = \underbrace{\frac{dx}{dt}}$

Bcoz of this term, amplitude of damped harmonic Oscillator is decaying.

Now, we take wave vector K is a complex quantity in the solutions of wave eqⁿ.

$$\vec{E} = E_0 e^{i(\vec{K}^*\cdot\vec{r} - \omega t)}$$

$$\vec{B} = B_0 e^{i(\vec{K}^*\cdot\vec{r} - \omega t)}$$

These solutions must satisfy the given diff. eqⁿ.

$$(5) \Rightarrow \quad (iK^*)^2\vec{E} - (-i\omega)\mu\sigma\vec{E} - (-i\omega)^2\mu\varepsilon\vec{E} = 0$$

$$-(K^*)^2\vec{E} + i\omega\mu\sigma\vec{E} + \mu\varepsilon\omega^2\vec{E} = 0$$

(from (6) → We get same eqⁿ)

$$\boxed{(K^*)^2 = \mu\varepsilon\omega^2 + i\omega\mu\sigma}$$

$$\boxed{K^* = [\mu\varepsilon\omega^2 + i\omega\mu\sigma]^{1/2}} \quad \text{——— (7)}$$

Wave vector is Complex.

Let $K^* = \alpha + i\beta$

$\alpha \rightarrow$ real part
$\beta \rightarrow$ imaginary part

$$\boxed{(K^*)^2 = \alpha^2 - \beta^2 + 2i\alpha\beta} \quad \text{——— (8)}$$

Compare (7) & (8) $\Rightarrow$

$$\alpha^2 - \beta^2 = \mu\varepsilon\omega^2 \quad \text{——— (9)}$$

$$2\alpha\beta = \omega\mu\sigma \quad \text{——— (10)}$$

We have 2 unknowns α & β.

$$(10) \Rightarrow \quad \beta = \frac{\omega\mu\sigma}{2\alpha}$$

$$(9) \Rightarrow \quad \alpha^2 - \frac{\omega^2\mu^2\sigma^2}{4\alpha^2} = \mu\varepsilon\omega^2$$

$$4\alpha^4 - \omega^2\mu^2\sigma^2 = 4\mu\varepsilon\omega^2\alpha^2$$

$$\cancel{4}\alpha^4 - \cancel{4}\mu\varepsilon\omega^2\alpha^2 - \frac{1}{4}\omega^2\mu^2\sigma^2 = 0$$

$$\alpha^2 = \frac{\mu\varepsilon\omega^2 \pm \sqrt{(\mu\varepsilon\omega^2)^2 + \omega^2\mu^2\sigma^2}}{2}$$

$$\alpha^2 = \frac{\mu\varepsilon\omega^2 \pm \mu\varepsilon\omega^2\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}}{2}$$

$$\alpha^2 = \frac{\mu\varepsilon\omega^2}{2}\left[1 + \sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}\right]$$

$$\Rightarrow \quad \alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2} \quad \text{——— (11)}$$

||ly for β, $\alpha = \frac{\omega\mu\sigma}{2\beta}$, we get

$$4\beta^4 + \mu\varepsilon\omega^2\beta^2 - \frac{1}{4}\omega^2\mu^2\sigma^2 = 0$$

$$\Rightarrow \quad \beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2} \quad \text{——— (12)}$$

$$\vec{K}^* = k^* \hat{n}$$
$$= (\alpha + i\beta)\hat{n}$$
$$\vec{E} = E_0 e^{i[(\alpha+i\beta)(\hat{n}\cdot\vec{r}) - \omega t]}$$
$$\boxed{\vec{E} = E_0 e^{-\beta(\hat{n}\cdot\vec{r})} e^{i[\alpha(\hat{n}\cdot\vec{r}) - \omega t]}} \quad \text{——— (13)}$$
$$\boxed{\vec{B} = B_0 e^{-\beta(\hat{n}\cdot\vec{r})} e^{i[\alpha(\hat{n}\cdot\vec{r}) - \omega t]}} \quad \text{——— (14)}$$

Now Amplitude is exponentially ↓ with space (Not with time).

Decay will be fast or slow, it will depand upon β.

→ β is called Atenuation Const. or cofficient.

→ α is propogation cofficient.

→ β depands upon certain properties of medium, depands upon → ε, μ, σ

If medium is dielectric → $\sigma = 0$ → $\beta \to 0$

then no attenuation in wave.

→ Attenuation in wave is due to σ & conductivity σ is due to free e^-s. {for dielectric → No free e^- so $\sigma = 0$}

→ Here α is like k (wave vector)

<u>Wave Velocity</u> $\quad v = \dfrac{\omega}{\alpha}$

$$v = \sqrt{\frac{2}{\varepsilon\mu}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^{2}} + 1\right]^{-1/2}$$

This is the wave velocity inside the conducting medium

If we put $\sigma = 0$ then we get expression for dielectric.

$$v = \sqrt{\frac{2}{\varepsilon\mu}}\,\frac{1}{\sqrt{2}}$$

$$\boxed{v = \frac{1}{\sqrt{\mu\varepsilon}}}$$

Wave velocity is decreased as compare to dielectric.

Distinction b/w Good Conductor & Bad Conductor :-

Conductor is good or bad - It depands upon its

- → Conductivity (σ)
- → Relaxation time (τ)

if we put a free static charge inside the conductor then how quickely that charge come out to the surface - that time will be the Relaxation time.

If conductor is bad, Its relaxation time will be more, it take more time to come out to the surface.

Relaxation time is given by permitivity to conductivity ratio.

$$\tau = \frac{\varepsilon}{\sigma}$$

Now If we fall an EM wave over a conductor of conductivity σ & per. ϵ, we know $\vec{E}$ inside the conductor is 0. so amplitude of EM wave inside the conductor must be zero. This E. field is not electrostatic, it is dynamic (changing with time) Here ω is the freq. of oscillation of Elec. & mag. field.

So Conductor is good or bad → it is Not only τ will depand of σ & τ but also depand on freq. ω.

→ If a conductor is good for a perticular freq. ω, It may be bad for some another freq.

If $\tau << \frac{1}{\omega}$

$\Rightarrow$ Good Conductor

If $\tau >> \frac{1}{\omega}$ $\Rightarrow$ Bad Conductor

(i) $\Rightarrow$ $\frac{\varepsilon}{\sigma} << \frac{1}{\omega}$ $\Rightarrow$ $\frac{\sigma}{\varepsilon\omega} >> 1$ $\Rightarrow$ good conductor

(ii) $\Rightarrow$ $\frac{\sigma}{\varepsilon\omega} << 1$ $\Rightarrow$ bad conductor

Inside the conductor, free volume charge density.

$\rho_f = 0$.

At $t=0$, we put ρ_f inside conductor & after some time ρ_f becomes 0. So have to find that time in which ρ_f become 0 from ρ_f.

Use Continuity eqn,

$$\vec{\nabla}\cdot\vec{J}_f = -\frac{\partial \rho_f}{\partial t}$$

We have, $J_f = \sigma E$

so $\sigma(\vec{\nabla}\cdot\vec{E}) = -\frac{\partial \rho_f}{\partial t}$

$$\nabla\cdot E = \frac{\rho_f}{\varepsilon}$$

$$\Rightarrow \frac{\sigma}{\varepsilon}\rho_f = -\frac{\partial \rho_f}{\partial t}$$

{ $\nabla\cdot E = 0$ only when all the charge come out of surface.

$$\Rightarrow \frac{\partial \rho_f}{\rho_f} = -\frac{\sigma}{\varepsilon}dt$$

$$\Rightarrow \ln \rho_f(t) = -\frac{\sigma}{\varepsilon}t + c$$

At time $t=0$, $c = \ln \rho_f(0)$

$$\Rightarrow \ln \rho_f(t) = -\frac{\sigma}{\varepsilon}t + \ln \rho_f(0)$$

$$\Rightarrow \ln \frac{\rho_f(t)}{\rho_f(0)} = -\frac{\sigma}{\varepsilon}t$$

$$\Rightarrow \rho_f(t)/\rho_f(0) = e^{-\frac{\sigma}{\varepsilon}t}$$

$$\Rightarrow \boxed{\rho_f(t) = \rho_f(0)e^{-\frac{\sigma}{\varepsilon}t}}$$

It defines - How the free charge decays with time exponentially inside the conductor.

If we have a Perfect Conductor,

$\sigma = \infty$

so free charge inside $= 0$

Relaxation time $\tau = \frac{\varepsilon}{\sigma}$ (if $\sigma = \infty$)

$\boxed{\tau = 0}$

i.e. No time lag in putting the charge & come out to the surface.

for *Insulator* $\sigma = 0$

$$\boxed{\tau = \frac{\varepsilon}{\sigma} = \infty}$$

i.e. charge take ∞ time to come out to the surface.
i.e. It can never come out to the surface.

Note :- $\vec{\nabla}\cdot\vec{D} = \rho_f$ is valid for every medium.
$\vec{\nabla}\cdot(\varepsilon\vec{E}) = \rho_f$ " " only for isotropic medium.

for semiconductor
Conductivity is small but finite.
σ = very small
so τ = large
i.e. It will take more time to come out.

- $\vec{\nabla}\cdot\vec{E} = 0$ is valid only for perfect conductor.

Wave velocity inside good & bad conductor :-

$$v_{good} = \frac{\omega}{\alpha}$$

$$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]^{1/2}$$

for good, $\frac{\varepsilon}{\sigma} << \frac{1}{\omega} \Rightarrow \frac{\sigma}{\varepsilon\omega} >> 1$ so neglect 1 as compare to $\frac{\sigma}{\varepsilon\omega}$

$$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]^{1/2}$$

$$\alpha_{good} = \beta_{good} = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left(\frac{\sigma}{\omega\varepsilon}\right)^{1/2}$$

$$= \sqrt{\omega}\sqrt{\frac{\varepsilon\mu}{2}\left(\frac{\sigma}{\varepsilon\omega}\right)^{1/2}} = \sqrt{\frac{\sigma\omega\mu}{2}}$$

$$\boxed{v_{good} = \frac{\omega}{\alpha} = \sqrt{\frac{2\omega}{\sigma\mu}}}$$

for bad cond. $\frac{\sigma}{\varepsilon\omega} << 1$

$$V_{bad} = \frac{\omega}{\alpha}$$

$$\alpha_{bad} = \omega\sqrt{\varepsilon\mu}$$

$$v_{bad} = \frac{\omega}{\omega\sqrt{\varepsilon\mu}}$$

$$\boxed{v_{bad} = \frac{1}{\sqrt{\varepsilon\mu}}}$$

This is similar to velocity inside the dielectric medium.

Skin Depth (δ) :-

If we incident EM wave on conductor. Inside the conductor, near the surface E-field is not zero but it will travell some distance & then E-field become zero.

Skin depth is the distance at which amplitud of EM wave become 1/e value of the value at the surface.

If E-field at surface is E_0 then after travelling some distance i.e. Skin depth, it become $\frac{E_0}{e}$.

More is β, less is the skin-depth.

$$\boxed{\delta = \frac{1}{\beta}}$$

Skin-depth of free space :- is ∞.

In free-space there is no decay in amp. of wave.

For Good Conductor,

$$\boxed{\delta_{good} = \frac{1}{\beta} = \sqrt{\frac{2}{\sigma\omega\mu}}}$$

for Bad Conductor,
(means it is Insulator)

$\boxed{\delta_{bad} = \infty}$ for perfect Insulator ($\sigma = 0$, $\beta = 0$)

for perfect dielectic (it is a " ")

If conductivity is finite but very small then there will be some skin depth. There will be decay so β will be finite.

$$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\cancel{1}+\frac{1}{2}\left(\frac{\sigma}{\varepsilon\omega}\right)^2-\cancel{1}\right]^{1/2}$$

$$= \omega\sqrt{\frac{\varepsilon\mu}{2}}\ \frac{1}{\sqrt{2}}\left(\frac{\sigma}{\varepsilon\omega}\right) = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\boxed{\delta_{bad} = \frac{1}{\beta_{bad}} = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}}$$

{for perfect insulator, $\sigma = 0$ so $\delta = \infty$}

Phase :- If wave impedence is complex, it defines the phase difference b/w $\vec{E}$ & $\vec{B}$.

$$Z = \left|\frac{E}{H}\right| = \frac{\mu E}{B} = \frac{\mu\omega}{K^*} \qquad \left\{\frac{E}{B} = v = \frac{\omega}{K^*}\right.$$

$$= \frac{\mu\omega}{(\alpha+i\beta)} = \text{complex quantity}$$

Inside the conducting medium $\vec{E}$ & $\vec{B}$ are out of phase.

We know $K^* = \alpha + i\beta = Ke^{i\phi}$

where $K \to$ Amp. $\Rightarrow K = (\alpha^2+\beta^2)^{1/2}$

$\phi \to$ phase difference $\Rightarrow \tan\phi = \frac{\beta}{\alpha}$

We know that

$$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{1/2}$$

$$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]^{1/2}$$

so $K = (\alpha^2+\beta^2)^{1/2} = [\omega$

$$\boxed{K = \omega\sqrt{\mu\varepsilon}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$$

and $\tan\phi = \frac{\beta}{\alpha}$

$$\boxed{\phi = \tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)}$$

This is the phase diff. b/w $\vec{E}$ & $\vec{B}$. If $\sigma = 0$ then $\phi = 0$ i.e. No phase diff.

We have $\vec{B} = \dfrac{\vec{K}^* \times \vec{E}}{\omega}$

$\left\{ \begin{array}{l} \vec{K}^* = K^* \hat{z} \\ \vec{E} = E\hat{x} \\ \text{so } \vec{B} \text{ will be in } \hat{y} \end{array} \right.$

$$\vec{B} = \frac{K^* E}{\omega} \hat{y}$$

$$\vec{B} = \frac{K e^{i\phi}}{\omega} E_0 e^{-\beta(\hat{n}\cdot\hat{r})} e^{i(Kz-\omega t)} \hat{y}$$

$\left\{ \begin{array}{l} K^* = K e^{i\phi} \\ E = E_0 e^{i(Kz-\omega t)} \end{array} \right.$

On putting the Values of K, We get

$$\boxed{\vec{B} = \sqrt{\mu\varepsilon}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4} E_0 e^{-\beta(\hat{n}\cdot\hat{r})} e^{i[Kz-(\omega t-\phi)]} \hat{y}}$$

Phase of $\vec{E}$ = $(kz-\omega t)$

$\vec{B}$ = $[Kz-(\omega t-\phi)]$

→ $\vec{B}$ is lagging behind $\vec{E}$ by phase diff ϕ i.e. $\vec{E}$ is leading by ϕ.

→ The amplitude of $\vec{B}$ is greater than $\vec{E}$ (as $\frac{\sigma}{\omega\varepsilon} >> 1$) becoz, Amp of $\vec{B}$ contains amp. of $\vec{E}$ & also a quantity multiplied by amp. of E (~~which~~ the quantity is greater than 1)

→ E decays faster that's why its amp. is small.

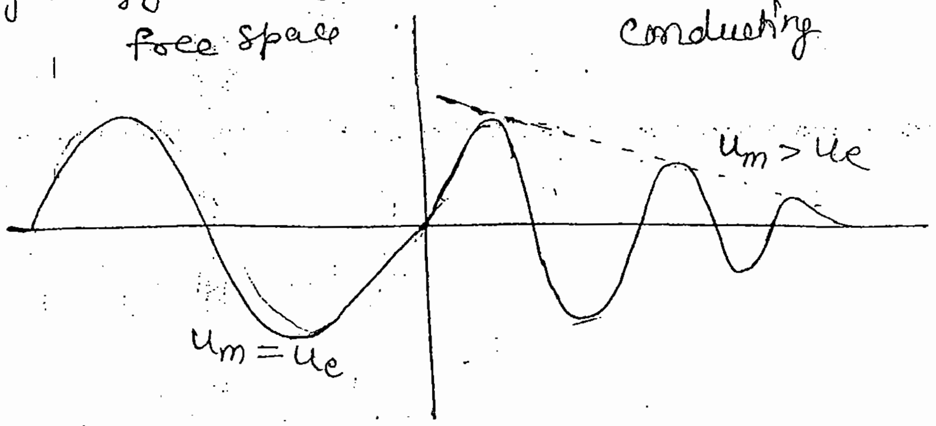
→ Energy density –

$u_m > u_e$

At any particular distance ?

$u_m \propto B^2$ & $u_e \propto E^2$

i.e. mag energy density is greater than electric energy density.



→ Poynting Vector :- $\vec{S} = \dfrac{\vec{E} \times \vec{B}}{\mu} \hat{z}$

$\because \vec{E} \to \hat{x}$, $\vec{B} \to \hat{y}$

$\therefore S \to \hat{z}$

i.e. dirn of energy flow is same as dirn of wave propagation.

We have $\vec{B} = \frac{K}{\omega} \vec{E}$

$$\frac{\vec{B}}{E} = \sqrt{\mu\varepsilon}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}$$

so $\vec{S} = \sqrt{\frac{\varepsilon}{\mu}}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4} E^2 \hat{z}$

$$\vec{S} = \sqrt{\frac{\varepsilon}{\mu}}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4} E_0^2 e^{-2\beta z}\left[e^{2i(Kz-\omega t)}\right]\hat{z}$$

Conclusions in conducting medium

① → Ele. & mag. field amplitude decays exponentially

$E_0 e^{-\beta z}$

$= \boxed{E_0 e^{-\frac{z}{\delta}}}$ $\quad (\delta = \frac{1}{\beta})$

$\vec{K} = K.\hat{n}$
$\hat{n}$ = dirn of prop.
$(\hat{r}.\hat{n} = \hat{z})$

e.g. – given that the skin depth for a certain material $\delta = 10\,nm$. Calculate the amp. of $\vec{E}$ after 100 nm distance into the conductor.

E_0 – field at surface.

z → distance travelled into the medium.

$$E_0 e^{-\frac{z}{\delta}} = E_0 e^{-\frac{100\,nm}{10\,nm}} = E_0 e^{-10}$$

e^{-10} → very very small (negligible)

* If skin depth is given & find the amp. after a distance the use above formula.

② → $E \perp K$, $B \perp K$

i.e. $\boxed{E \perp B \perp K}$

EM waves are transverse.

③ → There is a phase diff. b/w $\vec{E}$ & $\vec{B}$ fields.

$\vec{E}$ leading in phase by angle $\phi = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$

④ → $u_m \neq u_e$ but $u_m > u_e$

so most of the energy lies in the mag. field

& decay of u is $\boxed{u \propto e^{-\frac{2z}{\delta}}}$

(5) $\vec{S} \parallel \vec{K}$

i.e. dirn of Energy flow is along the dirn of wave propogation

i.e. $\vec{S} \propto e^{-2z/\delta}$

Relation b/w Conduction Current density & Displacement Current Density :-

for a medium having conductivity σ & permittivity ε then

$$\vec{J_c} = \sigma \vec{E}$$

$$\vec{J_d} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

If we incident a electromag. wave over such a medium $\vec{E} = E_0 e^{i(kz-\omega t)} \hat{x}$

then

$$\vec{J_c} = \sigma \vec{E}$$

$$\vec{J_d} = \varepsilon(-i\omega)\vec{E}$$

Then

$$\frac{J_c}{J_d} = \frac{\sigma}{\varepsilon(-i\omega)}$$

$$\left|\frac{\vec{J_c}}{\vec{J_d}}\right| = \left|\frac{\sigma}{\omega\varepsilon}\right|$$

for Good conductor, $\frac{\sigma}{\omega\varepsilon} >> 1$

So $\boxed{J_c >> J_d}$

for Poor Conductor, σ is less $\frac{\sigma}{\omega\varepsilon} << 1$

So $\boxed{J_c << J_d}$

{e.g. for a metal, $J_c = 10^6 A/m^2$ & $J_d = 10 A/m^2$ then it is Good Conductor.}

If $\sigma \uparrow$ then $J_c \uparrow$ (more)

If $\varepsilon \uparrow$ or $\omega \uparrow$ then $\vec{J_d} \uparrow$

i.e. $J_c \to$ depend on conductivity

$J_d \to$ " " permitivity & freq.

Propogation of EM Wave in Plasma :-

Plasma is the collection of charge particle, neutral particles & ions. It can consist of +ve & -ve ions.
Its condⁿ is that one of the charge type must be mobil
Plasma contains equal concentration of +ve & -ve ions
so Plasma as a whole is Neutral.
(One of charge type must be mobile but if both type mobile → then no problem)
Plasma is found - in upper side of atmosphere
Ionosphere contains plasma.
Plasma can be made in ~~library~~ laboratory
Use - Plasma is used in Communication. It can reflected only few freq EM waves.
~~It can~~ If we incident some EM wave on ionosphere, it will be reflected back if $\omega < \omega_p$
i.e freq of EM wave is less than a perticular freq called plasma freq (ω_p)
$\omega_p \rightarrow$ depands on free charge carrier concentration.
so Ionospher can reflect radio waves $(\omega < \omega_p)$ but can not reflect visible waves. $(\omega > \omega_p)$

Conductivity of Plasma Medium :- Condⁿ are of 2 types -

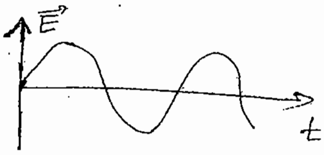
1) Static Conductivity
2) Dynamical "

Static Conductivity :- static means if we apply a const. elec. field on a conductor then its condictivity will be

$$\boxed{\sigma_s = \frac{ne^2\tau}{m}}$$

$n \rightarrow$ no. of free e^- per unit valume, $e \rightarrow e^-$ charge
$\tau \rightarrow$ relaxation time, $m \rightarrow e^-$ mass

Dynamical Cond. :- When we apply a variable E.field (i.e. varing with time) ~~then~~ on a conductor then its conductivity will be variable with freq. (Not const.).
This is called Dynamical conductivity.



$$\boxed{\sigma_d = \frac{ine^2}{m\omega}}$$

$\omega \rightarrow$ freq.

Total Conductivity,

$$\sigma_{Total} = \sigma_S + \sigma_d$$
$$= \frac{ne^2\tau}{m} + \frac{ine^2}{m\omega}$$
$$= \frac{ne^2}{m}\left(\tau + \frac{i}{\omega}\right)$$

$$\sigma_{Total} = \frac{ne^2}{m(\gamma - i\omega)} = \frac{ne^2}{m(\frac{1}{\tau} - i\omega)}$$

$\gamma \rightarrow$ damping factor.

$$\boxed{\sigma_{Total} = \frac{ne^2\tau}{m(1 - i\omega\tau)}}$$

This is the total condn of any conducting material.

For Static Conductivity, $\omega = 0$, condn reduces to then $\sigma_S = \frac{ne^2\tau}{m}$

Metal is also like plasma — In metal, there are +ve & −ve ions. One of charge type must be mobile.
But in actual plasma, charge types(e^-) moves ~~(e^-)~~ freely. & in metal, charge move freely but during ~~b/w~~ moving they scattered with each other. & due to this relaxation time comes.

Ionospher contains dilute plasma so e^- are completely free so there is No relaxation time i.e. No damping.

$\Rightarrow$ So Conductivity of a ionospher or plasma is

$$\sigma_d = \frac{ine^2}{m\omega}$$

For ionosphere or plasma medium, we have to put the value of σ (this is the diff. from conducting medium)

Maxwell's eq^n,

$$\vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \vec{J_f} + \frac{\partial\vec{D}}{\partial t}$$

Inside the plasma → no free charge so $\rho_f = 0$

i.e. if we put some external free charge then it will go on surface of plasma.

For Dilute plasma i.e. for Ionosphere,

$$\varepsilon \simeq \varepsilon_0$$
$$\mu \simeq \mu_0$$

$$\Rightarrow \quad \vec{\nabla}\cdot\vec{E} = 0 \quad \text{———} (1)$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{———} (2)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{———} (3)$$

$$\vec{\nabla}\times\vec{B} = \mu_0\sigma\vec{E} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \quad \text{———} (4)$$

$$\vec{J_f} = \sigma\vec{E}$$
$$D = \varepsilon_0\vec{E}$$

Taking curl of eq^n (3) & put the value of $\vec{\nabla}\times\vec{B}$ from (4)

we get $\quad \nabla\times\nabla\times E = -\frac{\partial(\vec{\nabla}\times\vec{B})}{\partial t}$

$$\nabla(\nabla\cdot E) - \nabla^2 E = -\frac{\partial}{\partial t}\left[\mu_0\sigma\vec{E} + \mu_0\varepsilon_0\frac{\partial E}{\partial t}\right]$$

$$\nabla^2\vec{E} - \mu_0\sigma\frac{\partial\vec{E}}{\partial t} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} = 0 \quad \text{———} (5)$$

Now take curl of (4), We get

$$\nabla^2\vec{B} - \mu_0\sigma\frac{\partial\vec{B}}{\partial t} - \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2} = 0 \quad \text{———} (6)$$

The sol^n is $\vec{E} = E_0 e^{i(\vec{K}\cdot r - \omega t)}$ Put in (5),

$\vec{\nabla} \to iK \ , \ \frac{\partial}{\partial t} \to -i\omega$

eqn (5) $\Rightarrow -K^2\vec{E} + i\mu_0\sigma\omega\vec{E} + \mu_0\varepsilon_0\omega^2\vec{E} = 0$

$\Rightarrow K^2 = i\mu_0\sigma\omega + \mu_0\varepsilon_0\omega^2$

Put $\sigma = \frac{ine^2}{m\omega}$

$$K^2 = -\frac{ne^2\mu_0}{m} + \mu_0\varepsilon_0\omega^2$$

$$= \mu_0\varepsilon_0\omega^2\left[1 - \frac{ne^2}{m\varepsilon_0\omega^2}\right]$$

where $\boxed{\omega_p = \left(\frac{ne^2}{\varepsilon_0 m}\right)^{1/2}}$ is called Plasma freq. —— (7)

$$K^2 = \frac{\omega^2}{c^2}\left[1 - \frac{\omega_p^2}{\omega^2}\right]$$

$$\boxed{K = \frac{\omega}{c}\left[1 - \frac{\omega_p^2}{\omega^2}\right]^{1/2}} \quad \text{—— (8)}$$

$K \to$ propogation coffi. or Amp. of wave vector

$\omega \to$ freq. of EM wave

(i) $\omega_p \to$ plasma freq., it is the property of medium it depend on e^- concentration (power 1/2)

$$\boxed{\omega_p \propto n^{1/2}}$$

(i) If any EM wave have freq ω & ($\omega < \omega_p$) then

$\omega < \omega_p$, $K \to$ imaginary $\to$ No propogation

if K is imaginary then ref. index will be imaginary.

& If ref. index is imaginary then wave propogation through that medium is Not possible & wave will be damped.

Wave can not propogate so it will reflect and go back into original medium.

All the EM wave having freq. $\omega < \omega_p$ can not travell through Plasma.

(ii) If $\omega > \omega_p$, $K \to$ Real, $n \to$ real

So wave will propogate. $\longrightarrow$ propogation

Refractive Index of plasma medium :-

$$n = \frac{c}{v}$$

and $v = \frac{\omega}{k} = c\left[1 - \frac{\omega_p^2}{\omega^2}\right]^{-1/2}$ (from 8)

So $n = \frac{c}{c}\left[1 - \frac{\omega_p^2}{\omega^2}\right]^{1/2}$

$$\boxed{n = \left[1 - \frac{\omega_p^2}{\omega^2}\right]^{1/2}}$$ (Real)

(for $\omega < \omega_p$ then $n \to$ imaginary) [if $\omega > \omega_p$ then $n \to$ real]

Expression for Cut off frequency in Plasma medium :-

Plasma freq. is behaving like cut off freq.

If $\omega < \omega_p \rightarrow$ wave can not propogate

$\omega > \omega_p \rightarrow$ wave can propogate.

So Plasma medium is High Pass filter.

$$\omega_p = \left(\frac{n e^2}{\varepsilon_0 m}\right)^{1/2}$$

Linear Plasma freq. $f_p = \frac{\omega_p}{2\pi} = \frac{1}{2\pi}\left(\frac{n e^2}{\varepsilon_0 m}\right)^{1/2}$

$n, e, \varepsilon_0, m \to$ all const. so $f_p \to$ const.

$f_c \to$ cut off freq. $\boxed{f_p = f_c \simeq 9\sqrt{n}}$

$n \to$ no. of e^- per meter^3 [$/m^3$] (in M.K.S.)

Maximum Penetration (skin) depth for plasma :-

$$\boxed{\delta_{max.} = \frac{c}{\omega_p}}$$

{ Word - skin depth is mostly used for metals }

Q. Find the skin depth for a typical metal $\sigma = 10^7\ \Omega^{-1} m^{-1}$ in visible range $\omega = 10^{15}/sec$. Assuming $\varepsilon \simeq \varepsilon_0$ & $\mu \simeq \mu_0$ & define why metals are opaque. Also find the phase diff. b/w Electric & magnetic field in this metal.

$\sigma = 10^7\ \Omega^{-1} m^{-1}$

$\omega = 10^{15}/sec$.

It is a good conductor, & for good conductor, skin depth is,

$$\delta_{good} = \sqrt{\frac{2}{\sigma \omega \mu}} = \sqrt{\frac{2}{10^7 \times 10^{15} \times 4\pi \times 10^{-7}}}$$

$$= \sqrt{\frac{10^{-16} \times 10}{2\pi}} = \sqrt{\frac{5}{\pi}} \times 10^{-8}\ m$$

$$= 1.3 \times 10^{-8}\ m$$

$$\delta_{good} = 13\ nm$$

Now why metal is opaque ⇒

at every 13 nm, amp. decay by 1/e. Hence amplitude decay very fast. So electromag. wave can not pass through the metals. Hence metals are opaque. But it has some limit.

$\omega > \omega_p$ then medium is transparent.

$\omega < \omega_p$, medium is opaque.

- Reflectivity of any metal remain ~~constant~~ upto UV region but beyond UV region it will ~~fall~~ drops.

Hence transmittance is also const. upto UV region.

$R + T = 1$

Reflectivity drops means transmitance increases.

- Hence, for X-rays & γ-rays, we can not make the mirrors. Because No metal exist which can reflect X-rays & γ-rays.

Visible ⟶ UV ⟶ X-ray ⟶ γ-ray.

- x-rays can be reflected from an atomic plane but not by metal surface. $R \simeq 1$; $T \simeq 0$

$$\phi = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$$

$$= \frac{1}{2}\tan^{-1}\left[\frac{10^{7}\ \Omega^{-1}m^{-1}}{10^{15}/\text{sec} \times 8.85\times10^{-12}}\right]$$

$$= \frac{1}{2}\tan^{-1}(10^{3}) = \frac{1}{2}\times(89) \qquad \{89 \simeq 90\}$$

$$\simeq \frac{1}{2}\,\frac{\pi}{2} \simeq \frac{\pi}{4}$$

$$\boxed{\phi \simeq 45^{\circ}}$$

Phase diff. b/w $\vec{E}$ & $\vec{B}$ inside a good conductor is approximately $\underline{45^{\circ}}$.

i.e. $\vec{E}$ leads $\vec{B}$ by 45°.

Q. In free space an electromagnetic wave is given by

$$\vec{E} = 20\cos(\omega t - 50x)\,\hat{y} \quad V/m$$

Calculate

(i) $\vec{B}$ (ii) $\vec{J}_d$ (iii) ω

$$\vec{E} = 20\cos(\omega t - 50x)\,\hat{y}$$

$$\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega} \Rightarrow \frac{\omega}{K} = c \quad \text{then } \vec{B} = \frac{E}{c}$$

$$\vec{K} = 50\,\hat{x}$$

$$\frac{\omega}{|\vec{K}|} = c = 3\times10^{8}\ m/\text{sec}$$

$$\omega = 3\times10^{8}\times 50 = \underline{1.5\times10^{10}/\text{sec}} \text{ Ans}$$

$$\vec{B} = \frac{20\cos(\omega t - 50x)\,\hat{z}}{3\times10^{8}}$$

$$\vec{B} = \frac{2}{3}\times10^{-7}\cos(\omega t - 50x)\,\hat{z} \quad Wb/m^{2} \text{ Ans}$$

$$\vec{J}_d = \varepsilon_0\frac{\partial\vec{E}}{\partial t} = -\varepsilon_0\, 20\sin(\omega t - 50x)\cdot\omega$$

$$\vec{J}_d = -20\,($$

$$\boxed{\vec{J}_d = \qquad A/m^{2}}$$

Q. An EM wave in free space is given by

$$\vec{E} = (10\hat{y} + 5\hat{z})\cos(\omega t + 2y - 4z) \; V/m$$

Calculate (i) $\vec{B}$ (ii) $\vec{K}$ (iii) ω (iv) $\vec{J_d}$

$$\vec{E} = (10\hat{y} + 5\hat{z})\cos(\omega t + 2y - 4z)$$

$$= (10\hat{y} + 5\hat{z})\cos\left[\omega t - \{(-2\hat{y}+4\hat{z})\cdot(x\hat{x}+y\hat{y}+z\hat{z})\}\right]$$

$\therefore \vec{k} = -2\hat{y} + 4\hat{z}$ $\quad \{\cos(\omega t - \vec{k}\cdot\vec{r})$ & $\vec{r} = x\hat{x}+y\hat{y}+z\hat{z}\}$

This is lying in y-z plane.

(ii) $|\vec{K}| = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

In free space, the wave velocity is

$$v = \frac{\omega}{K} = c$$

(iii) $\Rightarrow \frac{\omega}{|\vec{K}|} = c \Rightarrow \omega = 3\times10^{8} \times 2\sqrt{5}$

$$\omega = 6\sqrt{5}\times10^{8} \text{ rad/sec.}$$

Now (i) $\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}$

$\because \vec{K}\times\vec{E} = (-10\hat{x} - 40\hat{x})\cdot\cos(6\sqrt{5}\times10^{8}t + 2y - 4z)$

$$\therefore \vec{B} = \frac{-50\hat{x}}{6\sqrt{5}\times10^{8}}\cos(6\sqrt{5}\times10^{8}t + 2y - 4z)$$

$$\boxed{\vec{B} = \frac{5\sqrt{5}}{3\times10^{8}}\cos(6\sqrt{5}\times10^{8}t + 2y - 4z)\,(-\hat{x})} \; Wb/m^2$$

(ii) $J_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

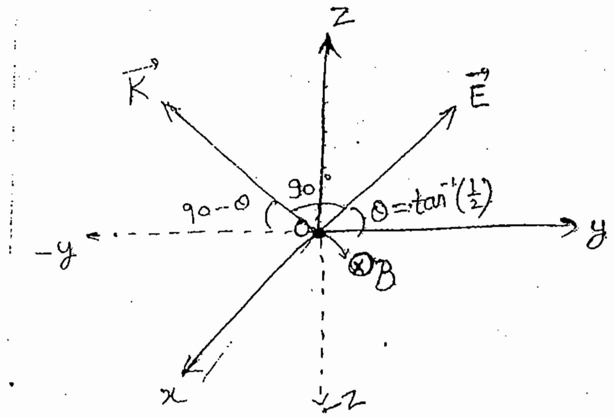
$$= \varepsilon_0(10\hat{y} + 5\hat{z})\left[-\sin(\omega t + 2y - 4z)\right]\times\omega$$

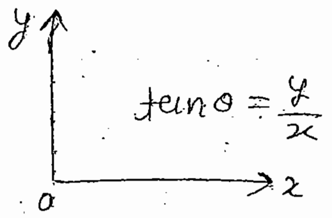
$$\boxed{\vec{J_d} = -\varepsilon_0(10\hat{y} + 5\hat{z})\left[6\sqrt{5}\times10^{8}\right]\sin(\omega t + 2y - 4z)}$$

Poyinting Vector →

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

dirn of $\vec{S}$ is same as dirn of $\vec{K}$.





$\vec{E} = 10\hat{y} + 5\hat{z}$

$\tan\theta = \frac{5}{10}$

$\theta = \tan^{-1}(\frac{1}{2})$

Mag. field is into the page.

{Angle b/w $\vec{E}$ & $\vec{K}$ = $90°$ (in free space)

Q. Mag. field of an EM wave is given by $\vec{B}(x,y,z,t) = B_0 \sin\left[\frac{(x+y)K}{\sqrt{2}} + \omega t\right]\hat{k}$ is given in free space. Find $\vec{E}$ and $\vec{S}$. Graphical rep^n = ?

$$\vec{B}(x,y,z,t) = B_0 \sin\left[\frac{(x+y)K}{\sqrt{2}} + \omega t\right]\hat{k}$$

$$= B_0 \sin\left[-\frac{(-x-y)K}{\sqrt{2}} + \omega t\right]\hat{k}$$

Standard from $\vec{B} = B_0 \sin(\omega t - \vec{K}\cdot\vec{r})$

$$= B_0 \sin\left[\omega t - \left\{\frac{(-\hat{x}-\hat{y})}{\sqrt{2}}K\cdot(x\hat{x}+y\hat{y}+z\hat{z})\right\}\right]$$

We get $\boxed{\vec{K} = \frac{K}{\sqrt{2}}(-\hat{x}-\hat{y})}$

We have
$$E = -\frac{c^2(\vec{K}\times\vec{B})}{\omega}$$

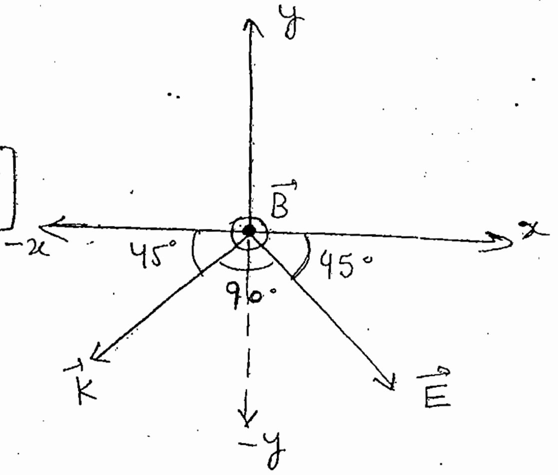
$$= -\frac{c^2}{\omega}\left[\frac{K}{\sqrt{2}}(-\hat{x}-\hat{y}) \times B_0 \sin\left[\frac{(x+y)K}{\sqrt{2}} + \omega t\right]\hat{z}\right]$$

$$= -\frac{c^2}{\omega}\frac{K}{\sqrt{2}}[-\hat{y}+\hat{x}]\, B_0 \sin\left[\frac{(x+y)K}{\sqrt{2}} + \omega t\right]$$

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

$$= \frac{1}{\mu_0}\left[-\frac{c^2}{\omega}\frac{K}{\sqrt{2}}[-\hat{y}+\hat{x}]\, B_0 \sin\left[\frac{(x+y)K}{\sqrt{2}} + \omega t\right] \times B_0 \sin\left[\frac{(x+y)K}{\sqrt{2}} + \omega t\right]\hat{z}\right]$$

$$\vec{S} = -\frac{c^2}{\mu_0 \omega}\frac{K}{\sqrt{2}} B_0^2 [-\hat{x} - \hat{y}] \sin^2\left[\frac{(x+y)K}{\sqrt{2}} + \omega t\right]$$



Q. Plane EM wave is propogating in lossless dielectric. Electric field of this wave is given by

$$\vec{E}(x,y,z,t) = E_0(\hat{x} + A\hat{z})\, e^{iK_0[-ct + (x+\sqrt{3}z)]}$$

Find (i) dielectric constant of the medium ε_r

(ii) Value of A

(iii) Poynting Vector S

$$\vec{E}(x,y,z,t) = E_0(\hat{x} + A\hat{z})\, e^{iK_0[(x+\sqrt{3}z) - ct]}$$

On comparing with $\vec{E} = E_0 e^{i(\vec{K}\cdot\vec{r} - \omega t)}$

We get $\vec{K} = K_0(\hat{x} + \sqrt{3}\hat{z})$ & so $|\vec{K}| = 2K_0$

$$\omega = K_0 c$$

Wave velocity $v = \frac{\omega}{|\vec{K}|} = \frac{K_0 c}{2K_0}$

$$\left.\begin{aligned} v &= \frac{c}{2} \\ \text{Also } v &= \frac{c}{n} \end{aligned}\right\} \text{On comparing} \Rightarrow \boxed{n = 2}$$

& $n = \sqrt{\varepsilon_r} \Rightarrow \varepsilon_r = n^2 \Rightarrow \boxed{\varepsilon_r = 4}$

for linear Isotropic dielectric (lossless)

$$\vec{K} \cdot \vec{E} = 0$$

$$K_0 (\hat{x} + \sqrt{3}\hat{z}) \cdot E_0 (\hat{x} + A\hat{z}) e^{iK_0(-ct + (x+\sqrt{3}z))} = 0$$

$$K_0 E_0 [1 + \sqrt{3}A] e^{iK_0[-ct + (x+\sqrt{3}z)]} = 0$$

$$\Rightarrow \quad 1 + \sqrt{3}A = 0$$

$$\Rightarrow \quad \boxed{A = -\frac{1}{\sqrt{3}}}$$

$$\vec{B} = \frac{\vec{K} \times \vec{E}}{\omega} \quad , \quad \omega = K_0 c$$

Now $\vec{E} = E_0 \left[\hat{x} - \frac{1}{\sqrt{3}}\hat{z}\right] e^{iK_0[-ct + (x+\sqrt{3}z)]}$ & $\vec{K} = K_0(\hat{x} + \sqrt{3}\hat{z})$

$$\vec{K} \times \vec{E} = K_0 E_0 \left[\frac{\hat{y}}{\sqrt{3}} + \sqrt{3}\hat{y}\right] e^{iK_0[-ct + (x+\sqrt{3}z)]}$$

$$\therefore \vec{B} = \frac{K_0 E_0}{K_0 c} \left[\frac{4}{\sqrt{3}}\right] \hat{y} \, e^{iK_0[-ct + (x+\sqrt{3}z)]}$$

$$\boxed{\vec{B} = \frac{4E_0}{\sqrt{3}c} e^{iK_0[-ct + (x+\sqrt{3}z)]} \hat{y}}$$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

dir^n of $\vec{S} \rightarrow$ dir^n of $\vec{K}$.

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{1}{\mu_0} \frac{4E_0^2}{\sqrt{3}c} \left[\hat{z} + \frac{1}{\sqrt{3}}\hat{x}\right] e^{2iK_0[-ct + (x+\sqrt{3}z)]}$$

Q. A Plane wave E_q^n is given by

$$\vec{E} = \left(\frac{1}{\sqrt{2}}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)\cos(10^8 t - \sqrt{3}x + y)$$

In Isotropic linear Non-magnetic medium. Find

i) Wave vector k

ii) Refractive index n

iii) Dielectric constant ε_r

iv) Magnetic field $\vec{B}$

v) Wavelength of the wave λ

vi) Poynting vector $\vec{S}$

vii) Angle θ, wave vector $\vec{k}$ making from $+x$ axis.

$$\vec{E} = \left(\frac{1}{\sqrt{2}}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)\cos(10^8 t - \sqrt{3}x + y)$$

$$\vec{E} = \left(\frac{1}{\sqrt{2}}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right)\cos\left[10^8 t - (\sqrt{3}x - y)\right]$$

On comparing with $\vec{E} = E_0 \cos(\omega t - k\cdot\vec{r})$, We get

$$\vec{K} = \sqrt{3}\hat{x} - \hat{y} \quad \& \quad \omega = +10^8$$

$$|\vec{K}| = \sqrt{3-1} = \sqrt{2} \Rightarrow \boxed{k = \sqrt{2}}$$

Wave velocity $v = \frac{\omega}{|\vec{K}|} = \frac{+10^8}{\sqrt{2}}$

$$n = \frac{c}{v} = \frac{3\times 10^8}{v} \Rightarrow v = \frac{3\times 10^8}{n}$$

$\frac{3}{n} = \frac{1}{\sqrt{2}}$

On comparing, $n = +\sqrt{2}\times 3 \Rightarrow \boxed{n = +3\sqrt{2}}$

$$n = \sqrt{\varepsilon_r} \Rightarrow \varepsilon_r = n^2 = 9\times 2$$

$$\boxed{\varepsilon_r = 18}$$

$$\vec{B} = \frac{K\times E}{\omega}$$

$\cancel{\vec{K}\times\vec{E}} \neq \vec{K} = \sqrt{3}\hat{x} - \hat{y} \quad \& \quad \vec{E} = \left(\frac{1}{\sqrt{2}}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right)\cos(10^8 t - \sqrt{3}x + y)$

$$\vec{K}\times\vec{E} = \left(\frac{3}{2}\hat{z} + \frac{1}{\sqrt{2}}\hat{z}\right)\cos(10^8 t - \sqrt{3}x + y)$$

$$\omega = +10^8$$

So $\boxed{\vec{B} = \frac{(3+\sqrt{2})\hat{z}}{\cdot 10^8}\cos(10^8 t - \sqrt{3}x + y)}$

Wave length $\lambda = ?$

$$K = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{K} \Rightarrow \lambda = \frac{2\pi}{\sqrt{2}}$$

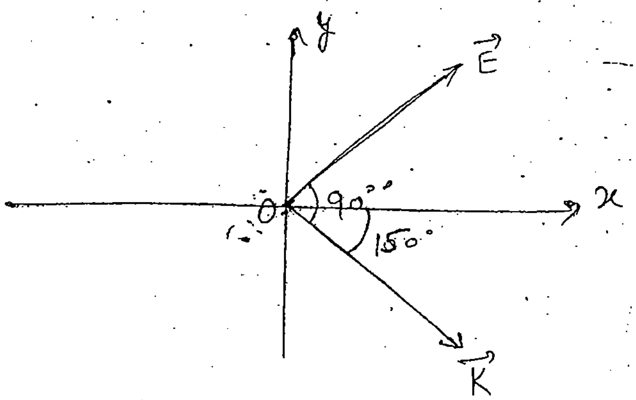
$$\boxed{\lambda = \sqrt{2}\,\pi} \text{ or } \lambda = 1.414 \times 3.14 \Rightarrow \boxed{\lambda = 4.44062}$$

Poyinting Vector $\vec{S} = ?$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$\vec{S} = \frac{1}{\mu_0}\left[\frac{3}{\sqrt{2} \times 10^8}\hat{y} - \frac{\sqrt{3}\cdot\sqrt{2}}{2 \cancel{\times} 10^8}\hat{x}\right] \cos^2(10^8 t - \sqrt{3}x + y)$$

$$\vec{S} = \frac{1}{\mu_0}\left[\frac{3}{\sqrt{2}}\hat{y} - \sqrt{\frac{3}{2}}\hat{x}\right]\frac{1}{10^8} \cos^2(10^8 t - \sqrt{3}x + y)$$



$$\vec{K} = \sqrt{3}\,\hat{x} - \hat{y}$$

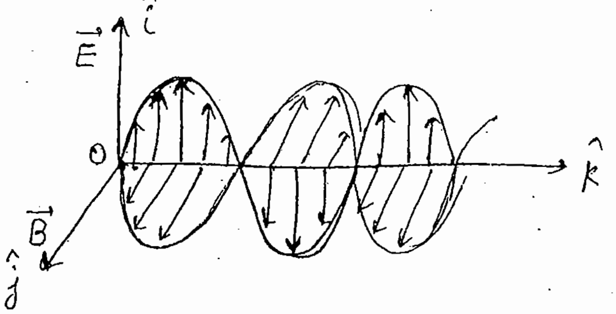
$$\tan\theta = \frac{-1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$$

$$\theta = 150^\circ$$

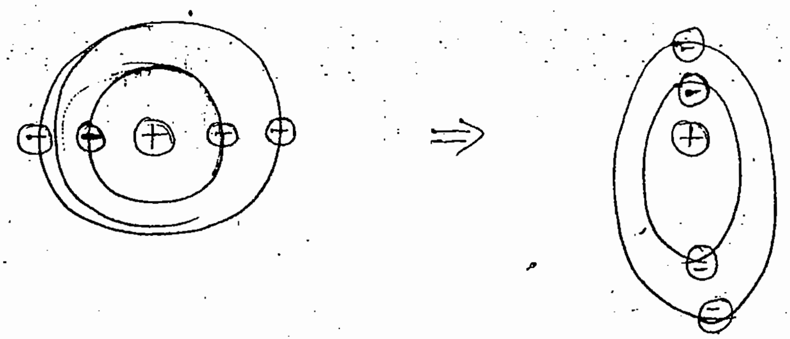
Polarisation in E.M. Waves :-

dirⁿ of polarisation vector is the dirⁿ of electric field $\vec{E}$.
suppose $\vec{E}$ is vibrating along $\hat{x}$-dirⁿ & wave is along $\hat{z}$.
so dirⁿ of polarisation is $\hat{x}$.



dirⁿ of mag. field $\vec{B}$ is $\perp^r$ to dirⁿ of $\vec{E}$. Hence $\perp^r$ to dirⁿ of polarisation.

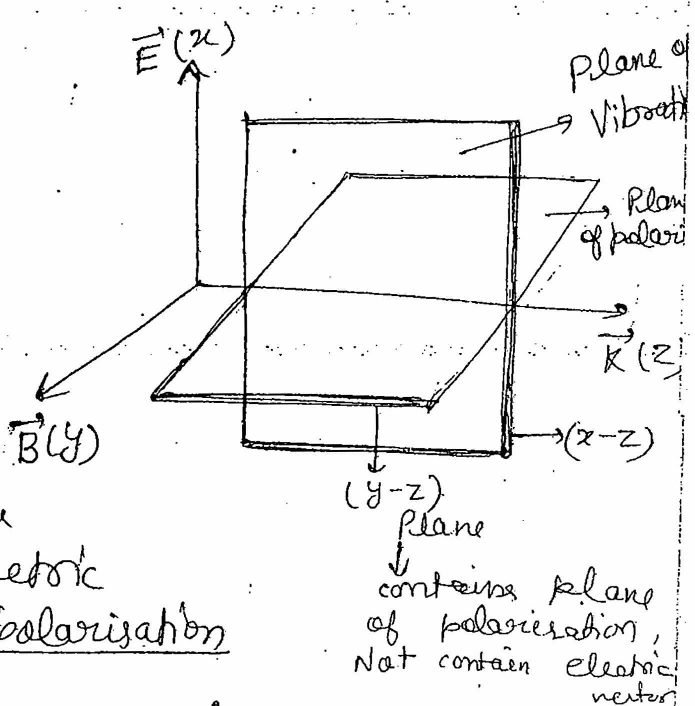
- If we fall EM waves on any material (metal) which contains atom, then electric field will polarise the atom. for I half cycle, $\vec{E}$ is upward so dirⁿ of e^- is downward & dirⁿ of nucleus is upward so atom will oscillate.



For lower half cycle
force of $e^- \to$ up.
nucleus $\to$ down
so atom will oscillate & shape will change

Plane of Vibration :-

The plane that contains the dirⁿ of propogation & the dirⁿ of electric field, is called plane of Vibration



- **Plane of Polarisation :-**

The plane that contains the dirⁿ of propogation & No electric vibration, is called plane of polarisation

Plane of Vibration is $\perp^r$ to plane of polarisation.

Unpolarised Light :-

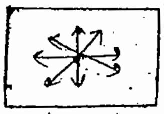
Light Waves → EM wave in visible region.

The concept of polarisation is valid only in Transverse wave. It is not found in longitudinal wave.

Sound wave (L.W.) can not be polarised but light waves can.

Plarisation is found in all electromag. wave (light sound ----). When light coming then $\vec{E}$ & $\vec{B}$ will confine in $\perp^r$ to the dir^n of propogation.

→ If a light wave is coming & electric ~~field~~ vector is not confine in perticular dir^n, i.e. $\vec{E}$ vibrate randomly in a plane. & dir^n of vibration is $\perp^r$ to $\vec{E}$.

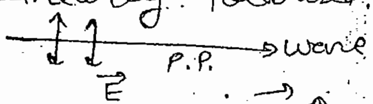
U.P.  $\vec{E}$ ~~so~~ it is called Unpolarised light

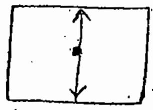
Polarisation may be of 3 types :-

Plane polarised , circularly pole. , eliptically pol.

for plane polarisation, $\vec{E}$ is confine along (i.e. vibrate along) a fix dir^n.

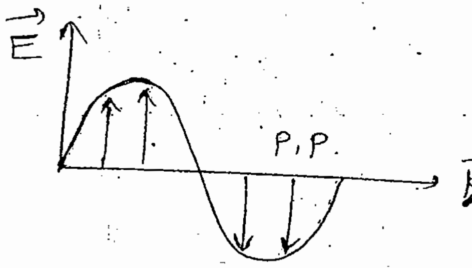
It is also called Linearly Polarised.



P.P. 

e.g. $E = E_0 e^{i(kz-\omega t)} \hat{x}$

This is Plane polarised wave.



If we superimpose, a another wave on it then

Dir^n of polarisation is depand upon the dir^n of polarisation of that other wave, their phase diff. & amplitude of both waves.

Note :- EM wave emit due to deexitaction of atoms. They will not produce only by single atom. When large no. of atoms deexite from higher to lower level then EM wave ~~b/w~~ will produce & there will be No phase corelation b/w the electric vector.

Sun gives unpolarised light bcoz light is not coming from single nuclei, it is coming from different nuclei so their electric vector will vibrate in diff. dirn

If beam & wave → const. phase then → Cohrent Wave.

<u>Plane Polarised</u> → Electric vector always propogate along a fixed dirn. ↕

Elliptical & Circular Polarisation :-

If wave is coming towards us & electric vector rotate & traces a circle then it is Circularly polarised.

If it traces a ellipse then it is elliptically polarised. & dirn of propagation is ⊥r to the field.

$$\vec{E_1} = E_0 e^{i(kz-\omega t)}\hat{x}$$

$$\vec{E_2} = E_0 e^{i(kz-\omega t)}\hat{x}$$

$$\vec{E_3} = E_0 e^{i(kz-\omega t)}\hat{y}$$

If we superimpose $\vec{E_2}$ on $\vec{E_1}$ then wave will be plane polarised as these waves have same amp., ~~same~~ phase (i.e. no phase diff. b/w them) & dirn of polarisation is same.

Amplitude of ~~Sump~~ Superimposed wave → $2E_0$

Q. Superimposition of which wave produce Interference.
(i) E_1 & E_2 (ii) E_2 & E_3 (iii) E_1 & E_3

<u>Condn of Interfecee</u> → The phase diff. b/w 2 waves should remain const. with time i.e. Cohrent.

Here all 3 waves are cohrent (i.e. have const. phase)

→ The superimposing ~~must~~ waves must have same state of polarisation.

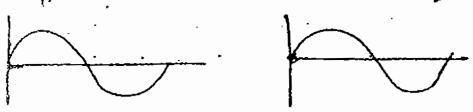
So E_1 & E_2 will produce Interference as they have same state of polarisation.

$\vec{E_1} = E_0 \, e^{i(ky-\omega t)} \hat{x}$
$\vec{E_2} = E_0 \, e^{i(kz-\omega t)} \hat{x}$

Now these waves can not produce Interfe-rence bcoz they can not superimpose bcoz both are propogating in different ~~state of polarisation~~ dirⁿ.

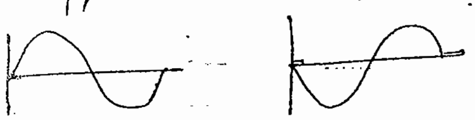
Condⁿ of Interference
i) propogation in same dirⁿ
ii) same freq.
iii) phase diff constant (cohrent)
iv) same state of polarisation.

If phase diff b/w 2 waves is 0 then → Constructive Inter.



→ No phas diff.

If phase diff b/w 2 waves is $\pi/2$ then → distructive Inte.



P.D. → $0, 2\pi, 4\pi$ .... Constructive
$\pi, 3\pi, 5\pi$ ..... Distructive

→ By the superposition of E_1 & E_2 waves then their resultant wave will be plane wave & amplitude of resultant wave will be

$$\boxed{E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\phi}}$$

ϕ → phase diff b/w 2 waves.

It is valid only when both waves have same state of polarisation

Here E_1 & $E_2 = E_0$ & ϕ = phase diff $= 0$

So $E = \sqrt{E_0^2 + E_0^2 + 2E_0^2} = \sqrt{4E_0^2}$

$E = 2E_0$

Superimposition of 2 wave which are different state of polarisation

We can never achieve circular or eliptical polarisation by the superposition of waves which are in same state of polarisation (only we get plane P.)

But ~~we can~~ By the superposition of waves which are in different state then we can achieve circularly or elliptically polarised wave under certain conditions & also can get plane P. wave.

First cheak the state of polarisation of 2 waves if same then → Plane wave.

Now if $\vec{E_1}$ & $\vec{E_3}$ superimpose then

$$\vec{E_1} = E_0 e^{i(kz-\omega t)}\hat{x} \quad , \text{P.P. in } \hat{x} \text{ dir}^n$$

$$\vec{E_3} = E_0 e^{i(kz-\omega t)}\hat{y} \quad \text{P.P. " } y \text{ "}$$

Resultant Wave $\vec{E} = \vec{E_1} + \vec{E_2}$

$$\boxed{\vec{E} = E_0(\hat{x}+\hat{y})e^{i(kz-\omega t)}} \quad \text{——(A)}$$

(i) State of polarisation

If $\measuredangle \parallel$ then plane polarised.

But If $\perp$ then cheak

(ii) phase difference ϕ

(a) If $\phi = 0, \pi, 2\pi, 3\pi$ then wave will be plane polarised

(b) If $\phi = \frac{\pi}{2}, \frac{3\pi}{2}$ ..... then ckeak amp. of both wave

→ amplitude

1. if amp. same → $\boxed{E_1 = E_2}$ → Circularly Polarised
2. if amp. is not same $E_1 \neq E_2$ → Elliptically "

(c) If phase diff $\phi \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ then

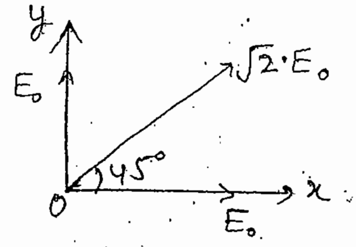
Surely it will Elliptically polarised.

Now for (A) $\Rightarrow$

$$\vec{E} = E_0(\hat{x}+\hat{y})\, e^{i(kz-\omega t)}$$ } state of polarisation $\to$ is sam both plane wave

This is plane polarised (No phase diff)

Superimposed wave is making angle 45° with x-axis & its magnitude is $\sqrt{2}E_0$.



- If we have

$$\vec{E} = \vec{E_1}+\vec{E_2} = E_0(\hat{x}+e^{i\phi}\hat{y})\, e^{i(kz-\omega t)}$$

(i) $\phi = 0$ then Plane polarised making angle 45° with x-axis.

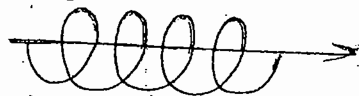
(ii) $\phi = \frac{\pi}{2}$ then

$$\vec{E} = E_0(\hat{x}+i\hat{y})\, e^{i(kz-\omega t)}$$

$\{e^{i\frac{\pi}{2}} = i\}$

$i \to$ phase diff of $\pi/2$

This is Circularly polarised wave.



amp. is const $\to E_0$

(iii) $\phi = \pi/4$

$$e^{i\pi/4} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}(i+1)$$

$$\vec{E} = E_0\left[\hat{x} + \frac{(i+1)}{\sqrt{2}}\hat{y}\right] e^{i(kz-\omega t)}$$

This is Elliptically polarised

In Ellipse – distance from centre is not same but in circle – " " is same so amplitude remain constant

* Most General type of Polarisation is Elliptical polarisation. Plane polarised & Circularly polarised waves are the special case of Elliptically polarised wave.

e.g. $\vec{E} = E_0(2\hat{x}+i\hat{y})\, e^{i(kz-\omega t)}$

phase diff $\to \pi/2$ & amp. is different

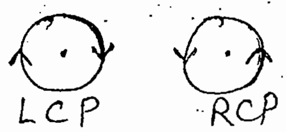
so Elliptically polarised wave.

Circularly Polarised	Elliptically Polarised
→ Right Circular Polarised RCP	→ Right Elliptical Polarised REP
→ Left " " LCP	→ Left " " LEP

- There are 2 type of sep^n
  one along ⟶ angular mom. (30 % Probability)
  2nd along ⟶ optics (70 % Probability)

If electric vector is moving towards us doing clockwise motion i.e. wave is coming towards us & electric vector is rotating clockwise then acc. to optics sep^n it is RCP (if amp. constant)
REP (if amp. Not constant)

acc. to Angular Mom. sep^n, it is



If electric vector is rotating anticlockwise then acc. to optics sep^n it is LCP (const. amp.)
LEP (Not " ")

e.g. $\vec{E}_1 = E_0 \cos(kz - \omega t)\,\hat{x} + E_0 \sin(kz - \omega t)\,\hat{y}$

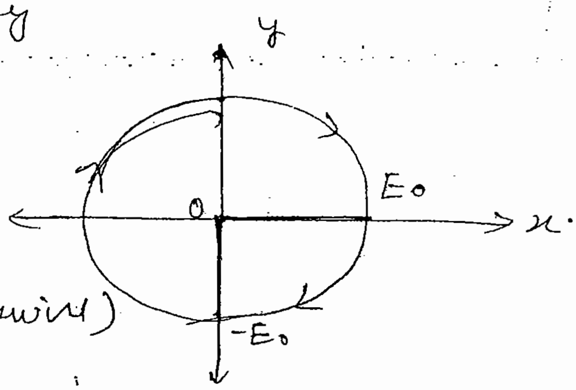
amp. same
Phase → $\pi/2$ } → Circularly Polarised

Take origin of wave $z = 0$ & dir of prop. $+\hat{z}$

$\vec{E} = E_0 \cos \omega t\,\hat{x} - E_0 \sin \omega t\,\hat{y}$

ωt → phase

At $\omega t = 0$, $\vec{E}$ along $\hat{x}$
$\omega t = \pi/2$, " $-\hat{y}$

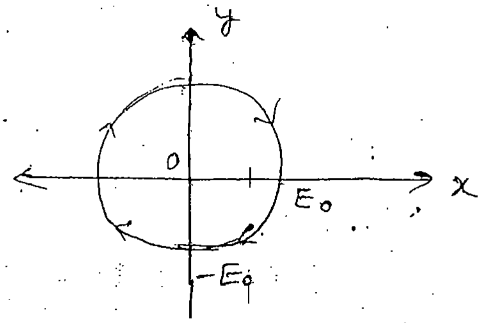


Acc. to ang. mom. LCP (clockwise)
so acc. to optics → RCP

RCP

Q. $\vec{E} = E_0 (\hat{x} + e^{i\frac{\pi}{2}} \hat{y}) e^{i(\omega t - kz)}$

Wave propogation along +z dirⁿ
elec. vector is vibrating in x-y plane.



$z = 0,$

$\vec{E} = E_0 (\hat{x} + i\hat{y}) e^{i\omega t}$

for $\omega t = 0$, $\vec{E} = E_0 (\hat{x} + i\hat{y})$

We need to take real amplitude only so

$\vec{E} = E_0 \hat{x}$

for $\omega t = \frac{\pi}{2}$, $\vec{E} = E_0 (\hat{x} + i\hat{y}) e^{i\pi/2} = E_0 (\hat{x} + i\hat{y}) i$

$= iE_0 \hat{x} - E_0 \hat{y}$

$\vec{E} = -E_0 \hat{y}$ (real)

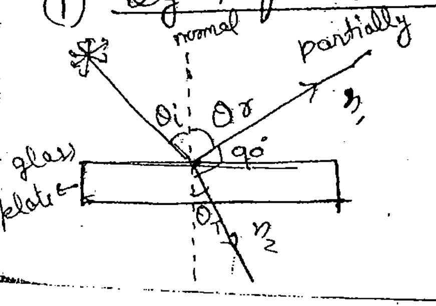
Rotation is clockwise → so LCP acc. to ang mom. rep.
so RCP acc. to optics.
So <u>RCP</u>.

2/9/2012

Production of Plane Polarised Light :-

Plane polarised light can be produced by no. of methods.
i) Reflection method
ii) Refraction " (Transmission)
iii) By scattering
iv) By phase retardation (doubly refracting crystals)
v) Beam splitter

(i) <u>By Reflection</u> :- If we have a transparent glass plate. If an unpolarised light is incident onto a crystal at an angle θ_i. Reflected light is partially plane polarised. θ_r is the reflection angle. And for a certain angle of incident, ~~refle~~ θ_i reflected



light is completely plane polarised with vibrations $\perp^r$ to plane of incidence. & that perticular angle of incidence is called <u>Brewstor's angle</u> θ_B. $\boxed{\theta_i = \theta_B}$

This will happen When reflacted light wave & transmitted light wave are $\perp^r$ to each other.

$$\boxed{\theta_r + \theta_T = \frac{\pi}{2}}$$

$$\theta_i = \theta_r \quad \text{(angle of incident = angle of reflection)}$$

$$\therefore \quad \theta_i + \theta_T = \frac{\pi}{2}$$

By <u>Snell's law</u>, $\dfrac{\sin\theta_i}{\sin\theta_T} = \dfrac{n_2}{n_1}$

$n_1 \rightarrow$ refractive index of medium 1.
$n_2 \rightarrow$ " " " " 2

$$\Rightarrow \frac{\sin\theta_B}{\sin\theta_T} = \frac{n_2}{n_1} \Rightarrow \tan\theta_B = \frac{n_2}{n_1}$$

$$\left\{ \begin{aligned} &\sin\theta_T \\ &= \sin\left(\tfrac{\pi}{2} - \theta_r\right) \\ &= \sin\left(\tfrac{\pi}{2} - \theta_B\right) \\ &= \cos\theta_B \end{aligned} \right.$$

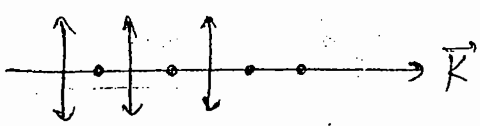
$$\Rightarrow \boxed{\theta_B = \theta_P = \tan^{-1}\left(\frac{n_2}{n_1}\right)}$$

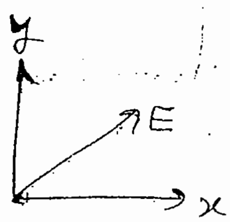
This is <u>Brewstor angle</u> & also called <u>polarisation angle</u> i.e. angle at which unpolarised light is completely ~~polar~~ plane polarised.

<u>Note</u> :- for air-glass interface and glass-air interface, this angle θ_B or θ_P is different as their ref. index has been inverted.

(ii) <u>Refraction method</u> :-

We can break any vector of unpolarised light into 2 components - horizontal & vertical.

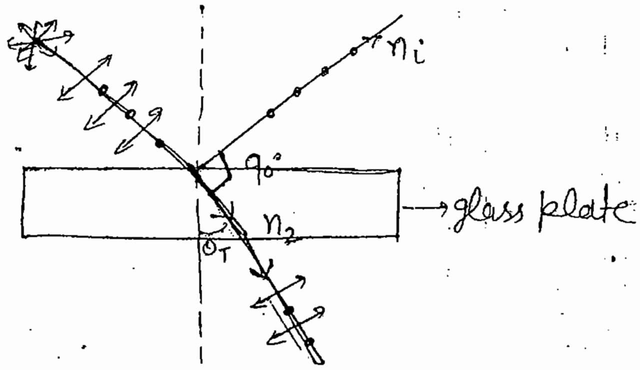


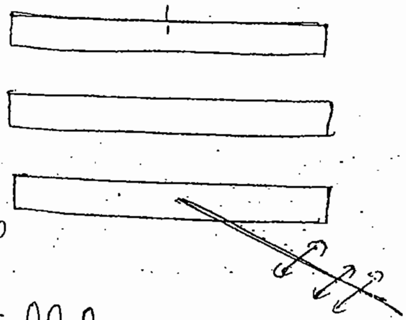


$\updownarrow \rightarrow$ parallel to the plane of incidence

$\bullet \rightarrow \perp^r$ " " "

Interface plane → plane which seperates 2 medium.





If we place a larger no. of plates then there will be polarisation by refraction. & this will be parallel to the plane of incidence.

⇒ After passing through large no. of plates, ⊥r wave will filter & only parallel wave will remain.

- This law is valid for both rare to dencer & dencer to rare

air → glass (rare to dencer) $n_{glass} > n_{air}$

glass → air

Polarisation by Scattering :- When a EM wave is incident on atom or molecule then reflacted light will be partially plane polarised.

Sky is blue due to the scattering of blue colour — Raleigh scattering (it is $\propto \frac{1}{\lambda^4}$)
Red colour sca. least as it wavelength is large.

When unpolarised light is scattered by small particles, the scattered light is partially polarised. The blue light recived from the sky is accordingly partially polarised light.

(iv) By phase retardation :- In nature there are certain types of crystals which splitt the electromagnetic waves into 2 parts.

(i) E-ray (extraordinary ray)

(ii) O-ray (Ordinary ray)

→ E-ray does not follows, the laws of refraction while O-ray follows the laws of refraction.

→ E-ray travells with different speeds in different directions ~~will~~ while O-ray travells with same speed in different dir^n.

→ Along a perticular dir^n in a crystal E-ray & O-ray travells with same speed, this dir^n is called Optic axis of the crystal.

If a medium is s.t. in which a ray split into E-ray & O-ray after entering in it. then there must be some angle at which E-ray & O-ray travell with same speed. The crystals in which this phenomenon occur are known as Doubly Reflecting Crystals. These are Anisotropic medium as they have diff. properties in diff. dir^n. as Refractive index is different in different dir^n.

• Doubly reflecting Crystals are of 2 types -

(i) Uniaxial → one optic axis

(ii) Biaxial → two optic axis

uniaxial

negative | positive

Negative uniaxial crystal - In -ve crystal,

$$\boxed{n_o > n_e}$$

Hence $\boxed{v_e > v_o}$

example → Calsite

n_o → Ref. index of ordinary ray

n_e → " extra " "

v_e → speed of ~~ex~~ extraordinary ray

v_o → " " ordinary ray

Positive Uniaxial crystal

$$n_e > n_o$$

Hence $\boxed{v_o > v_e}$

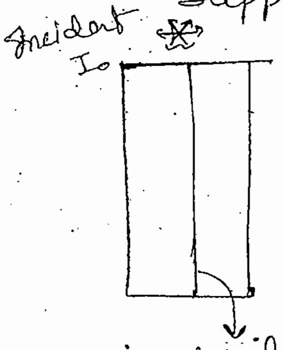
example - Quartz.

Polarisers - Those produces plane polarised light.

Nikol prism is a device which convert unpolarised light into plane polarised light.

Nikol prism is made up of doubly reflecting crystal

Suppose we have a polariser,



If we incident some unpolarised light on polariser then we get vibrations ∥ to the polarisation axis of polariser.

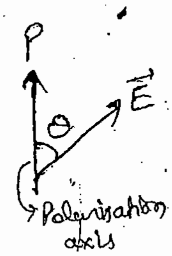
If intensity of unpolarised light is I_o then transmitted intensity of polarised light is always half of I_o. $\left(\frac{I_o}{2}\right)$.

Transmitted intensity is always found by the law of Malus.

Transmitted intensity is given by

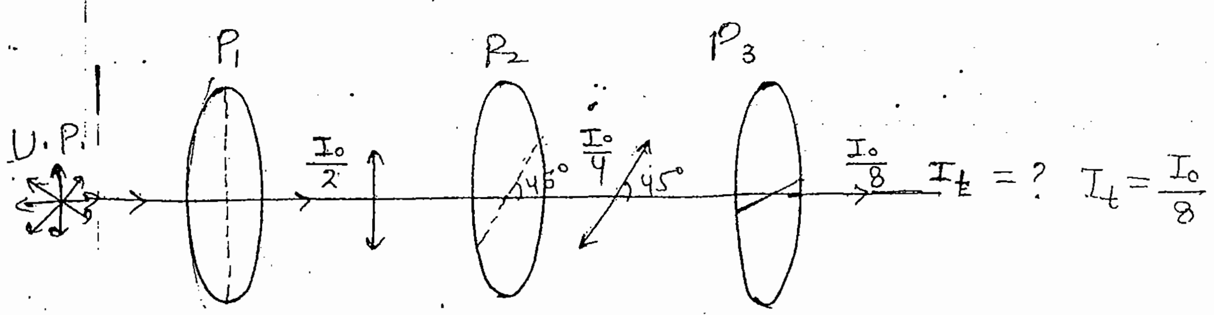
$$\boxed{I_t = I_o \cos^2\theta}$$

where, θ is the angle b/w dirn of electric field & plarisation ~~angle~~ axis.



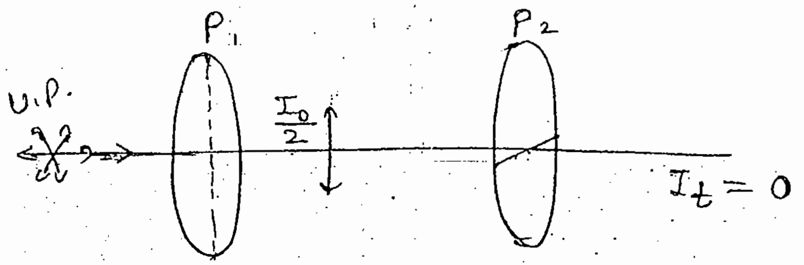
For unpolarised light we need to take average value of $\cos^2\theta$. (i.e. $\frac{1}{2}$) so $\boxed{I_t = \frac{I_o}{2}}$

Polarisation axis is also called Transmission Axis.



$I_t = \frac{I_0}{2}$ $\quad$ $I_t = \frac{I_0}{2}\cos^2 45^\circ$ $\quad$ $I_t = \frac{I_0}{4}\cos^2 45^\circ$

$I_t = \frac{I_0}{4}$ $\quad$ $I_t = \frac{I_0}{8}$



$I_t = \frac{I_0}{2}\cos 90^\circ$

$I_t = 0$

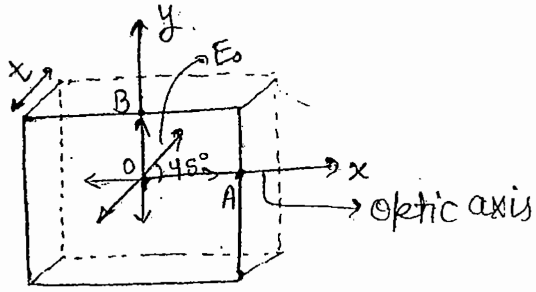
How to produce Circularly & Elliptically polarised light :- We use Quarter wave plate & quater wave crystals. Plates is made up of doubly reflecting ~~waves~~ crystals s.t. it produce a path difference of $\lambda/4$ i.e. phase diff of $\pi/2$ b/w E-ray & O-ray. That's why it is called $\frac{\lambda}{4}$ - Plate (Quarter wave plate)

$$\Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4}$$

$$\boxed{\Delta\phi = \frac{\pi}{2}}$$

Vibrations of E-ray & O-rays are $\perp^r$ to each other. Suppose we have a plate of doubly reflecting crystal, of thickness t is placed in x-y plane.

If we incident an EM wave on it which is in -z dir^n.
& Elec field.



$$\vec{E} = E_0\, e^{i(Kz+\omega t)} \left(\frac{\hat{x}+\hat{y}}{\sqrt{2}} \right)$$

As ele. field enter in crystal EM wave split into 2 parts - one is along x, $E_0 \cos 45°$ & one is along y - $E_0 \sin 45°$.

Vibrations of Electric field parallel to optic axis is <u>E-Ray</u>. & Vibration " " perpendicular " " " "
<u>O-Ray</u>.

$$OA = E_0 \cos 45° \quad (E\text{-ray})$$
$$OB = E_0 \sin 45° \quad (O\text{-ray})$$

As E-rays travell with diff speed ~~so~~ when these ray comes ~~co~~ out of this crystal then there must be some ~~phase~~ difference in ~~these ways~~. The path travelled by the wave is called optical path,

Optical path = ref. index of that medium × distance travelled

optical path $= n t$

for E-ray, $n_e t$
for O-ray, $n_o t$

So path diff. $= (n_e t - n_o t)$ (+ve)

for circularly polarised light

Phase diff → $\pi/2$
Path → $\lambda/4$

$$\therefore \boxed{t = \frac{\lambda}{4(n_o - n_e)}}$$

for plane polarised light

Phase diff → π
Path → $\lambda/2$

$$\boxed{t = \frac{\lambda}{2(n_o - n_e)}}$$

Now this is known as half wave plate or $\frac{\lambda}{2}$-plate.
It depends upon thickness t, that how much path difference is produced by wave.
If t is in b/w these two values then phase diff is not π & $\pi/2$

i.e. phase diff. $\Delta\phi \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

Then Elliptically Polarised. & thickness t lies b/w these 2.

Detection of Light Wave :-

If a given light is passing through a rotating Nicol prism then, there are 3 posibilities,

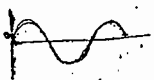
↓ given light

rotating Nicol Prism

There may be 3 posibilities

variation in intensity with 0 minima then Light wave is surely plane polarised.

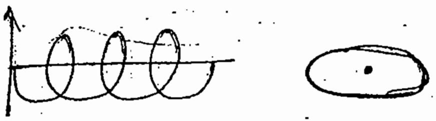
$I \propto E_0^2$

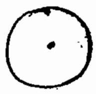


Variation in intensity with Non-zero minima then 2 posibilites
→ Elliptical
→ Partially plane P.
i.e. mixture of unpolarised & plane polarised.

No variation in intensity. then
→ Circularlly P.
→ Unpolarised

When Elec. field $= 0$, then Intensity $= 0$ (I can't be −ve as it is square of amp. E_0)

for Elliptical  In ellipse radius can't be zero.

for Circular 

In (b) case, there is some doubt, so ~~we have~~ use a $\lambda/4$ plate.

↓ (b) Partially P.

$\lambda/4$

No effect

t_1

variation int. with non-zero minima

↓ (b) elliptically light

$\lambda/4$ plate

↓ P.P.

rotating Crystal

variation int. with 0 minima

If elliptically polarised light passes through λ/4 plate then it will produce a phase diff of π/2 b/w E-ray & O-ray. It'll become Plane polarised & →|

If partially plane polarised ligh passes through λ/4 plate then it will remain partially plane polarii (no effect) & after appassing through nicol crystal there will be variation intensity with non-zero minima. It'll become Partially Plane polarised.

(→ & after passing through rotating crystal, there will be variating in int. with 0 minima i.e. it was elliptical)

If C.P. light passes through λ/4 plate then it'll become ~~circul~~ plane polarised. If we get variation in intensity with zero minima. This confirms it was circularly P. light.

C.P. → $\boxed{\lambda/4}$ → P.P. → $\boxed{\text{rotating c}}$ → variation int. with 0 min

If ~~for~~ unpolarised light passes through λ/4 plate. It remains ". If un.P. light passes through rotating crystal then No variation in intensity. This confirms it was Unpolarised.

Un.P. → $\boxed{\lambda/4}$ → $\boxed{\text{rotating C.}}$ → No variation in intensity.

e.g. P.P. → $\boxed{\lambda/2}$ → P.P.

C.P. → $\boxed{\lambda/2}$ → C.P.

R.C.P. → $\boxed{\lambda/2}$ → L.C.P.

L.C.P. → $\boxed{\lambda/2}$ → R.C.P.

E.P. → $\boxed{\lambda/2}$ → E.P.

R.E.P. → $\boxed{\lambda/2}$ → ~~R~~ L.E.P.

L.E.P. → $\boxed{\lambda/2}$ → R.E.P.

P.P. → $\boxed{\lambda/4}$ → C.P. / E.P.

depends on amp. of wave
if amp. is same then C.P.
" " " not " " E.P.

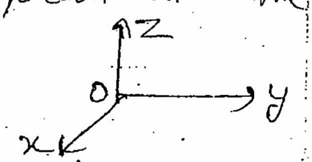
C.P. → $\boxed{\lambda/4}$ → P.P.

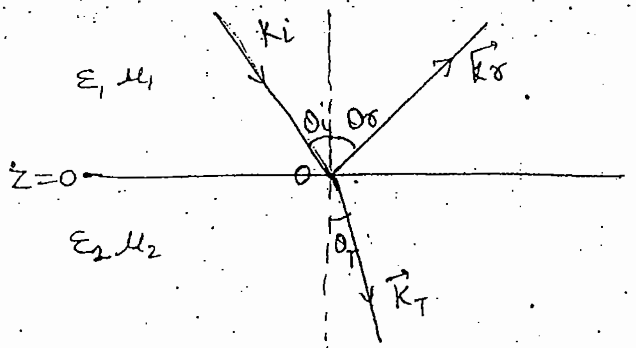
E.P. → $\boxed{\lambda/4}$ → P.P.

- Plane polarised light or Linearly Polarised light is the combination of L.C.P & R.C.P.

Reflection & Refraction of EM Waves at the Dielectric Interface :-

If we have a (dielectric) interface having different dielectric above & below the interface.





(interface is in x-y plane & z dirⁿ is normal to the surface.)

EM waves carries energy & when EM waves fall on an interface then there will be redistribution of energy & some part will reflect & some part will Transmit.

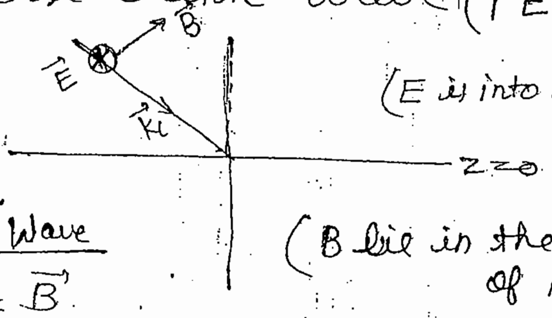
Interface Plane :- The plane deviding the 2 mediums.
In present case → x-y plane

Plane of Incidence :- The plane that contains propogation vector & normal to the plane
In present case → y-z plane of incidence

(i) $\vec{E}$ (polarisation Vector) is $\perp$ to plane of incident.
This is also called Transverse electric wave (TE Wave)

- $\vec{B}$ & $\vec{K_i}$ both lie in x-y plane
- $\vec{E}$ is lying in $-x$ plane.
- $\vec{E}$ is $\perp^r$ to both $\vec{K}$ & $\vec{B}$.

So $\vec{E}$ is transverse electric wave
i.e. $\vec{E}$ is transverse to $\vec{K}$ & $\vec{B}$.

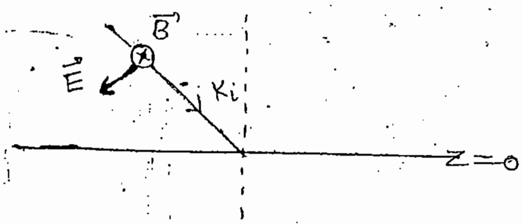


(E is into the page)

(B lie in the plane of k.)

& Normal waves are Transverse electromagnetic wave
i.e. transverse electric & transverse magnetic wave both.

(ii) $\vec{E}$ ~~vec~~ parallel to plane of incident.
Here $\vec{E}$ is not transvers.
& $\vec{B}$ is into the page.
$\vec{E}$ vector is in y-z plane i.e. $\parallel$ to the plane of incidence.
It is called Transverse Magnetic Wave
$\vec{B}$ is $\perp$ to the plane of $\vec{E}$ & $\vec{K}$



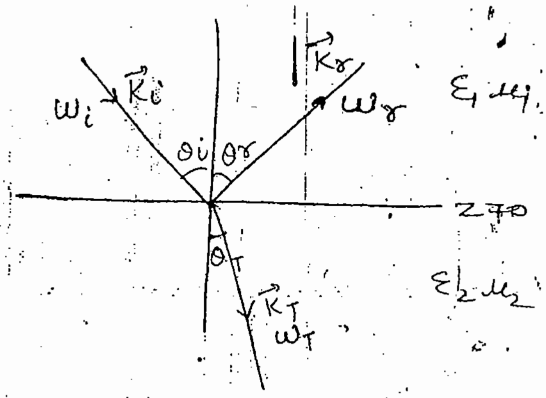
Static properties :- Same for TE & TM waves.

(i) $\omega_i = \omega_r = \omega_t$

(ii) Wavelengths & wave velocity changes s.t. freq. remains same.

(ii) $\theta_i = \theta_r$

(iii) $$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\sqrt{\varepsilon_2\mu_2}}{\sqrt{\varepsilon_1\mu_1}}$$



~~Dynamical properties~~ :-

Boundary Cond^n :- (i) $E_1^{\parallel} = E_2^{\parallel}$

(on interface, no free charge so $\varepsilon_1^f = \varepsilon_2^f$)

(ii) $\varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp}$

(iii) $B_1^{\perp} = B_2^{\perp}$ (iv) $\frac{B_1^{\parallel}}{\mu_1} = \frac{B_2^{\parallel}}{\mu_2}$

On the basis of these 4 B.C., we can derive the Dynamical properties.

Dynamical Properties :- (Fresnel's Relations) :-

1. For TE Waves ($\perp$):-

- $$\boxed{\left(\frac{E_{or}}{E_{oi}}\right)_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

$E_{or} \rightarrow$ Amp of E-field of reflected wave

$E_{oi} \rightarrow$ " " " incident "

$\perp$ means electric field is $\perp^r$ to the plane of incidence.

where $\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$ & $\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$

- $$\boxed{\left(\frac{E_{ot}}{E_{oi}}\right)_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

$E_{ot} \rightarrow$ amp of E-field of transmitted wave.

- $\frac{H_{or}}{H_{oi}} \Rightarrow$ $$\boxed{\left(\frac{E_{or}}{E_{oi}}\right)_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = -\left(\frac{H_{or}}{H_{oi}}\right)_{\perp}}$$

$H_{or} \rightarrow$ Amp of Mag. field of reflected wave (-ve sign indicate phase change)

$H_{oi} \rightarrow$ " " " incident "

$\perp \rightarrow$ decides from electric field i.e. E-vector is $\perp$ to the plane of incidence.

When reflected vector of electric field do not change phase then " " Mag. " changes the phase of π.

- & $$\boxed{\left(\frac{H_{ot}}{H_{oi}}\right)_{\perp} = \frac{2\eta_1 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

2. For TM Wave (||) :-

$$\left(\frac{E_{or}}{E_{oi}}\right)_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = -\left(\frac{H_{or}}{H_{oi}}\right)_{\parallel}$$

$$\left(\frac{E_{ot}}{E_{oi}}\right)_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\left(\frac{H_{ot}}{H_{oi}}\right)_{\parallel} = \frac{2\eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

I) For $\perp^r$ components :-

When medium is Non-magnetic,

$$\mu_1 = \mu_2 = \mu_0$$

$$-\left(\frac{H_{or}}{H_{oi}}\right) = \left(\frac{E_{or}}{E_{oi}}\right)_{\perp} = -\frac{\sin(\theta_i - \theta_T)}{\sin(\theta_i + \theta_T)}$$

$$\left(\frac{E_{ot}}{E_{oi}}\right)_{\perp} = \frac{2\sin\theta_T \cos\theta_i}{\sin(\theta_i + \theta_T)}$$

$$\left(\frac{H_{ot}}{H_{oi}}\right)_{\perp} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \cdot \left(\frac{E_{ot}}{E_{oi}}\right)_{\perp}$$

If $\varepsilon_1 < \varepsilon_2$ then $\boxed{n_2 > n_1}$ $\quad (n = \sqrt{\varepsilon_r})$

if EM wave going from Rare to dencer (air-glass) then phase change will be

$$\frac{\sin\theta_i}{\sin\theta_T} > 1 \qquad (\because n_2 > n_1)$$

i.e. $\boxed{\theta_i > \theta_T}$

If EM wave going from Rare-dencer then it will bend toward normal & reflected component of $E_\perp$ will suffer a phase change of π. And Reflected comp. of mag. field will not suffer a phase change.

Transmitted Components of electric & mag. field suffers No phase change.

If $\varepsilon_1 > \varepsilon_2$ then $\boxed{n_1 > n_2}$

$\boxed{\theta_i < \theta_T}$

When a EM wave is going from dencer to rare (glass-air) or (water-air) then reflected component of mag. field will suffer a phase change of π & Reflected comp. of Electric field vector will not suffer any phase change.

Transmitted components in this case suffer no phase change either go from rare to dencer or dencer to rare.

② <u>For Parallel Components</u> :-

$$-\left(\frac{H_{0r}}{H_{0i}}\right)_{\parallel} = \left(\frac{E_{0r}}{E_{0i}}\right)_{\parallel} = -\frac{\tan(\theta_i - \theta_T)}{\tan(\theta_i + \theta_T)}$$

$$\left(\frac{E_{0T}}{E_{0i}}\right)_{\parallel} = \frac{2\sin\theta_T \cdot \cos\theta_i}{\sin(\theta_i+\theta_T)\cdot\cos(\theta_i-\theta_T)}$$

$$\left(\frac{H_{0T}}{H_{0i}}\right)_{\parallel} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\left(\frac{E_{0T}}{E_{0i}}\right)_{\parallel}$$

Transmitted components never suffers a phase change in any case [if $\theta_i > \theta_T$ or $\theta_T > \theta_i$ then $\cos(\theta_i - \theta_T) \Rightarrow \cos(-\theta) = \cos\theta$] i.e. either go from rare to dencer or dencer to rare.

below 90°, tan = +ve

If $(\theta_i + \theta_T) < 90^\circ$

$\tan(\theta_i+\theta_T) > 0$

$\theta_i > \theta_T \Rightarrow \varepsilon_1 < \varepsilon_2 \Rightarrow n_2 > n_1$

EM wave is going from Rare → dencer & $(\theta_i + \theta_T) < 90^\circ$ then reflected vector of electric vector will suffer a phase change of π & Mag. field will Not and for $(\theta_i+\theta_T) < 90^\circ$

$\varepsilon_1 > \varepsilon_2 \Rightarrow n_1 > n_2$

$\theta_T > \theta_i$

EM wave is going from dencer to rare & ref. comp. of Mag. field H will suffer a phase change of π.

No component of Electric or mag. field will be reflected which is parallel to the plane of incidence.