$\tan 90^\circ = \infty$ then $\left(\frac{E_{0r}}{E_{0i}}\right) = 0$

Hence reflected light will contain only $\perp^r$ comp. of electric field which is normal to the plane of incidence.

$$\theta_i + \theta_T = 90^\circ$$

$$\theta_i = \theta_B$$

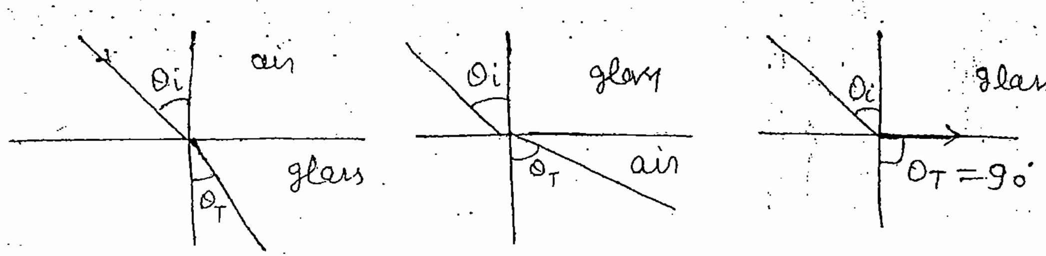
$$\boxed{\theta_P = \theta_B = \tan^{-1}\frac{n_2}{n_1} = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}} \longrightarrow \text{Brewster's law}$$

So Reflected comp. of elec. field will contain only $\perp^r$ vibrations of ele. field.

This is valid for both rare to dencer & dencer to rare medium.

dencer to rare $\rightarrow$ away from normal

rare to dencer $\rightarrow$ towards the normal



for any angle of incidence ($\theta_i > \theta_c$) then there will be Total internal reflection. It can only observe for denser to rare medium.

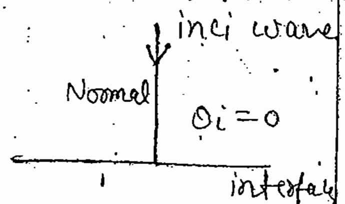
$$\frac{\sin\theta_i}{\sin\theta_T} = \frac{n_2}{n_1}$$

$$\sin\theta_i = \frac{n_2}{n_1}$$

$$\boxed{\theta_i = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

Relation b/w the amplitude of mag. & electric field for Normal Incidence :-

for Normal incidence $\theta_i = 0$



$$\mu_1 = \mu_2 = \mu_0$$

then $-\left(\frac{H_{0r}}{H_{0i}}\right) = \left(\frac{E_{0r}}{E_{0i}}\right)_{\perp \text{ or } \parallel} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)$

- If EM wave is incident from Rare → dencer then reflected comp of e-field will suffer a phase change & If wave is inci from dencer to rare then reflected comp of mag. field will suffer a phase change.

- $\frac{E_{or}}{E_{oi}}$ ⇒ This indicate → if E_{oi} is the amp. of incident wave then how much part of incident wave has been reflect

  $\frac{E_{ot}}{E_{oi}}$ → " " transmit

$$\left(\frac{E_{oT}}{E_{oi}}\right) = \frac{2\eta_1}{(\eta_1+\eta_2)} \quad \& \quad \left(\frac{H_{oT}}{H_{oi}}\right) = \frac{2\eta_2}{(\eta_1+\eta_2)}$$

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Reflection & Transmission cofficients for normal incidence :-

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$T = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

It indicate How much part of energy/power / Intensity will be reflected or transmitted.

Intensity is,

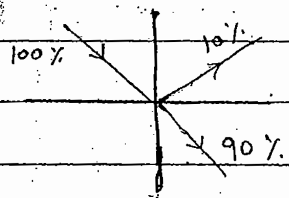
$$\langle \vec{S} \rangle = I = \vec{E} \times \vec{H}$$
$$= E_0 H_0$$

At Interface, energy conservation must be followed.

If 100% energy is incident & 10% is reflected then 90% energy should be transmitted.

$R = 10\%$

$T = 90\%$



$$\boxed{R + T = 1}$$

Q. - A EM wave $\vec{E} = 4 \cos(\omega t - Kz)\,\hat{x}$ incident from air to glass ($n = 1.5$) interface normally. Calculate reflected & transmitted amplitude of electric field & also their phase relation. Calculate % of energy reflected & transmitted at the interface. θ_B for air-glas interface = ?

$\vec{E} = 4 \cos(\omega t - Kz)\,\hat{x}$

$n = 1.5$

$E_{0i} = 4$

$$\left(\frac{E_{0T}}{E_{0i}}\right) = \frac{2n_1}{n_1+n_2} = 0.8 \qquad \begin{bmatrix} n_1 = 1 \\ n_2 = 1.5 \end{bmatrix}$$

$$\frac{E_{0T}}{4} = 0.8 \Rightarrow E_{0T} = 0.8 \times 4$$

$$\boxed{E_{0T} = 3.2}$$

$$\left(\frac{E_{0r}}{E_{0i}}\right) = \frac{n_1-n_2}{n_1+n_2} = \frac{-1}{5} = -0.2$$

$$E_{0r} = -0.2 \times 4$$

$$\boxed{E_{0r} = -0.8}$$

$$R = \left(\frac{n_1-n_2}{n_1+n_2}\right)^2 = (-0.2)^2 = 0.04$$

$$\boxed{R = 4\%}$$

$$R+T = 1 \Rightarrow \boxed{T = 96\%}$$

Brewster angle = ?

$$\theta_P = \theta_B = \tan^{-1}\left(\frac{1.5}{1}\right)$$

$$\boxed{\theta_B = 56.30^\circ}$$

* Brewster angle for air-glass interface is 56.30°.

If we incident a EM wave at this angle then reflected comp. of electric field will be plane polarised.

$\theta_T = ?$

$$\theta_T = \frac{\pi}{2} - \theta_B$$

$$\theta_T = 90^\circ - 56.30^\circ$$

$$\boxed{\theta_T = 33.7^\circ}$$

Q. Do the same for air-Water ($n=1.33$) interface. Calculate critical angle θ_c for water-air interface.

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for air-water interface, $n_1 = 1$, $n_2 = 1.33$

$$\left(\frac{E_{oT}}{E_{oi}}\right) = \frac{2n_1}{n_1+n_2} = \frac{2}{1+1.33} = \frac{2}{2.33}$$

$$E_{oT} = 0.85837 \times 4$$

$$\boxed{E_{oT} = 3.43348}$$

$$\left(\frac{E_{or}}{E_{oi}}\right) = \frac{n_1-n_2}{n_1+n_2} = \frac{1-1.33}{1+1.33} = \frac{-0.33}{2.33} = -0.14163$$

$$E_{or} = -0.14163 \times 4$$

$$\boxed{E_{or} = -0.56652}$$

$$R = \left(\frac{n_1-n_2}{n_1+n_2}\right)^2 = (-0.14163)^2 = 0.02006$$

$$\boxed{R = 2.006\ \%}$$

$\because R+T=1 \Rightarrow \boxed{T = 97.994\ \%}$

$$\theta_P = \theta_B = \tan^{-1}\frac{n_2}{n_1}$$

$$\theta_B = \tan^{-1}\left(\frac{1.33}{1}\right) = 53.06 \Rightarrow \boxed{\theta_B = 53.06^\circ}$$

$$\theta_T = \frac{\pi}{2} - \theta_B = 90^\circ - 53.06$$

$$\boxed{\theta_T = 36.94}$$

Critical angle for water-air interface,

$$\theta_c = \sin^{-1}\frac{n_2}{n_1}$$

water	$n_1 = 1.33$
air	$n_2 = 1$

$$\theta_c = \sin^{-1}\left(\frac{1}{1.33}\right)$$

$$\boxed{\theta_c = 48.75^\circ}$$

Dielectric - Conductor

$\varepsilon_1, \mu_1, \sigma_1 = 0$

dielectric

conductor

$\varepsilon_2, \mu_2, \sigma_2$

$\sigma_1 = 0$ always wheather for air or dielectric.

For perfect conductor

$\sigma_2 \to \infty$

For Normal Incidence

$$\boxed{E_{0r} = -E_{0i}}$$

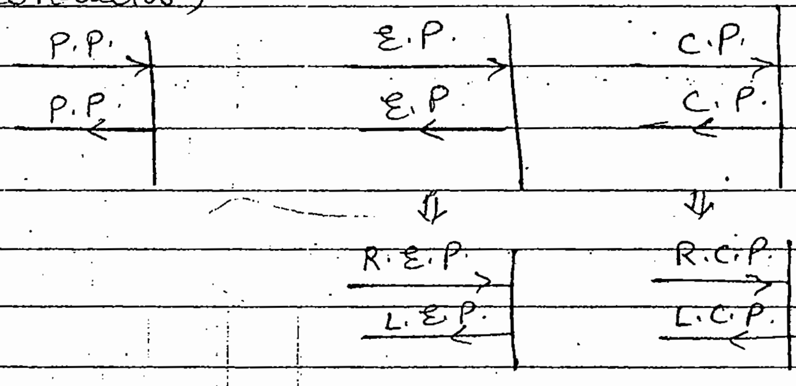
$$\boxed{E_{0T} = 0}$$

$-ve \Rightarrow$ Phase change

Inside the conductor Ele. field $= 0$
Bcoz $E_{0T} = 0$ i.e. all the components will be reflected back.

Reflected comp. of Ele. field will suffer a phase change of π. And Reflected comp. of mag. field will not suffer a phase change.

For Any Rare to Denser Medium, (air to conductor)



When Conductivity is finite then

$$\left(\frac{E_{0r}}{E_{0i}}\right) = \frac{1 - (1+i)\Delta}{1 + (1+i)\Delta}$$

$$\left(\frac{E_{0T}}{E_{0i}}\right) = \frac{2}{1+(1+i)\Delta}$$

where

$$\Delta = \left(\frac{\sigma_2 \mu_1}{2\mu_2 \varepsilon_1 \omega}\right)^{1/2}$$

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for perfect Conductor, $\sigma_2 \to \infty$ i.e. $\Delta \to \infty$

$$\left(\frac{E_{or}}{E_{oi}}\right) = -1$$

Reflectance :- How much part of energy will be reflected back

$$R = \left|\frac{E_{or}}{E_{oi}}\right|^2 = 1 \quad \text{(for perfect conductor)}$$

$$T = 0$$

$$\boxed{R + T = 1}$$

No energy can transmit in perfect conducting medium, all energy will be reflected

Reflectance if conductivity is finite

$$\boxed{R = 1 - \frac{2}{\Delta}}$$

$$T = 1 - R$$

Q. Calculate the reflectance coefficient for light at an air-silver interface $\mu_1 = \mu_2 = \mu_0$, $\varepsilon_1 = \varepsilon_0$, $\sigma = 6 \times 10^7 \ \Omega^{-1}\text{-}m^{-1}$ & $\omega = 4 \times 10^{15}$.

$$\Delta = \left(\frac{\sigma_2 \mu_1}{2 \mu_2 \varepsilon_1 \omega}\right)^{1/2}$$

$$= \left(\frac{\cancel{6}3 \times 10^7 \cancel{\mu}}{\cancel{2} \cancel{\mu} \varepsilon_0 \times 4 \times 10^{15}}\right)^{1/2} = \left(\frac{3}{4\varepsilon_0} \times 10^{-8}\right)^{1/2}$$

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Conservation laws in Electrodynamics :-

1. Conservation of charge :- Cons. of charge follows from continuty eqⁿ

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$$

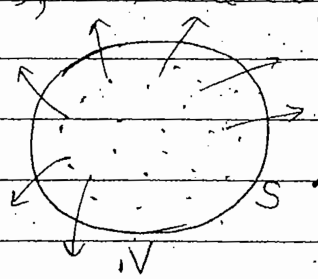
2. Cons. of Energy (Poynting theorem) :-

$$\frac{dw}{dt} = -\frac{d}{dt}\int_V \frac{1}{2}\left(\frac{B^2}{\mu_0} + \varepsilon_0 E^2\right)d\tau - \frac{1}{\mu_0}\oint(\vec{E}\times\vec{B})\cdot d\vec{s}$$

This is the integral form of Poynting theorm.
This is the Work - Energy theorem of electrodynamics.

Statement - Work done on the charges by the electromagnetic forces is equal to decrease in the energy stored in the fields, less the energy flow out through the surface.

If we have a close surface filled charge inside. vol. of close surface → V then energy



If EM fall on surface, then charges displaced. EM wave contain energy
This energy can be used in 2 ways

(i) Working on the charges
(ii) Energy flowing out through the surface S.

$$\boxed{\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}} \Rightarrow \text{Energy flux density}$$

Poynting Vector tells
→ Per unit area, per unit time, how much energy is flowing out through the surface S.

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Generally $\vec{S} = \vec{E} \times \vec{H}$

$$\frac{dW}{dt} = \frac{d}{dt}\int_V u_{mech} \cdot d\tau$$

$u_{mech} \rightarrow$ Mechanical energy density.
By incident EM wave, there will be mechanical motion.
By Multiplying $u_{mech} \times d\tau \Rightarrow$ Energy

$$\frac{d}{dt}\int_V \frac{1}{2}\left(\frac{B^2}{\mu_0} + \varepsilon_0 E^2\right) d\tau = \frac{d}{dt}\int_V u_{em}\, d\tau$$

We can write

$$\oint_S \left(\frac{\vec{E} \times \vec{B}}{\mu_0}\right) da = \int_S \vec{S} \cdot d\vec{a} = \int_V (\vec{\nabla} \cdot \vec{S}) d\tau$$

$d\tau$ = Arbitrary so

$$\frac{d}{dt}\left(u_{mech} + u_{em}\right) = -\vec{\nabla} \cdot \vec{S}$$

$u_{mech} + u_{em} = u_{Total}$
This S is equivalent to energy density $\vec{J}$ & energy density is equivalent to charge density $\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \vec{J}$

Cons. of Linear Mom. :- Total linear mom. is always conserved in electrodynamics & linear momentum density of Electromagnetic field is

$$\boxed{p_{em} = \mu_0 \varepsilon_0 \vec{S}}$$

Angular Mom. in Electromag. field :-
(Cons. of Ang. mom) :- Total ang. mom. is conserved in electromag. field. Angular

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mom. density of EM field is given by

$$\boxed{\vec{l}_{em} = (\vec{r} \times \vec{p}_{em})}$$

$$\vec{p}_{em} = \mu_0 \varepsilon_0 \vec{S} = \mu_0 \varepsilon_0 \left(\frac{\vec{E} \times \vec{B}}{\mu_0} \right)$$

$$\boxed{\vec{l}_{em} = (\vec{r} \times \vec{p}_{em}) = \varepsilon_0 [\vec{r} \times (\vec{E} \times \vec{B})]}$$

exercise - 1

Q.① If $\vec{E} = \hat{i} A \sin(12y) \sin(\alpha z) \cos \omega t$, describes an electric field in free space. What should be the numerical value of α for which electric field satisfies Maxwell's eqn. Given that, $\omega = 15 \times 10^8$ rad/sec.

Q.② In a region of Empty space, the mag. field is described by $\vec{B} = B_0 e^{ax} \sin(ky - \omega t) \hat{z}$. Find Electric field.

Q.③ If 2 EM waves given by $\vec{E}_1(z,t) = (E_0 \hat{x} + E_0 e^{-i\frac{\pi}{2}} \hat{y}) e^{i(kz-\omega t)}$ & $\vec{E}_2(z,t) = (E_0 \hat{x} + E_0 e^{i\frac{\pi}{2}} \hat{y}) e^{i(kz-\omega t)}$ superimposed. Find polarisation of resultant wave.

Q.④ The elec. vector of a wave is given by $\vec{E} = E_0 \left(\frac{8\hat{x} + 6\hat{y} + 5\hat{z}}{\sqrt{125}} \right) e^{i(\omega t + 3x - 4y)}$ & its freq. is 1 GHz. (ν).

a) Investigate weather wave is plane wave or not ?

b) Find wave vector $\vec{K}$.

c) Phase velocity along y-dir^n

d) dielectric const. & refractive index assuming medium is Non-magnetic.

Q. Calculate Plasma freq & maxi. penetration depth for a plasma containing 10^8 e's per m^3.

Q. A He-Ne laser beam of wavelength $\lambda = 632\,nm$ of intensity $1\ W/m^2$ is travelling along x-axis in vaccum. (a) Find amplitudes of electric & magnetic fields associated with laser beam

(b) Express $\vec{E}(x,t)$ & $\vec{B}(x,t)$ if electric field is polarised along y-dir^n.

<u>Potential Formulation</u> :- We have 2 type of pot^n,

(i) Scalar Pot^n $V(r,t)$

(ii) ~~Scalar~~ Vector Pot^n $\vec{A}(r,t)$

Both pot^n are func^n of time in electrodynamics. In electrostatics & Magnetostatics, they are independent of time.

We have $\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \boxed{\vec{B} = \vec{\nabla}\times\vec{A}}$ —— (A)

$$\vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t} \Rightarrow \vec{E} = -\vec{\nabla}V$$

$$\vec{\nabla}\times\vec{E} = -\vec{\nabla}\times\left(\frac{\partial\vec{A}}{\partial t}\right) \quad \text{(On putting } \vec{B} = \vec{\nabla}\times\vec{A}\text{)}$$

$$\vec{\nabla}\times\left(\vec{E} + \frac{\partial\vec{A}}{\partial t}\right) = 0 \quad \text{(in electrodynamics)}$$

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$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\vec{\nabla} V$$

$$\boxed{\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}} \quad \text{——(B)}$$

for same (unique) value of $\vec{E}$ & $\vec{B}$, we can have different value of $\vec{V}$ & $\vec{A}$. ($\vec{E}$ & $\vec{B}$ are dependent on each other).

If we choose a new mag. vector potⁿ $\vec{A}'$ s.t

$$\boxed{\vec{A}' = \vec{A} + \vec{\nabla}\lambda}$$

$\vec{A}$ is corresponding to $\vec{B}$
$\vec{A}'$ " " " $\vec{B}'$

We have to proof for different $\vec{A}$ & $\vec{A}'$, we will get same $\vec{B}$.

Simultaneously, we have to substract time derivative from scalar potⁿ.

$$\boxed{V' = V - \frac{\partial \lambda}{\partial t}}$$

$\lambda \rightarrow$ Gauge function

Corresponding to $\vec{A} \longrightarrow V, \vec{E}$
" $\vec{A}' \longrightarrow V', \vec{E}'$

$$\vec{B}' = (\vec{\nabla} \times \vec{A}')$$

$$= \vec{\nabla} \times \vec{A} + 0$$

$$\boxed{\vec{B}' = \vec{B}}$$

On changing $\vec{A}$ by $\vec{A}'$, $\vec{B}$ (mag. field) remains same.

$$\vec{E}' = -\vec{\nabla} V' - \frac{\partial \vec{A}'}{\partial t}$$

$$\vec{E}' = -\vec{\nabla} V + \vec{\nabla}\left(\cancel{\frac{\partial \lambda}{\partial t}}\right) - \frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\left(\cancel{\frac{\partial \lambda}{\partial t}}\right)$$

$$\vec{E}' = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

* To solve Maxwell's eqⁿ in form of potⁿ, we use these restrictions (gauge) [C.G. & L.G.]

$$\boxed{\vec{E}' = \vec{E}}$$

Hence, for same value of $\vec{E}$ & $\vec{B}$, we get different sets of $\vec{A}$ & V.

Coulomb Gauge :-

Condition of Coulomb gauge is

$$\boxed{\vec{\nabla}\cdot\vec{A} = 0}$$

We have

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

subs. the value of $\vec{E}$, from (B),

$$\Rightarrow \vec{\nabla}\cdot\left(-\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}\right) = \frac{\rho}{\varepsilon_0} \qquad (\vec{\nabla}\cdot\vec{A} = 0)$$

$$\Rightarrow \boxed{\nabla^2 V = -\frac{\rho}{\varepsilon_0}} \Rightarrow \text{Poission's eq}^n.$$

This is the wave eqⁿ of V under Coulomb gauge.

We have $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}$

subs. the value of $\vec{B}$ & $\vec{E}$ from (A) & (B)

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial}{\partial t}\left(-\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}\right)$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A} = \mu_0\vec{J} + \mu_0\varepsilon_0\vec{\nabla}\left(\frac{\partial V}{\partial t}\right) - \mu_0\varepsilon_0\frac{\partial^2\vec{A}}{\partial t^2}$$

$$\Rightarrow \underbrace{\vec{\nabla}(\vec{\nabla}\cdot\vec{A})}_{=0} - \nabla^2\vec{A} + \mu_0\varepsilon_0\frac{\partial^2\vec{A}}{\partial t^2} = \mu_0\vec{J} - \mu_0\varepsilon_0\vec{\nabla}\left(\frac{\partial V}{\partial t}\right)$$

$$\Rightarrow \boxed{\nabla^2\vec{A} - \mu_0\varepsilon_0\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{J} + \mu_0\varepsilon_0\vec{\nabla}\left(\frac{\partial V}{\partial t}\right)}$$

This is the wave eqⁿ for vector potⁿ A.

Under Coulomb Gauge, it is easy to find V but difficult to find $\vec{A}$.

Hence this gauge is of little use.

Lorentz Gauge :-

Cond^n of Lorentz gauge is

$$\boxed{\vec{\nabla}\cdot\vec{A} + \mu_0 \varepsilon_0 \frac{\partial V}{\partial t} = 0}$$

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$$

$$\Rightarrow \vec{\nabla}\cdot\left(-\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}\right) = \frac{\rho}{\varepsilon_0}$$

$$\Rightarrow -\nabla^2 V - \frac{\partial}{\partial t}(\nabla\cdot\vec{A}) = \frac{\rho}{\varepsilon_0}$$

$$\boxed{\nabla^2 V - \mu_0 \varepsilon_0 \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon_0}} \quad \text{--- (1)}$$

This is the wave eq^n for scalar pot^n V.

Now, $\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(-\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}\right)$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A} = \mu_0 \vec{J} - \mu_0 \varepsilon_0 \vec{\nabla}\left(\frac{\partial V}{\partial t}\right) - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$\Rightarrow -\nabla^2\vec{A} + \mu_0 \varepsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} + \vec{\nabla}\Big(\underbrace{\vec{\nabla}\cdot\vec{A} + \mu_0 \varepsilon_0 \frac{\partial V}{\partial t}}_{=0}\Big) = \mu_0 \vec{J}$$

$$\Rightarrow \boxed{\nabla^2\vec{A} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}} \quad \text{--- (2)}$$

This is the wave eq^n of vector pot^n $\vec{A}$.
eq^n (1) & (2) are the wave eq^n under Lorentz Gauge.

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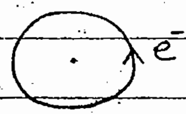
Leinard-Weichard potential :- This is pot^n of a moving point charge moving at velocity very close to speed of light (relativistic)

If charge q is moving in straight line with const. vel. $\vec{v}$, then it can not emit electromag. radiation

$q \longrightarrow \vec{v}$ const.

'Only accelerating charge can emit EM radiation.

If charge q is moving with constant vel. v in a circle then it will emit EM radiation bcoz v is const. but dir^n is changing. So it is accelerated motion



v const.

$$\vec{a} = \frac{d\vec{v}}{dt}$$

It depand on both dir^n & velocity.

Rotarted time (t_r) :- (means time lag)

L-W pot^n is Retarted pot^n,

Pot^n (not func^n of time)

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(r)}{r}\, d\tau$$

Pot^n at time t is due to charge of retarded time,

$$V(r,t) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(r, t_r)}{r}\, d\tau$$

Sun-light reaches on earth in 8 min (ret. time) & light of starts are the concept of ret. time.

Rotarted time, $\boxed{t_r = t - \frac{r}{c}}$

$t \rightarrow$ present time

$\frac{r}{c} \rightarrow$ time of travel

||ly. for $\vec{A}$,

$$\vec{A}(r,t) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(r,t_r)}{r}\,d\tau$$

→ We can not find $\vec{E}$ & $\vec{B}$ directely through retarted time but we can find pot^n

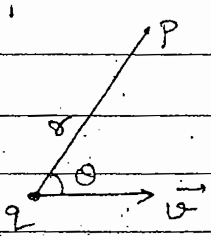
Becoz $\vec{E}$ & $\vec{B}$ are not linear with $\frac{1}{r}$.

→ We can find $\vec{E}$ & $\vec{B}$ by $\vec{B} = \vec{\nabla}\times\vec{A}$

$$\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}$$

- A point charge q moving with const. rel^v velocity in a straight line then its pot^n at r distance

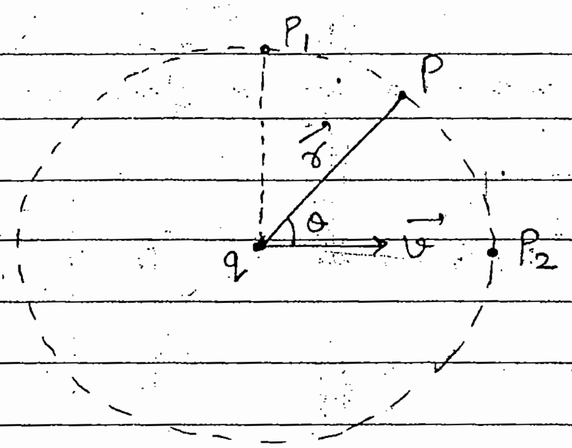
$$V(r,t) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r\sqrt{1-\frac{v^2\sin^2\theta}{c^2}}}$$



If charge is stationary then pot^n will be same on the surface of sphere (if we sketch a sphere of radius r) at a inst of time.

But if charge is moving then pot^n is not same. Pot^n at P_1 is more than at P_2.

$$\boxed{V_{P_1} > V_{P_2}}$$



Here $v \simeq c$, if $v \ll c$ then no effect of pot^n V.

$V_{static} = \frac{q}{4\pi\varepsilon_0 r}$ Static means $v=0$

Vector potⁿ of a point charge

$$\vec{A} = \frac{\vec{v}}{c^2} V(\vec{r}, t)$$

if charge is static then $v=0 \Rightarrow \vec{A}=0$

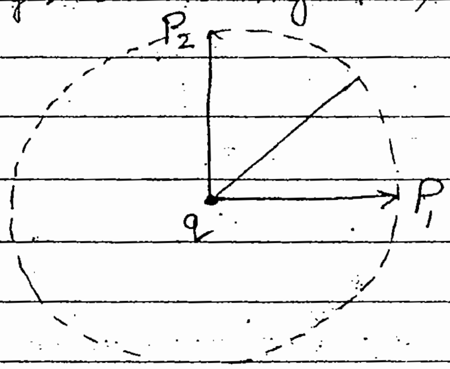
Vector potⁿ $\vec{A}$ depends on both strength of charge q as well as velocity $\vec{v}$.

Electric field of a pt. charge moving in straight line with const. relᵛ vel. v,

$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon_0} \frac{\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2} r^2}\hat{r}}$$

If charge particle q is stationary then $\vec{E}$ will be same But if q is moving then

$$\vec{E}_{static} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$



for P_1, $\theta = 0$

$$\vec{E}_{P_1} = \frac{q}{4\pi\varepsilon_0} \frac{\left(1-\frac{v^2}{c^2}\right)}{r^2}\hat{r}$$

At P_2, $\theta = 90°$

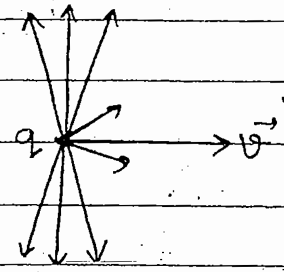
$$\vec{E}_{P_2} = \frac{q}{4\pi\varepsilon_0} \frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)} r^2}\hat{r}$$

$$\vec{E}_{P_2} > \vec{E}_{P_1}$$

$$\left\{ \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \right.$$

$$\boxed{E_{P_1} = \frac{\vec{E}_{static}}{\gamma^2}} \quad \& \quad \boxed{E_{P_2} = \gamma \vec{E}_{static}}$$

Field lines :- will be symmetrical but their strength will be different in diff. dirⁿ. If charge particle q move with c then in the dirⁿ of motion of q there will be No field.



$E_{P_1} \longrightarrow 0$

$E_{P_2} \longrightarrow \infty$

i.e. electric field will have no component.

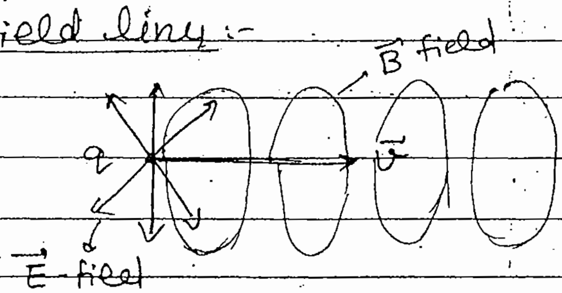
Mag. field,

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi} \frac{qv\left(1-\frac{v^2}{c^2}\right)\sin\theta}{\left[1-\frac{v^2}{c^2}\sin^2\theta\right]^{3/2} r^2}\hat{\phi}}$$

if $\theta = 0$ then $\vec{B} = 0$

Mag. field can never exist in dirⁿ of motion of q.

Field line :-



Thumb will be in the dirⁿ of motion of particle. Mag. field will curl around dirⁿ of motion

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Radiation :- Radiation is propogation of energy in the form of EM waves.

Radiation means EM wave or photon.

For a stationary charge, energy / intensity / power / poyinting vector is same thing.

	E	B	$\vec{S}$	Power
Stationary charge	$\propto \frac{1}{r^2}$	0	0	0
charge moving with const. vel $\vec{v}$	$\propto \frac{1}{r^2}$	$\propto \frac{1}{r^2}$	$\propto \frac{1}{r^4}$	$\propto \frac{1}{r^2}$

($\vec{E}$ & $\vec{B}$ are independent)

We know,

Intensity, $I = \langle \vec{S} \rangle$ = energy per unit area/unit time

Power, $\boxed{P = \int_S \langle \vec{S} \rangle \, d\vec{a}}$

For charge particle moving with const. v

Power is dec as $\frac{1}{r^2}$. After some time $P = 0$.

Accelerated Motion, $\vec{E}$ & $\vec{B}$ of an accelerated charge are $\propto \frac{1}{r}$.

E	B	S	P
$\propto \frac{1}{r}$	$\propto \frac{1}{r}$	$\propto \frac{1}{r^2}$	constant

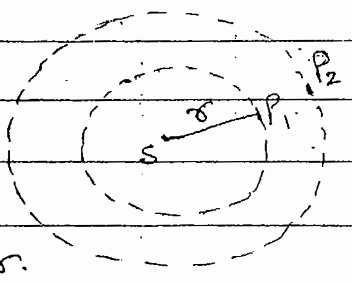
Quantised Radiation / Accelerated motion :- Mag. of $\vec{E}$ & $\vec{B} \propto \frac{1}{r}$

EM wave coming from Sun, $E \propto \frac{1}{r}$ & $B \propto \frac{1}{r}$.

Both spheres have same power at surface

But energy/unit area at P_1 is more than " " " P_2

But power is const. indipendent on r.



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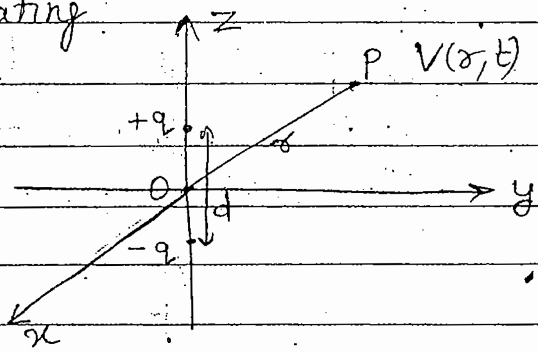
* Power dec. by some value of r & area is ↑ by the same value of r.
i.e. Power is constant over a spherical surface.
(if power decay then we can't recieve sun rays)

For a, point charge, dipole, Quadroupole,
Accelerated part of electric & mag. field defined as $E_a \propto \frac{1}{r}$ & $B_a \propto \frac{1}{r}$

$S \propto \frac{1}{r^2}$, $P \rightarrow$ const. over a given surface.

Electric dipole Radiation :-

Consider a electric dipole along z-axis.
dipole will be oscillating if q is func^n of time t.
& if q is not a func^n of t then dipole will be static & radiation will not emit in that case.



If dipole i.e. charge is oscillating with freq. ω then

$$q(t) = q_0 \cos \omega t$$

Its dipole moment $\vec{p} = q d$

$$\boxed{\vec{p} = q_0 d \cos \omega t \, \hat{z}}$$

$$\boxed{\vec{p} = p_0 \cos \omega t \, \hat{z}}$$

& freq. of oscillation is ω.
Now find at point P, what is the

scalar & vector pot^n, Ele. & mag. field, poynting vector S & power ~~de~~ radiated by the dipole.

Use the concept of Retarted time.

$$t_r = t - \frac{r}{c}$$

The changes in dipole at time t will happen in retarted time t_r. We have to take approximation.

<u>Approximation</u> :- P is at a far distance so Power at P will be due to <u>Accelerated</u> pot^n.

$$E = \underbrace{E_s}_{\text{static}} + \underbrace{E_v}_{\text{velocity}} + \underbrace{E_a}_{\text{acceleration}}$$

Static & velocity part is very very less then accelerated part at far distance

(i) $d << r$

(ii) $d << \frac{c}{\omega}$

(iii) ~~d~~ $r >> \frac{c}{\omega}$

$\Rightarrow$ Radiation Zone or far - field Zone

Combinely, $d << \frac{c}{\omega} << r$

$$\boxed{d << \lambda << r}$$

- <u>Scalar Pot^n</u>,

$$V(r,\theta,t) = \frac{-p_0 \omega}{4\pi\epsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin\left(\omega t - \frac{\omega r}{c}\right)$$

- <u>Vector Pot^n</u>,

$$\vec{A}(r,\theta,t) = -\frac{\mu_0 p_0 \omega}{4\pi r}\sin\left(\omega t - \frac{\omega r}{c}\right)\hat{z}$$

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$\boxed{V, \vec{A}, E, B \propto \frac{1}{r}}$ for accelerated charge or radiation.

- Electric field

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{E} = -\frac{\mu_0 p_0 \omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega t - \frac{\omega r}{c}\right]\hat{\theta}$$

by grad V → radial part → drop thi
θ part

- Magnetic field

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{B} = -\frac{\mu_0 p_0 \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega t - \frac{\omega r}{c}\right]\hat{\phi}$$

- Poynting vector

$\vec{E}$ is along $\hat{\theta}$, $\vec{B}$ is along $\hat{\phi}$ so $\vec{S}$ is along $\Rightarrow$ $\hat{\theta} \times \hat{\phi} = \hat{r}$

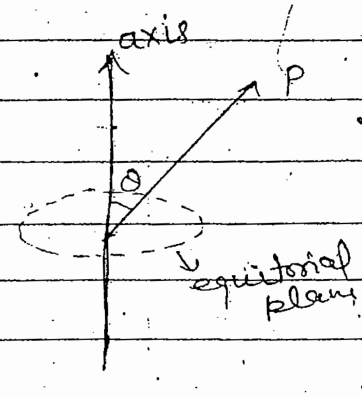
$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$\vec{S} = \frac{\mu_0 p_0^2 \omega^4}{16\pi^2 c}\left(\frac{\sin^2\theta}{r^2}\right)\cos^2\left[\omega t - \frac{\omega r}{c}\right]\hat{r}$$

$\hat{r}$ means power (energy) is going out.

- Intensity

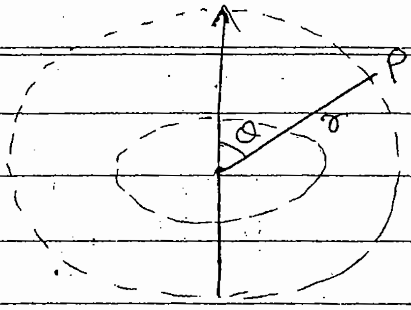
$$I = \langle \vec{S} \rangle = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c}\,\frac{\sin^2\theta}{r^2}$$



\# If P is along axis then $\theta = 0$
i.e. Power = 0

✓ No power is radiated along axis of the dipole.

✓ Maxi. power radiated along equitorial plane



If we sketch a sphere then

Power radiated over the surface of sphere

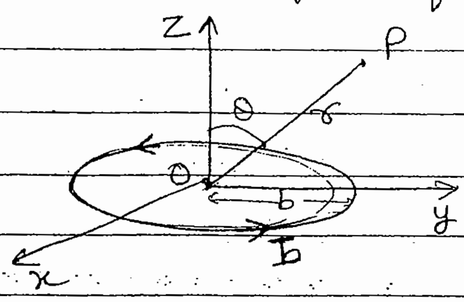
$$P = \int_S \langle \vec{S} \rangle \, da \qquad d\vec{a} = r^2 \sin\theta \, d\theta \, d\phi$$

$$P = \frac{\mu_0 p_0^2 \omega^4}{32 \pi^2 c} \int_0^{2\pi} d\phi \int_0^{\pi} \sin^3\theta \, d\theta$$

$$\boxed{P = \frac{\mu_0 p_0^2 \omega^4}{12 \pi c}} = \text{const.}$$

So Power radiated of a dipole is proportional to ω^4. $\boxed{P \propto \omega^4}$

Magnetic Dipole Radiation :- Suppose we have a magnetic dipole in the form of circular ring of radius b & current flowing I(t) which is time dependent (if I is not time dependent then charge it will not radiate.



This circular ring is uncharged.

$$I(t) = I_0 \cos \omega t$$

Mag. mom. $\vec{m} = I \cdot \text{Area}$

$$\vec{m} = I \cdot \pi b^2 \hat{z}$$

$$\vec{m} = I_0 \pi b^2 \cos\omega t\, \hat{z}$$

$$\boxed{\vec{m} = m_0 \cos\omega t\, \hat{z}}$$

m_0 is the amp. of mag. mom.

Because loop is uncharged so scalar pot^n $\boxed{V = 0}$ but $A \neq 0$ bcoz V arrise from charge & A arrise from current.

Approximation

$$b << \frac{c}{\omega} << r$$

$$b << \lambda << r$$

far-field zone / Radiation Zone

- Vector pot^n :- $\vec{A}$ is $\parallel$ or anti $\parallel$ to current.

$$\vec{A}(r,\theta,t) = -\frac{\mu_0 m_0 \omega}{4\pi c}\left(\frac{\sin\theta}{r}\right)\sin\left(\omega t - \omega\frac{r}{c}\right)\hat{\phi}$$

- Electric field :-

$$\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}$$

Since $V = 0$ so $\vec{E} = -\frac{\partial\vec{A}}{\partial t}$

$$\vec{E} = \frac{\mu_0 m_0 \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega t - \omega\frac{r}{c}\right]\hat{\phi}$$

- Mag. field :-

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{B} = -\frac{\mu_0 m_0 \omega^2}{4\pi c^2}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega t - \omega\frac{r}{c}\right]\hat{\theta}$$

$\vec{B}$ at centre will be in $\hat{z}$ dir^n.

- Poynting Vector :-

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

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$$\vec{S} = \frac{\mu_0 m_0^2 \omega^4}{16\pi^2 c^3}\left(\frac{\sin^2\theta}{r^2}\right)\cos^2\left[\omega t - \frac{\omega r}{c}\right]\hat{r}$$

Again, No energy radiated along z-dir.

- Intensity :- average of $\vec{S}$ over one cycle of time.

$$I = \langle \vec{S} \rangle = \frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3}\left(\frac{\sin^2\theta}{r^2}\right)\hat{r}$$

- Power Radiated :-

$$P = \int_S \langle \vec{S} \rangle \, d\vec{a}$$

$$\boxed{P = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3}}$$

Comparision of $P_{electric\,dipol}$ & $P_{mag\,dipole}$:-

$$\boxed{\left(\frac{P_{ele}}{P_{mag}}\right) = \left(\frac{p_0 c}{m_0}\right)^2}$$

This is the energy comparision.

\# Energy around a dipole weather electric or magnetic forms a Donut shape.
⇒ shape in which energy is maxi. along eq equitorial plane & no energy along the axis.

Power radiated due to Quadrupole :-

$$\boxed{P \propto \omega^6}$$

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Power Radiated by an Accelerated point charge,

$$\boxed{P = \frac{\mu_0 q^2 \gamma^6}{6\pi c}\left[a^2 - \left|\frac{\vec{v}\times\vec{a}}{c}\right|^2\right]} \Rightarrow \text{General formula}$$

where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

$q \longrightarrow \vec{v}$ (accelerated)

$a \rightarrow$ acceleration

$v \rightarrow$ velocity

This formula is called Leinard Formula.

There may be 3 cases,

(i) $v << c$, (ii) v & a are $\perp$ (iii) v & a are colinear.

(i) $v << c$

charge particle is moving with non-relativistic speed. i.e. $\gamma \rightarrow 1$, power

$$\boxed{P = \frac{\mu_0 q^2 a^2}{6\pi c}}$$

This is called Larmour formula.

Here power radiated $P \propto q^2$ & a^2.

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \times \frac{\epsilon_0}{\epsilon_0} \qquad \mu_0\epsilon_0 = \frac{1}{c^2}.$$

$$\boxed{P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3}}$$

This is an another form of Larmour formula.

$\Rightarrow$ If e^- & α particle are accelerated with same acceleration then α particle will be accelerated more. α-particle will emit 4 times power than e^- bcoz it depends of magnitude of charge.

{ e^- & p will emit same power if they acc. with same acc. }

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(ii) Sychrotron Radiation
v & a are $\perp$ to each other.
If charge particle moves in a circle then v & a are $\perp$ to each other.

$$P = \frac{\mu_0 q^2 \gamma^6 a^2 (1 - v^2/c^2)}{6\pi c \cancel{(1 - v^2/c^2)}}$$

$$P = \frac{\mu_0 q^2 \gamma^6 a^2}{6\pi c \gamma^2}$$

$$\boxed{P = \frac{\mu_0 q^2 \gamma^4 a^2}{6\pi c}}$$

(iii) v & a are colinear (Brehmstrauhlung Radiation)
then angle b/w them = 0
this is possible if charge particle is moving in a straight line.

$$\boxed{P = \frac{\mu_0 q^2 \gamma^6 a^2}{6\pi c}}$$

This is also called Braking Radiation.
This is found, when high speed e^-s deaccelerated.

$\Rightarrow$ When high speed e^- fall on metal surface then e^- will be deaccelerated & energy will radiate, it will be Continous x-ray. & this radiation is called Brehmstrauhlung Radiation.

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Exercise - 2

Q. 1 :- Find the fields, charge & current distributions corresponding to $V(r,t) = 0$

$$A(r,t) = -\frac{1}{4\pi\epsilon_0} \frac{qt}{r^2} \hat{r}$$

Q. 2 :- Suppose $V = 0$ & $\vec{A} = A_0 \sin(kx - \omega t)\hat{y}$ where A_0, ω & k are constants. Find $\vec{E}$ & $\vec{B}$ & cheak weather they satisfies Maxwell's eqn in vaccum or not & what condition is to be imposed on ω & k.

Q. 3 :- Use Gauge fuc^n $\lambda = -\frac{1}{4\pi\epsilon_0}\frac{q.t}{r}$ to transform the pot^{n}s given by $V(r,t) = 0$

& $\vec{A}(r,t) = \frac{-1}{4\pi\epsilon_0} \frac{qt}{r^2} \hat{r}$

i.e. V' & $\vec{A}' = ?$

Q. 4 :- Found which is Coulomb Gauge, Lorentz Gauge or both in the fallowing potentials :-

(i) $V(r,t) = 0$

$$\vec{A}(r,t) = -\frac{1}{4\pi\epsilon_0} \frac{qt}{r^2} \hat{r}$$

(ii) $V = 0$, $\vec{A} = A_0 \sin(kx - \omega t)\hat{y}$

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Exercise - 1

Q.1 :- $\vec{E} = \hat{i}\, A \sin(12y)\sin(\alpha z)\cos\omega t$

$(\alpha = 13)$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A\sin(12y)\sin(\alpha z)\cos\omega t & 0 & 0 \end{vmatrix}$$

$$\vec{\nabla}\times\vec{E} = \hat{j}\left[\frac{\partial}{\partial z}\{A\sin(12y)\sin(\alpha z)\cos(\omega t)\}\right] - \hat{k}\left[\frac{\partial}{\partial y}\{A\sin(12y)\sin(\alpha z)\cos(\omega t)\}\right]$$

$$= \hat{j}\,A\alpha\sin(12y)\cos(\alpha z)\cos(\omega t) - \hat{k}\,A(12)\cos(12y)\sin(\alpha z)\cos(\omega t)$$

$$-\frac{\partial\vec{B}}{\partial t} = A\cos\omega t\left[\hat{j}\,\alpha\sin(12y)\cos(\alpha z) - \hat{k}\,12\cos(12y)\sin(\alpha z)\right]$$

$$\vec{B} = -\int dt\left[A\cos\omega t\{\hat{j} \quad '' \quad -\hat{k} \quad ''\}\right]$$

$$\vec{B} = -\frac{A\sin\omega t}{\omega}\left[\hat{j}\alpha\sin(12y)\cos(\alpha z) - \hat{k}\,12\cos(12y)\sin(\alpha z)\right]$$

Now $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$ —— (1)

$$\Rightarrow \vec{\nabla}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & -\alpha\sin(12y)\cos\alpha z & 12\cos(12y)\sin\alpha z \end{vmatrix}$$

$$= \hat{i}\left[-144\sin(12y)\sin(\alpha z) - \alpha^2\sin(12y)\sin(\alpha z)\right]$$

$$(1) \Rightarrow -\frac{A\sin\omega t}{\omega}\,\hat{i}\left[-144-\alpha^2\right]\sin(12y)\sin(\alpha z)$$

$$= +\mu_0\varepsilon_0\omega\,\hat{i}\,A\sin(12y)\sin(\alpha z)\sin\omega t$$

$$\Rightarrow \frac{1}{\omega}\left[-144-\alpha^2\right] = \omega\, \mu_0\varepsilon_0$$

$$-144-\alpha^2 = \frac{\omega^2}{c^2}$$

$$\Rightarrow \alpha^2 = -144-\left(\frac{15\times10^8}{3\times10^8}\right)^2 = -144-25$$

$$\Rightarrow \alpha^2 = -169$$

$$\Rightarrow \boxed{\alpha = 13i}$$

Q.2 :- $\vec{B} = B_0 e^{ax}\sin(ky-\omega t)\hat{z}$ in empty space.

If for a plane wave $\vec{K}\cdot\vec{B}=0$

then $\vec{E} = -\frac{c^2}{\omega}(\vec{K}\times\vec{B})$

from $\vec{B}$, we get $\vec{K} = K\hat{y}$

& $\vec{K}\cdot\vec{B} = 0$ Here, so

$$\vec{E} = -\frac{c^2}{\omega}\left[K B_0 e^{ax}\sin(Ky-\omega t)\right]\hat{x}$$

$$\{\hat{y}\times\hat{z} = \hat{x}\}$$

$$\boxed{\vec{E} = \frac{c^2}{\omega}K B_0 e^{ax}\sin(Ky-\omega t)\,(-\hat{x})}$$

<u>Another Method</u>,

$$\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\Rightarrow \vec{E} = \int\frac{1}{\mu_0\varepsilon_0}(\vec{\nabla}\times\vec{B})\,dt$$

$$\vec{\nabla}\times\vec{B} = \hat{i}\left[\frac{\partial}{\partial y}B_z\right]-\hat{j}\left[\frac{\partial}{\partial x}B_z\right]+\hat{k}(0)$$

$$= \hat{i}\left[B_0 e^{ax}K\cos(Ky-\omega t)\right]-\hat{j}\left[B_0 a e^{ax}\sin(Ky-\omega t)\right]$$

$$\text{So } \vec{E} = c^2\int\left[\hat{i}\,B_0 e^{ax}K\cos(Ky-\omega t)-\hat{j}\,B_0 a e^{ax}\sin(Ky-\omega t)\right]dt$$

$$\vec{E} = c^2\left[\hat{i}\,B_0 e^{ax}K\frac{\sin(Ky-\omega t)}{(-\omega)}+\hat{j}\,B_0 a e^{ax}\frac{\cos(Ky-\omega t)}{-\omega}\right]$$

$$\vec{E} = \frac{c^2}{\omega}\left[(-\hat{i})B_0 e^{ax}K\sin(Ky-\omega t)+(-\hat{j})B_0 a e^{ax}\cos(Ky-\omega t)\right]$$

Q.3 :- $\vec{E}_1(z,t) = \left(E_0\hat{x} + E_0 e^{-i\frac{\pi}{2}}\hat{y}\right)e^{i(kz-\omega t)}$

$\vec{E}_2(z,t) = \left(E_0\hat{x} + E_0 e^{i\frac{\pi}{2}}\hat{y}\right)e^{i(kz-\omega t)}$

Put $z=0$, in $\vec{E}_1$ then

$\vec{E}_1 = \left[E_0\hat{x} + E_0(-i)\hat{y}\right]e^{-i\omega t}$

→ At $\omega t = 0$ then

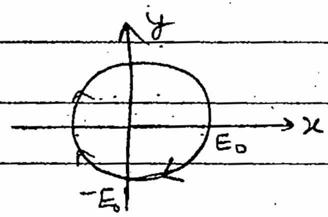
$\vec{E}_1 = E_0\hat{x} - E_0 i\hat{y}$

$\vec{E}_1 = E_0\hat{x}$

→ At $\omega t = \pi/2$, $\vec{E}_1 = (E_0\hat{x} - iE_0\hat{y})(-i)$

$\vec{E}_1 = -iE_0\hat{x} - E_0\hat{y}$

$\vec{E}_1 = -E_0\hat{y}$



So acc. to Ang. mom. → LCP

So Acc. to optics RCP. clockwise

Now, Put $z=0$ in $\vec{E}_2$

$\vec{E}_2 = (E_0\hat{x} + iE_0\hat{y})e^{-i\omega t}$

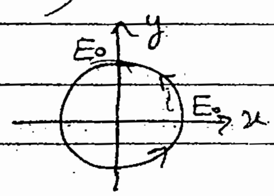
At $\omega t = 0$, $\vec{E}_2 = E_0\hat{x} + iE_0\hat{y}$

$\vec{E}_2 = E_0\hat{x}$

At $\omega t = \frac{\pi}{2}$, $\vec{E}_2 = (E_0\hat{x} + iE_0\hat{y})(-i)$

$\vec{E}_2 = -iE_0\hat{x} + E_0\hat{y}$

$\vec{E}_2 = E_0\hat{y}$



So acc. to optics LCP

Now $\vec{E}_1(z,t) = (E_0\hat{x} - iE_0\hat{y})e^{i(kz-\omega t)}$

$\vec{E}_2(z,t) = (E_0\hat{x} + iE_0\hat{y})e^{i(kz-\omega t)}$

$\vec{E} = \vec{E}_1 + \vec{E}_2$

$\vec{E} = 2E_0\hat{x}\,e^{i(kz-\omega t)}$

Resultant wave polarised along x-dirⁿ.

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Q.4 :- $\vec{E} = \frac{E_0(8\hat{x}+6\hat{y}+5\hat{z})}{\sqrt{125}} e^{i(\omega t+3x-4y)}$

freq. $\nu = 1\,GHz$

(a) $\vec{E} = \frac{E_0(8\hat{x}+6\hat{y}+5\hat{z})}{\sqrt{125}} e^{i\{\omega t-[(-3\hat{x}+4\hat{y})(x\hat{x}+y\hat{y})]\}}$

$\vec{K} = -3\hat{x}+4\hat{y}$

$\vec{K}\cdot\vec{E} = -24+24 = 0 \Rightarrow \vec{K}\cdot\vec{E} = 0$

So It is a plane wave.

(b) $\boxed{\vec{K} = -3\hat{x}+4\hat{y}}$

(c) $v_y = \frac{\omega}{K_y} = \frac{2\pi\nu}{K_y}$

$v_y = \frac{2\times 3.14\times 1\times 10^9}{4} = \frac{6.28}{4}\times 10^9$

$\boxed{(v_p)_y = 1.57\times 10^9}$

This is more than c. v_p may be $> c$ but v_g is $< c$ s.t. $v_p v_g = c^2$

(d) $\varepsilon_r, n = ?$

$v = \frac{\omega}{K} = \frac{2\times 3.14\times 1\times 10^9}{\sqrt{9+16}} = \frac{6.28\times 10^9}{5}$

$v = 1.256\times 10^9$

$v = \frac{c}{n} \Rightarrow n = \frac{c}{v} = \frac{3\times 10^8}{1.256\times 10^9}$

$n = 2.38854\times 10^{-1} \Rightarrow \boxed{n = 0.23885}$

& $\varepsilon_r = n^2 = (0.23885)^2 = 0.05705$

$\boxed{\varepsilon_r = 0.05705}$

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Q.5 :- Plasma freq. = ? & δ_{max} = ?

$n = 10^{18}\ e^{-1}/m^3$

$f_c = 9\sqrt{n}$

$\simeq 9\sqrt{10^{18}} = 9 \times 10^9$ Hz

$f_c = \frac{\omega_P}{2\pi} \Rightarrow \omega_p = 2\pi f_c$

$\omega_p = 2 \times 3.14 \times 9 \times 10^9$

$\omega_p = 5.66 \times 10^9$

$\omega_p = 5.66 \times 10^{10}$ Hz

Now Maxi. penetration depth

$\delta_{max} = \frac{c}{\omega_p}$

$= \frac{3 \times 10^8}{5.66 \times 10^{10}}$

$= 0.53004 \times 10^{-2}$

$\delta_{max} = 0.0053$

or $\boxed{\delta_{max} = 5.3 \times 10^{-3}}$

Note :- $\omega_p = \left(\frac{ne^2}{\varepsilon_0 m}\right)^{1/2}$

Q.6 :- He-Ne Laser

$\lambda = 632$ nm , $I = 1\ W/cm^2$

$= 10000\ W/m^2$

$\vec{E}$ & $\vec{B}$ = ?

$I = \langle \vec{S} \rangle = \frac{1}{2} c \varepsilon_0 E_0^2$

$\Rightarrow E_0^2 = \frac{2\langle \vec{S} \rangle}{c \varepsilon_0}$

$$E_0 = \sqrt{\frac{2\langle S\rangle}{c\,\varepsilon_0}}$$

$$= \sqrt{\frac{2\times 10^4}{3\times 10^8 \times 8.85\times 10^{-12}}} \quad V/m$$

$$= 0.2744 \times 10^4$$

$$= 2744 \quad V/m$$

Now $B_0 = \dfrac{E_0}{c}$

$$= \frac{0.2744\times 10^4}{3\times 10^8} = 0.09147 \times 10^{-4}$$

$$B_0 = 9.146 \times 10^{-6}$$

(b) Express $\vec{E}(x,t)$ & $\vec{B}(x,t)$ in polarised along y-dir^n.

$$\vec{E}(x,t) = E_0 e^{i(kx-\omega t)}\hat{y}$$

$$K = \frac{2\pi}{\lambda} \quad \& \quad \frac{\omega}{K} = c$$

$$K = \frac{2\pi}{\lambda} = \frac{2\times 3.14\times 10^9}{632} = 0.00994\times 10^9$$

$$= 9.9\times 10^6$$

$$\omega = cK = 3\times 10^8 \times 9.9\times 10^6$$

$$\omega = 29.7\times 10^{14}$$

$$\vec{E}(x,t) = 2744\, e^{i(9.9\times 10^6 x - 2.97\times 10^{14} t)}\hat{y}$$

& $\vec{B}(x,t) = B_0 e^{i(kx-\omega t)}\hat{z}$

$$\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}$$

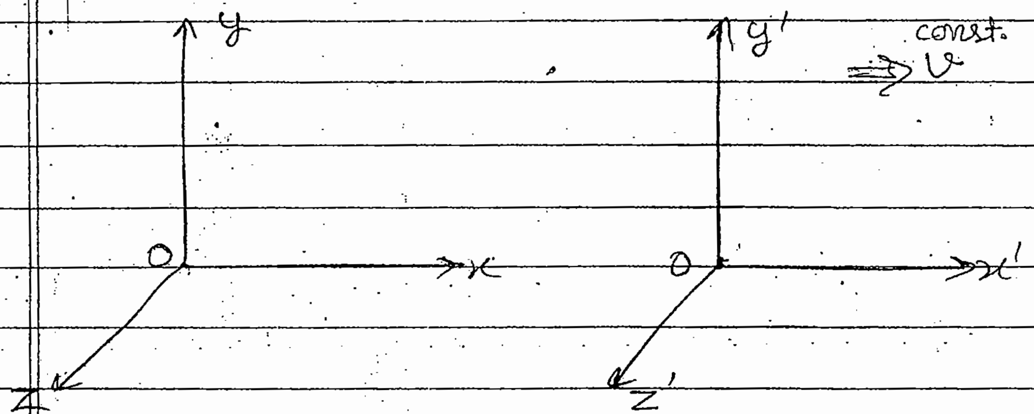
Special Theory of Relativity & Electrodynamics :-

It is apply only on inertial fram of references. Inertials fram of ref. are non accelerated & non rotated.

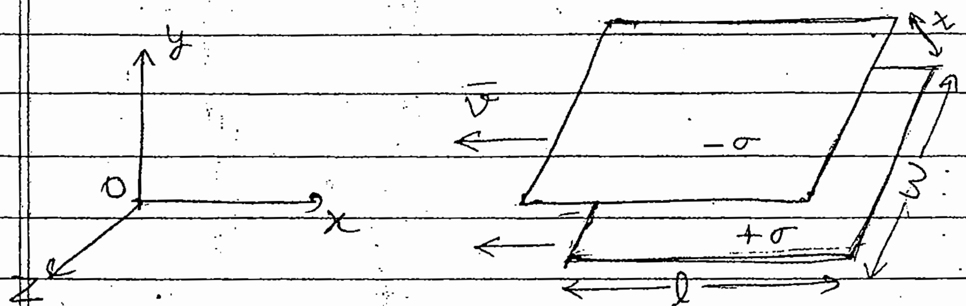
Postulates :-

1) Speed of light is same for all inertial frame
2) law of physics are " " "

Inertial frame – Newtons Ist law is valid.
How the fields are modified from one frame of reference to another.



We know ele. & mag. field in rest frames & have to find in moving frame S'.
Let us consider a parallel plate capacitor,



charge q is relativistically (Lorentz) invariant.

In Rest frame, electric field

$$\boxed{\vec{E}_0 = \frac{\sigma}{\varepsilon_0}\hat{y}}$$

When this capacitor moves then elec field will be increased γ times as shown below

$$\sigma_0 = \frac{q}{A} = \frac{q}{\ell w}$$

When capacitor is moving then $\sigma = \frac{q}{\frac{\ell}{\gamma} w}$

$$\sigma = \frac{\gamma q}{\ell w} = \gamma \sigma_0 \Rightarrow \boxed{\sigma = \gamma\sigma_0}$$

$$\therefore \vec{E} = \frac{\gamma\sigma_0}{\varepsilon_0}\hat{y}$$

$$\boxed{\vec{E} = \gamma\vec{E}_0}$$

$\Rightarrow$ When Elec. field is $\perp^r$ to the motion then it $\uparrow$ γ times & when Elec field is $\parallel$ or anti $\parallel$ to the motion then E will be No change in ~~eff~~ electric field. (length contraction in ℓ & w but Not in t)

Hence,

$$\boxed{E^{\parallel}_{motion} = E^{\parallel}_{rest}}$$

&

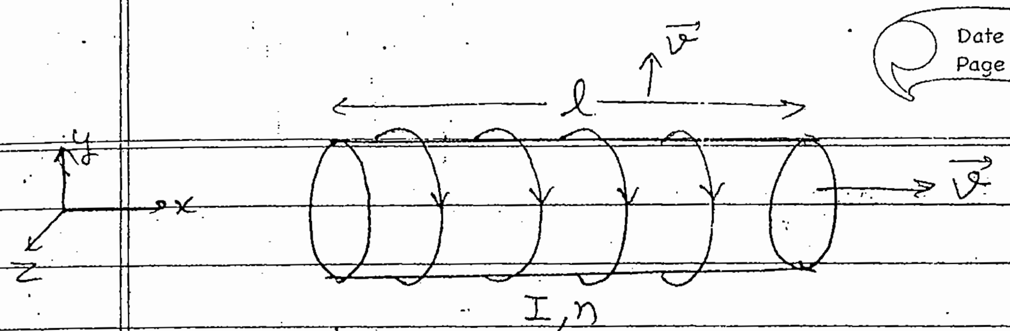
$$\boxed{E^{\perp}_{s'} = \gamma E^{\perp}_{s}}$$

This is for independent E. field.

For Independent Mag. field

Consider a solenoid & current flowing in its wires I & n be the no. of wires per unit length.

When it is at rest then its mag field will be $\boxed{\vec{B} = \mu_0 n I \hat{x}}$



when solanoid is in motion they current in it (charge is invarient)

$$I_{s'} = \frac{q}{t}$$

$(q = It)$

$$\boxed{I_{s'} = \frac{I}{\gamma}}$$

& No. of turns per unit length while it is moving, ↑ by γ times i.e.

$$n_{s'} = \gamma n$$

So $\vec{B_{s'}} = \mu_0 n I \hat{x}$

$$\boxed{\vec{B_{s'}} = \vec{B_s}}$$

Conclusion :- when solanoid is moving in x-dir. If Any comp. of $\vec{B}$ is in x-dir^n then it will be unaffected. i.e.

$$\boxed{B^{\parallel}_{s'} = B^{\parallel}_{s}}$$

(only 1 vector z will contract)

Now If solanoid is moving in upward dir^n then $\perp^r$ comp of mag. field is also ↑ by γ times.

$$\boxed{B^{\perp}_{s'} = \gamma B^{\perp}_{s}}$$

bcoz in this case simultanouly length of 2 vectors ↕ ↓ (contract)
(s, ϕ)

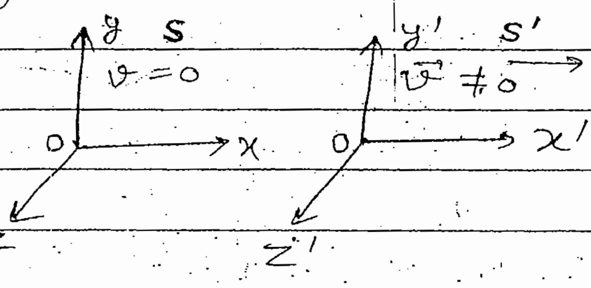
$$n' = \gamma^2 n$$

$$B_s' = \mu_0 \gamma^2 n \frac{I}{\gamma} \hat{x} \Rightarrow \vec{B_s}' = \gamma \vec{B_s}$$

i.e. $\boxed{B^{\perp}_{s'} = \gamma B^{\perp}_{s}}$

Hence, No change in ∥ comp. of $\vec{E}$ & $\vec{B}$ &, γ times increase the $\perp^r$ " " " ".

Combinely for $\vec{E}$ & $\vec{B}$:-
Transformation eqn :-
If motion is along x-dirn (i.e. comp. of E. field which is in the dirn of motion — No change) i.e.



$$\boxed{E_x' = E_x}$$

& $$\boxed{B_x' = B_x}$$

If v is the velocity of motion then

$$\boxed{E_y' = \gamma(E_y - vB_z)}$$

$$\boxed{E_z' = \gamma(E_z + vB_y)}$$

$$\boxed{B_y' = \gamma\left(B_y + \frac{vE_z}{c^2}\right)}$$

$$\boxed{B_z' = \gamma\left(B_z - \frac{vE_y}{c^2}\right)}$$

These are the modifications in Elec. & mag. field when motion is in x-dirn.
All these eqns are known as Transformation eqns.

If Motion is in Y-dirn then

$E_x' = \gamma(E_x + vB_z)$

$E_y' = E_y$

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Special Cases :-

1.) If Mag. field is 0 in rest frame

$\vec{B_s} = 0$ but $\vec{E_s} \neq 0$

i.e. $B_x = B_y = B_z = 0$

then in motion frame

$$\boxed{\vec{B_{s'}} = \frac{-1}{c^2}(\vec{v} \times \vec{E_{s'}})} \quad \text{or} \quad \vec{B_{s'}} = \frac{-\gamma}{c^2}(\vec{v} \times \vec{E_s})$$

It is not necessary if $\vec{B} = 0$ in rest place then it is 0 in motion frame bcoz mag. field & E. field are not Lorentz invarient itself.

2.) If $\vec{E} = 0$, $\vec{B} \neq 0$

then $$\boxed{\vec{E_{s'}} = \vec{v} \times \vec{B_{s'}}}$$

Lorentz Invarient Quantities :-

(i) charge q

(ii) $\vec{E} \cdot \vec{B}$ — if product E & B is 0 in rest frame then it will be 0 in moving frame.

(iii) $E^2 - c^2B^2$

or $E^2 - \frac{1}{\mu_0 \varepsilon_0} B^2$

or $\left(\varepsilon_0 E^2 - \frac{B^2}{\mu_0}\right)$

or $E^2 - B^2$ (in Gaussian Unit i.e. CGS)

or $|\vec{E}|^2 - |\vec{B}|^2$

(iv) $x^2 + y^2 + z^2 - c^2t^2$

(v) $\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$

(vi) $E^2 - p^2c^2$ (relv energy)

Conversions :-

from M.K.S. to Gaussian, the replacements are

$$\varepsilon_0 \longrightarrow \frac{1}{4\pi}$$

$$\mu_0 \longrightarrow \frac{4\pi}{c^2}$$

$$\vec{B} \longrightarrow \frac{\vec{B}}{c}, \quad \vec{A} \longrightarrow \frac{\vec{A}}{c}$$

$$\left\{ \begin{array}{l} \vec{B} = \vec{\nabla}\times\vec{A} \\ \frac{\vec{B}}{c} = \vec{\nabla}\times\frac{\vec{A}}{c} \end{array} \right\}$$

$$\vec{E} \longrightarrow \vec{E}$$

	S.I.	Gaussian
1.	$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$	$\vec{\nabla}\cdot\vec{E} = 4\pi\rho$
2.	$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$	$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$
3.	$\vec{\nabla}\cdot\vec{B} = 0$	$\vec{\nabla}\cdot\vec{B} = 0$
4.	$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$	$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$
5.	Lorentz force $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$	$\vec{F} = q\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right)$
6.	energy density $u = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$	$u = \frac{1}{8\pi}(E^2 + B^2)$
7.	Poynting vector $\vec{S} = \left(\frac{\vec{E}\times\vec{B}}{\mu_0}\right)$; $\vec{S} = \varepsilon_0\frac{\vec{E}\times\vec{B}}{\mu_0\varepsilon_0} = \frac{\varepsilon_0 c^2\,\vec{E}\times\vec{B}}{\cancel{c}}$	$\vec{S} = \frac{c}{4\pi}(\vec{E}\times\vec{B})$
8	Lorentz Gauge $\vec{\nabla}\cdot\vec{A} = 0$	$\vec{\nabla}\cdot\vec{A} = 0$
9.	Colomb Gauge $\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial V}{\partial t} = 0$	$\vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial V}{\partial t} = 0$
10	Power radiated by a point charge, $P = \frac{\mu_0 q^2 a^2}{6\pi c}$ or $P = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3}$ Larmour formula	$P = \frac{2}{3}\frac{q^2 a^2}{c^3}$
11	$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$	$\vec{D} = \vec{E} + 4\pi\vec{P}$

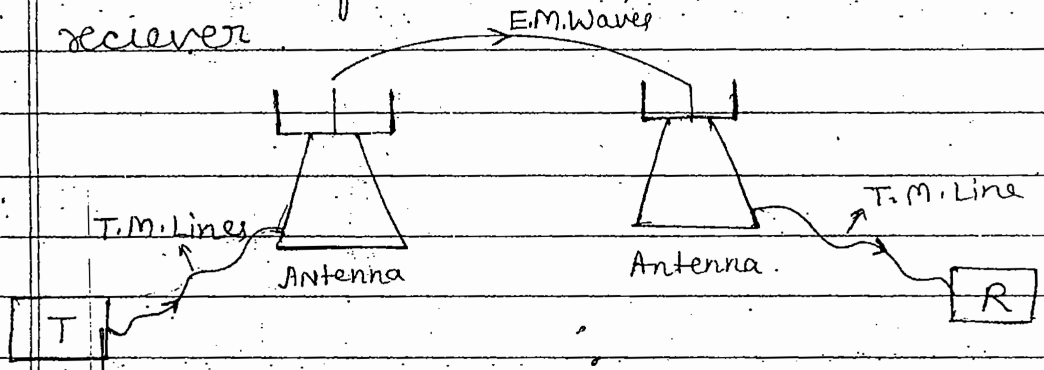
Exercise-3

Q.1:- Let $\vec{B} = B\hat{y}$ & $\vec{E} = 0$ in a rest frame S. Find all components of $\vec{E}$ & $\vec{B}$ in a frame S' which is moving with a constant velocity along x dirⁿ $\vec{v} = v\hat{x}$.

Q.2:- Suppose in a rest frame $\vec{E} = \alpha\hat{x}$ & $\vec{B} = \beta\hat{y}$. Find electric & mag. fields in a frame moving with $\vec{v} = v\hat{y}$.

Transmission Lines :-

Transmission lines are the connector b/w i.e. connecting medium b/w transmitter & reciever



In T.M. lines, current & voltages are in the form of transverse electromagnetic i.e. TEM waves.

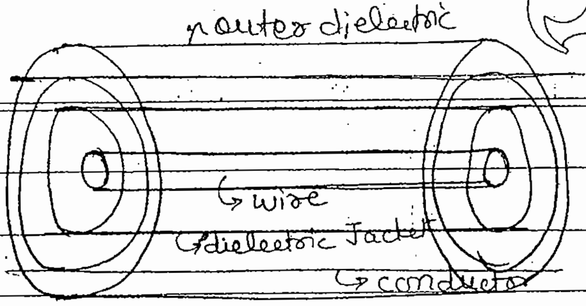
Transmission lines are of 2 types

i) Two ~~ze~~ wire line

ii) Co-axial cable

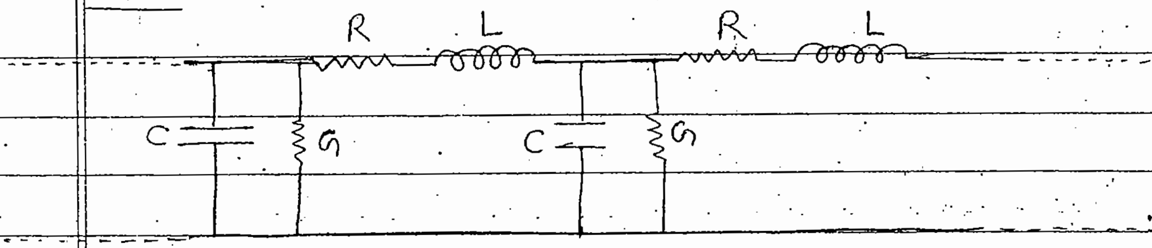
→ In Two wire line, 2 wires are always conected ∥ to each other.

→ Now a days, Two wire lines are replaced by coaxial cable in which there is a central wire, over which dielectric & then conductor.



- Transmission of information of transmission line always occur in the form of TEM wave. Transmission lines are commonly used in power distribution at low frequencies & in communication at high frequencies. They are of two type – Two wire line & coaxial cable.
  - Two wire line — balance line
  - coaxial cable — Unbalance line

Equivalent Circuit-diagram of Transmission Line :-



These are distributed parameters. They are distributed over all the trans. line.

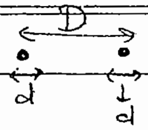
G → Conductance / unit length
L → Inductance / " "
R → Resistance / " "
C → capacitance / " "

Expressions for L & C ;

(i) for two wire line,

$$L = \frac{\mu}{\pi} \ln\left(\frac{2D}{d}\right) \ H/m$$

$$C = \frac{\pi \epsilon}{\ln\left(\frac{2D}{d}\right)} \ F/m$$

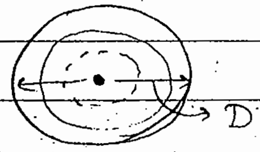


d → diameter of each wire
D → distance b/w 2 wires

(ii) Coaxial (Line) Cable :-

$$L = \frac{\mu}{2\pi} \ln\left(\frac{D}{d}\right) \ H/m$$

$$C = \frac{2\pi\epsilon}{\ln\left(\frac{D}{d}\right)} \ F/m$$



d → is the diameter of inner conductor (wire)
D → " " " outer dielectric

Phase Velocity of T.M. Line :-
for any medium which have permiability μ & permitivity ϵ then phase vel.

$$\boxed{v_p = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{\sqrt{\epsilon_r}}} \quad \text{—— (1)}$$

Wavelength :- $\nu = \frac{c}{\lambda_0} = \frac{v_p}{\lambda}$ (freq. is same in all medium)

$$\boxed{\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}}} \quad \text{—— (2)}$$

If wave freq. vel. ↓ then W.L will also ↓ s.t. their ratio must be same.

Product of L & C for both cases,

$$LC = \mu\epsilon$$

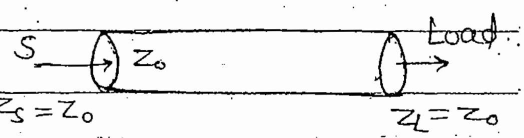
so $\boxed{v_p = \frac{1}{\sqrt{LC}}}$

where L → inductance / unit length
C → capacitance / " "

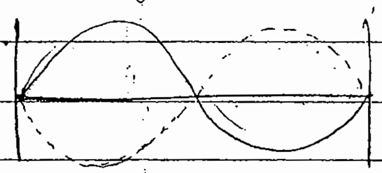
Characterstic Impedence (Z_0) :-

for maxi power transmission, impedence matching is required. i.e. impedence of T.M. Line of source & impedence of T.M. Line of reciver will match.

$$Z_L = Z_0$$
$$Z_S = Z_0$$



If impedence is not matching & wave will be reflected back but phase change - This is called Standing wave.



Expression for characterstic impedence,

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}\ \Omega}$$

(i) When T.M. Line is loss-less,

$R = 0$ & $G = 0$ then

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

(ii) Distortions less transmission line

$$\boxed{\frac{R}{G} = \frac{L}{C}}$$

$$Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

(iii) At low frequencies,

$R >> \omega L$ & $G >> \omega C$

$$\boxed{Z_0 = \sqrt{\frac{R}{G}}}$$

(iv) At high frequencies,

$R << \omega L$ & $G << \omega C$

$$\therefore \boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

Propogation Coffiicient (γ) :-

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

For Loss-less transmission line,

$$\gamma = j\omega\sqrt{LC}$$

For Distortion less T.M. line, $\left(\frac{R}{G} = \frac{L}{C}\right)$

$$\gamma = \sqrt{L\left(\frac{R}{L}+j\omega\right) C\left(\frac{G}{C}+j\omega\right)}$$

$$\gamma = \sqrt{LC\left(\frac{R}{L}+j\omega\right)^2}$$

$$\gamma = \left(\frac{R}{L}+j\omega\right)\sqrt{LC}$$

$$\gamma = \frac{1}{L}(R+j\omega L)\sqrt{LC}$$

$$\boxed{\gamma = \sqrt{\frac{C}{L}}(R+j\omega L)}$$

Reflection Coffiicient :- Γ_L

i.e How much part of incident voltage will be reflected back.

$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Matched Load

(i) If $Z_L = Z_0$ then $\boxed{\Gamma_L = 0}$

i.e. No part will be reflected back.

(ii) Short Circuit, $Z_l = 0$
then $\boxed{\Gamma_L = -1}$
(iii) Open Circuit (load), $Z_l = \infty$
then $\boxed{\Gamma_L = 1}$

So Range of Reflection coefficient is
$\boxed{-1 \leq \Gamma_L \leq 1}$

Voltage Standing Wave Ratio (VSWR) :-

$$\boxed{VSWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}}$$

Case (i) → Matched Load
$Z_L = Z_0$ then $\Gamma_L = 0$
$VSWR = 1$
(ii) Short Circuit, $Z_L = 0$, $\Gamma_L = -1$
$VSWR = \infty$
(iii) Open circuit,
$Z_L = \infty$, $\Gamma_L = 1$.
$VSWR = \infty$

Range of VSWR $\boxed{1 \leq VSWR \leq \infty}$

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Wave Guides :- Wave guides are hallow mattalic pipes of uniform cross-section. They can transmit electromag. waves by succesive reflection ~~fop~~ inner walls of wave guide.

Optical fiber → dielectric wave guide.

Reason for using Wave Guide :- At high frequencies above 3 GHz, transmission of electromag. wave through transmission line becomes difficult due to the losses occurs in dielectric jacket as well as on the conductor.

If signal is of less than 3 GHz then for transmission of EM wave, transmission lines can be used. Working Range of TM lines ~~30a~~ 0 → 3 GHz

TM Line ~~Waveguide~~ support TEM waves.

→ Working range of WG, 300 MHz – 300 GHz (Microwave region)

→ Waveguide can not support TEM waves But It supports TE & TM waves.

→ Waveguide acts as a high pass filter. i.e it can not support the EM. waves whose freq. is less than the cutt off freq. ω_c. i.e. $\omega < \omega_c$.

It will support only those EM waves for which $\boxed{\omega > \omega_c}$

→ Hence we can not transmit dc.

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Boundry Conditions on the wave guide surface :-

At the surface

$E^{\parallel} = 0 \Rightarrow \boxed{E^t = 0}$

Tangential comp. of E-field.

$B_1^{\perp} = B_2^{\perp}$

$B_1^{\perp} = 0 \Rightarrow \boxed{B^n = 0}$.

i.e. Tangential comp. of E-field & normal comp. of mag fd must be zero.

(i) Transverse Electric (TE) wave :-

E-field is entirely transverse to the dir^n of wave propogation.

If wave propogation is ~~only~~ along z-dir^n

$E_z = 0$

$B_z \neq 0$

This is also called H-Wave.

(ii) Transverse Magnetic (TM) wave :-

$B_z = 0$, $E_z \neq 0$

This is also called E-Wave

(iii) TEM Wave :-

$E_z = 0$, $B_z = 0$

(iv) If $E_z \neq 0$, $B_z \neq 0$ then wave is called Mixed wave or Hybrid Wave.

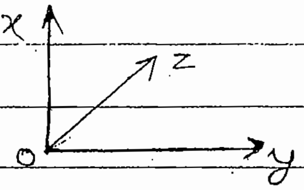
TE :- $E_z = 0$, $H_z \neq 0$

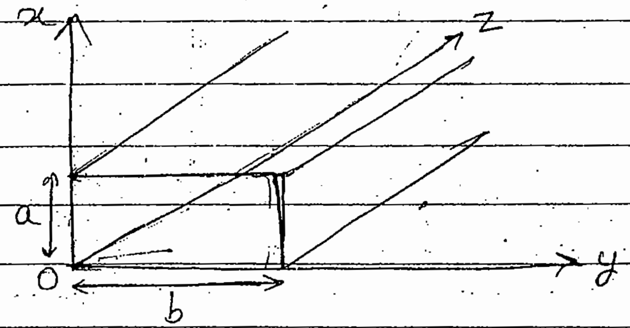
TM :- $E_z \neq 0$, $H_z = 0$

Transverse Electric Mode :-

$E_z = 0$ & $B_z \neq 0$

Consider Ractangular WG (most useful)





(a > b)

Comp. of mag. field along dir^n of wave propogation.

$$\boxed{B_z = B_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}.}$$

By using B_z we can find other comp. of elec. & mag. field → E_x, E_y, B_x, B_y

z → dir^n of wave propogation

x → dimension along x-dir^n

y → " " y- "

mn, n&m → decides modes supported by the WG.

- **Dominent Mode :-** TE_{mn}

$TE_{10} \Rightarrow m=1, n=0$

for TE_{10}, $\boxed{B_z = B_0 \cos\left(\frac{\pi x}{a}\right) e^{-\gamma z}}$

WG behaves as high pass filter.

- **Cut off freq. :-** $f_c = \frac{\omega_c}{2\pi}$

$$\boxed{f_c = \frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2}}$$

Modes - how many waves can propogate at a time through WG.

depends upon modes & dimensions.

- Cut off Wavelength :-

$$\nu = \frac{c}{\lambda} \Rightarrow \lambda = \frac{c}{\nu}$$

$$\boxed{\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}}$$

- Guide Wavelength :- Wave travel inside the WG is called Guide wave. And WL of Guide wave is

$$\boxed{\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

where, $\lambda_0 \rightarrow$ free space WL.

- Phase Velocity :- phase vel. of Guide wave is

$$\boxed{v_p = \frac{c}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

- Group Velocity :-

$$\boxed{v_g = c\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

such that $\boxed{v_p v_g = c^2}$

v_p may be $> c$ but v_g always $< c$.

- Wave Guide Impedance :-

$$TE \rightarrow \boxed{Z_E = \frac{Z_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

$$TM \rightarrow \boxed{Z_M = Z_0\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

s.t. $\boxed{Z_E Z_M = Z_0^2}$

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Note: For different wave, cut off freq. will be different.

- If incident freq. $f = 10^9$ Hz

& cut off freq. $(f_c)_{mn}$

$(f_c)_{10} = 10^8$ Hz

$(f_c)_{11} = 10^9$ Hz

So 10 mode will propogate but 11 not.

bcoz $f > (f_c)_{10}$ & $f < (f_c)_{11}$

- For TE Mode; Dominent Mode:-

$$B_z = B_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

1.) Lowest value of mode,

$m = 0,\ n = 0$

for these values all B_x, B_y, E_x, E_y are zero

So TE_{00} mode is not possible.

2.) $m = 0,\ n = 1$

$m = 1,\ n = 0$

for $m = 0, n = 1$, $(f_c)_{10} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$(f_c)_{10} = \frac{c}{2a}$

for $m = 1, n = 0$, $(f_c)_{01} = \frac{c}{2b}$

As $a > b$, so $(f_c)_{10} < (f_c)_{01}$

lower freq. mode or longest WL mode is called dominent mode.

$\Rightarrow$ So dominent mode $\rightarrow$ TE_{10} if $a > b$

for rectangular WG $\qquad TE_{01}$ if $b > a$

for Square WG → both TE_{10} & T_{01} will be dominant mode.

TM Wave :-

$E_z \neq 0$, $H_z = 0$

Comp. of elec. field along the dirn of wave propogation.

$$E_z = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

By this expression, we can find out E_x, E_y, B_x, B_y, $B_z = 0$

Dominent Mode :- Mode 00, 01, 10 are not possible.

So Lowest possible mode for TM wave is TM_{11} Mode i.e.

Dominent Mode for TM wave → TM_{11}

- If we compare TE & TM then,

Lowest possible freq. :- is for TE mode either TE_{01} or TE_{10}. If we combinely propogate both TE & TM wave in a WG then TE wave will start to propogate first.

{ Dominent Mode → for which mode, the wave propogate first. }

* $$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

This comes from $K_c^2 + K_g^2 = K_0^2$

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$K_c \rightarrow$ Cut-off Wave vector

$K_g \rightarrow$ Guide "

$K_o \rightarrow$ free space "

Now, $K_c^2 + K_g^2 = K_o^2$

$$\Rightarrow \left(\frac{2\pi}{\lambda_c}\right)^2 + \left(\frac{2\pi}{\lambda_g}\right)^2 = \left(\frac{2\pi}{\lambda_o}\right)^2$$

$$\Rightarrow \boxed{\frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2} = \frac{1}{\lambda_o^2}}$$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c} = \frac{\omega}{c}$$

$K \propto \omega$

So $K_c^2 + K_g^2 = K_o^2 \Rightarrow \omega_c^2 + \omega_g^2 = \omega_o^2$

$\omega_o \rightarrow$ free space freq.

$\omega_o = c K_o$ $(K = K_o)$

So $\omega_c^2 + \omega_g^2 = c^2 K_o^2$

$$\boxed{\omega_g^2 = c^2 K^2 - \omega_c^2}$$

Q.2 What modes are possible at an operating freq. of 3 GHz in a hollow rectangular WG of inner dimensions $3.44 \times 7.22 \text{ cm}^2$ and corresponding values of phase velocity, group velocity, cut off W.L., cut off freq. & phase constant.

$f = 3$ GHz

TE_{01} is the dominent mode as $(3.44 < 7.22)$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \quad \& \quad \lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

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$(f_c)_{10} = \frac{c}{2a}$ | $(\lambda_c)_{10} = 2a = 2 \times 3.44 = 6.88$ cm

$(f_c)_{01} = \frac{c}{2b}$ | $(\lambda_c)_{01} = 2b = 2 \times 7.22 = 14.44$ cm

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m}$$

or $\lambda = 10$ cm

for Propogation, $\lambda < \lambda_c$

$\lambda < (\lambda_c)_{01}$

The only mode that can propogate is $\underline{\underline{TE_{01}}}$

Cut off freq.

$$(f_c)_{01} = \frac{c}{2b} = \frac{3 \times 10^8}{2 \times 7.22} =$$

$v_p = ?$

$$v_p = \frac{c}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - \left(\frac{10}{14.44}\right)^2}} \text{ m/sec.}$$

$$v_p = \frac{3 \times 10^8}{0.72139793} = 4.1586 \times 10^8 \text{ m/sec}$$

$v_g = ?$

$$v_p v_g = c^2 \Rightarrow v_g = \frac{c^2}{v_p}$$

$$v_g = \frac{(3 \times 10^8)^2}{4.1586 \times 10^8} = 2.1642 \times 10^8 \text{ m/sec}$$

Phase const. $\beta = \frac{2\pi}{\lambda}$

$$\beta = \frac{2 \times 3.14}{10} = 0.628$$

Q.2:- A hollow rectangular WG has $a = 6\,cm$, $b = 4\,cm$. Freq. of the incident signal is 3 GHz. Calculate (i) λ_c of dominating mode.
(ii) λ_g of dominent mode
(iii) Intrinsic wave impedence
(iv) v_p (v) v_g

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 0.1\ m = 10\ cm$$

(i) $(\lambda_c)_{10} = 2a = 12\ cm$

$(\lambda_c)_{01} = 2b = 8\ cm$

Only Mode which can propogate i.e. Dominent Mode $\rightarrow$ ...... TE_{10}

(ii)
$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \frac{10}{\sqrt{1-\left(\frac{10}{12}\right)^2}}\ cm$$

$$\lambda_g = \frac{10}{0.55277} = 18.0907\ cm$$

(iii) Intrinsic Impedance $Z_E = \frac{Z_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$

$Z_E =$

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Hz.

Q.3 :- At 15 GHz an air field $5 \times 2\ cm^2$ WG has $E_z = 20 \sin 40\pi x \sin 50\pi y\, e^{-j\beta z}$ V/m. Find the mode of propogation and cut off W.L. of this mode.

$f = 15$ GHz (No Need)

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{15\times10^9} = \frac{1}{50} = 0.02\ \cancel{cm}\ m$$

$\lambda = 2$ cm

$\because E_z \neq 0$ so TM wave can propogate

dimension $\rightarrow 5 \times 2\ cm^2 = 0.05 \times 0.02\ m^2$

$E_z = 20 \sin(40\pi x) \sin(50\pi y)\, e^{-j\beta z}$

Compare with $E_z = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$

$$\Rightarrow \frac{m\cancel{\pi x}}{a} = 40\cancel{\pi x}$$

$$\Rightarrow m = 40 \times a = 40 \times 0.05$$

$$\boxed{m = 2}$$

$\& \quad \frac{n\cancel{\pi y}}{b} = 50\cancel{\pi y} \Rightarrow n = 50 \times 0.02$

$$\boxed{n = 1}$$

So Dominent Mode $\rightarrow TM_{21}$

Cut off W.L.

$$(\lambda c)_a = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$(\lambda c)_{21} = \frac{2}{\sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{1}{2}\right)^2}} = \frac{2}{\sqrt{\frac{4}{25} + \frac{1}{4}}} = \frac{20}{\sqrt{16+25}}$$

$$(\lambda c)_{21} = \frac{20}{6.4031}\ cm$$

$$(\lambda c)_{21} = 3.12348\ cm$$

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Q.4:- In a rectangular W.G. for which $a = 1.5$ cm $b = 0.8$ cm, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$,

$$H_x = 2 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{3\pi y}{b}\right) \sin\left(\pi \times 10^{11} t - \beta z\right) \text{ A/m}$$

Determine Mode of operation, cut off freq. & cut off wavelength for this mode.

Q.5:- A standard air filled rectangular WG with dimensions $a = 8.6$ cm & $b = 4.3$ cm fed by 4 GHz carrier from a coaxial cable. Determine weather TE_{10} mode will be propogated or not. If Yes, determine its group & phase velocity.

Q.6:- A loss-less transmission line has a characterstic impedence of 70 ohms & phase velocity as 2×10^7 m/sec. Calculate inductance per meter & capacitance per meter of the T.M. Line.

$$Z_0 = 70\,\Omega$$

$$v_p = 2 \times 10^7 \text{ m/s}$$

for loss-less TM Line,

$$Z_0 = \sqrt{\frac{L}{C}} \quad \text{———(1)}$$

$$\& \quad v_p = \frac{1}{\sqrt{LC}} \quad \text{———(2)}$$

eqn (1) X (2) $\Rightarrow$ $$Z_0 v_p = \sqrt{\frac{L}{C}} \frac{1}{\sqrt{LC}} = \frac{1}{C}$$

$$\Rightarrow \quad C = \frac{1}{Z_0 v_p}$$

$$C = \frac{1}{70 \times 2 \times 10^{7}} = \frac{1}{140 \times 10^{7}} \text{ F/m} = \frac{100 \times 10^{-10}}{14}$$

$$\boxed{C = 7.14 \times 10^{-10} \text{ F/m}}$$

$$\& \ (2) \Rightarrow v_p^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{C v_p^2}$$

$$L = \frac{(2 \times 10^{7}) \times 70}{(2 \times 10^{7})^{2}}$$

$$\boxed{L = 3.5 \times 10^{-6} \text{ H/m}}$$ (Henri/m)

Q.7:- A loss-less T.M. Line of length 50 cm with $L = 10\ \mu H/m$ & $C = 40\ pF/m$ is operated at 30 MHz. If T.M. Line is assumed to be non-magnetic. Find the dielectric constant of the T.M. Line.

$$L = 10\ \mu H/m \qquad , \qquad C = 40\ pF/m$$
$$= 10 \times 10^{-6}\ H/m \qquad\qquad = 40 \times 10^{-12}\ F/m$$

$$f = 30\ MHz = 30 \times 10^{6}\ Hz$$

We know, $v_p = \frac{1}{\sqrt{LC}}$ & Also $v_p = \frac{c}{\sqrt{\varepsilon_r}}$

i.e. $$\frac{c}{\sqrt{\varepsilon_r}} = \frac{1}{\sqrt{LC}}$$

$$\frac{c^2}{\varepsilon_r} = \frac{1}{LC} \Rightarrow \cancel{\varepsilon_r = LC^3}$$

$$\varepsilon_r = (3 \times 10^{8})^{2} \times L \times C$$
$$= 9 \times 10^{16} \times 10^{-5} \times 40 \times 10^{-12}$$
$$= 360 \times 10^{-1}$$
$$\varepsilon_r = 36$$

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Q.4 :- $H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\sin(\pi\times10^{11}t - \beta z)$ A/m

$a = 1.5$ cm & $b = 0.8$ cm

$\frac{m\pi x}{a} = \frac{\pi x}{a} \Rightarrow m = 1$

$\frac{n\pi y}{b} = \frac{3\pi y}{b} \Rightarrow n = 3$

Dominant mode will be TE_{13} or TM_{13}

Cut off freq. $f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$(f_c)_{13} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{1.5}\right)^2 + \left(\frac{3}{0.8}\right)^2}$

$= \frac{3\times10^8}{2}\left(\sqrt{3.80879}\right)$

$= 5.7132\times10^8$

Cut off W.L. $\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$

$(\lambda c)_{13} = \frac{2}{3.80879}$

$(\lambda c)_{13} = 0.52510115$

Q.5 :- $a = 8.6$ cm, $b = 4.3$ cm

$f = 4$ GHz

$\lambda = \frac{c}{f} = \frac{3\times10^8}{4\times10^9} = \frac{0.75}{10} = 0.075$ m

$= 7.5$ cm

$(\lambda c)_{10} = 2a = 2\times8.6 = 17.2$ cm

Here $(\lambda c)_{10} \ngtr \lambda_c$

TE_{10} mode will ~~not~~ propagate.

Phase & group vel. = ?

$$v_p = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \frac{3\times10^8}{\sqrt{1-\left(\frac{7.5}{17.2}\right)^2}} = \frac{3\times10^8}{\sqrt{1-0.19014}}$$

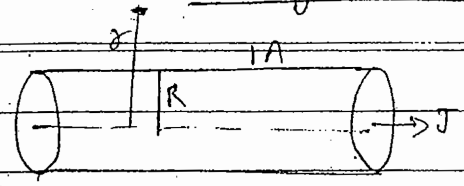
$$v_p = \frac{3\times10^8}{0.89992} = 3.33363\times10^8 \text{ cm/s}$$

Now, $v_p v_g = c^2$

$$\Rightarrow v_g = \frac{c^2}{v_p} = \frac{(3\times10^8)^2}{(3.33363\times10^8)} = 2.69976\times10^8 \text{ cm/s}$$

Assignment

(36)



$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

$\vec{B} = \frac{\mu_0}{2\pi \cancel{s}} \hat{\phi}$ at distance r.

J is uniformly distributed.

$$\vec{\nabla} \times \vec{B} = \frac{1}{s} \begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & s\frac{\mu_0}{2\pi s} & 0 \end{vmatrix}$$

$\vec{\nabla} \times \vec{B} = \frac{1}{s}(0) = 0$

(38) force that compensate the ele. field

$\vec{F} = q\vec{E}$

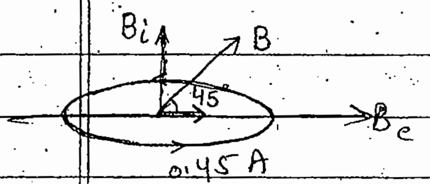
(c) ✓

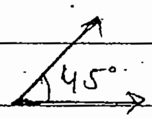
(65) $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

In Gaussian units $\vec{D} = \vec{E} + 4\pi \vec{P}$

(88) Circular coil is $\parallel$ to the earth's mag field

No





$\tan 45^\circ = \frac{y}{x} = \frac{B_i}{B_e}$

$1 = \frac{B_i}{B_e}$

$B_e = B_i = \frac{\mu_0 I}{2R}$ (at centre mag. field)

If N no. of turns then

$B_e = \frac{\mu_0 N I}{2R}$

$= \frac{10 \times 0.45 \times 4\pi \times 10^{-7}}{2 \times 0.12}$ T

$= 75 \times 3.14 \times 10^{-7}$ T

$= 23.5\ \mu T$

in terms of Maxwell's eqn

- Kirchoff's Current law $\Rightarrow \Sigma i = 0 \Rightarrow \vec{\nabla}\cdot\vec{J} = 0$

  Voltage law $\Rightarrow \Sigma V = 0 \Rightarrow \vec{\nabla}\times\vec{E} = 0$

(108) $\vec{E} = E_0$, $\vec{B} = B_0$

for a uniform mag. field , $\vec{A} = \frac{1}{2}(\vec{B_0}\times\vec{r})$

$\vec{A}' = \vec{A} + \vec{\nabla}\lambda$

$V' = V - \frac{\partial\lambda}{\partial t}$

for Lorentz T. $\vec{E}' = E$ & $\vec{B}' = B$

$E = -\nabla V$ or $-\nabla\phi$

(i) If $\phi = 0$ then $\nabla\phi = 0$ i.e. $E = 0$, so ϕ must not be 0.

(ii) ✓

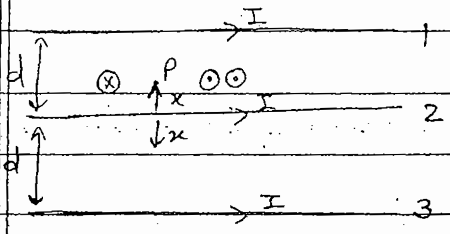
$$\nabla\phi = \vec{\nabla}(\vec{E_0}\cdot\vec{r}) = E_0$$

<u>Note</u> :- For a constant uniform Electric field $\vec{E}$, mag. field $\vec{B}$

$\vec{A} = \frac{1}{2}(\vec{B_0}\times\vec{r})$ & $\phi = -\vec{E_0}\cdot\vec{r}$

<u>Level - II</u>

(2)



$B_1 = \frac{\mu_0 I}{2\pi(d-x)}$ ⊗

$B_2 = \frac{\mu_0 I}{2\pi x}$ ⊙

$B_3 = \frac{\mu_0 I}{2\pi(d+x)}$ ⊙

$$\vec{B} = \frac{\mu_0 I}{2\pi}\left[\frac{1}{d-x} - \frac{1}{x} - \frac{1}{d+x}\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{x(d+x) - (d+x)(d-x) - x(d-x)}{x(d-x)(d+x)}\right]$$

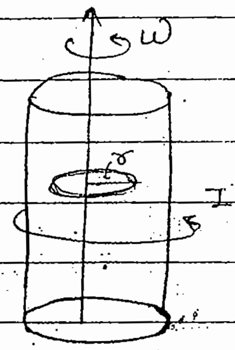
$$= \frac{\mu_0 I}{2\pi}\left[\frac{dx + x^2 - d^2 + x^2 - dx + x^2}{x(d-x)(d+x)}\right]$$

$dx^2 + x^2 - d^2 + x^2 - dx + x^2 = 0$

$3x^2 = d^2$

$\boxed{x = \pm \frac{d}{\sqrt{3}}}$ At this distance above & below mag. field $= 0$

(10)



$\vec{K} = \sigma v \hat{\phi}$

$= \sigma \omega R \hat{\phi} \quad (v = R\omega)$

(11) $\vec{B}_{in} = \mu_0 K$

$= \mu_0 \sigma \omega R \, \hat{z}$

It'll behave as a solenoid.

$\oint \vec{A}\cdot d\vec{l} = \int_S \vec{B}\cdot d\vec{S}$

$= \int_S (\vec{\nabla}\times\vec{A})\cdot d\vec{S}$

$\phi = B_{in} A$

Flux pass through the loop, $\Phi_m = \mu_0 \sigma \omega R \cdot \pi r^2$

$\vec{A} = \frac{\mu_0 \sigma \omega R r}{2} \hat{\phi}$

(12) $\phi = \frac{\mu_0 I a}{2\pi} \ln\left(\frac{s+a}{a}\right)$

$= \frac{\mu_0 I a}{2\pi} \ln(2)$

for induced current, 1st find induced emf,

$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{\mu_0 I a}{2\pi}\ln(2)\right) = -\frac{d}{dt}\frac{\mu_0 a}{2\pi} I_0 \cos\omega t \ln(2)$

$= \cancel{\frac{\mu_0 I \omega a}{2\pi R}\ln(2)} = \frac{\mu_0 I_0 \omega a}{2\pi R}\ln(2)\sin(\omega t)$

$I = \frac{\mathcal{E}}{R} = -\frac{d\phi}{dt}\frac{1}{R}$

$I = \frac{\mu_0 I_0 \omega a}{2\pi R}\ln(2)\sin(\omega t)$

(13)

$$E_{or} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right) E_{oi} \qquad n_1 = 1$$

$$E_{ot} = \left(\frac{2n_1}{n_1 + n_2}\right) E_{oi} \qquad n_2 = 1.5, \quad E_{oi} = 2$$

$$E_{or} = \frac{1 - 1.5}{1 + 1.5} \times 2 = \frac{-0.5}{2.5} \times 2 = \frac{-2}{5}$$

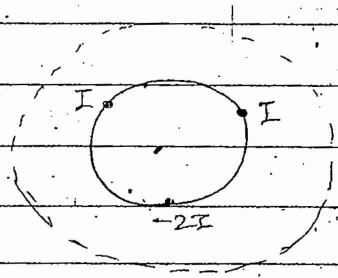
$$E_{ot} = \frac{2 \times 1}{1 + 1.5} \times 2 = \frac{2}{2.5} \times 2 = \frac{8}{5}$$

(17)

$$\oint B \cdot dl = \mu_0 I_{enc}$$

$$I_{enc} = 0$$

$$\text{so } \oint B \cdot dl = 0$$



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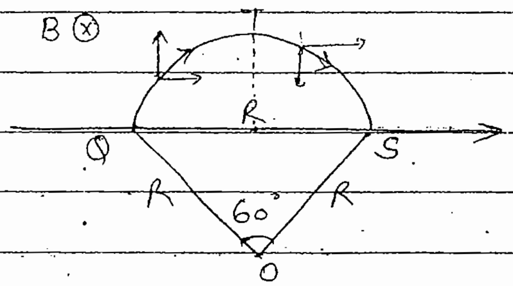

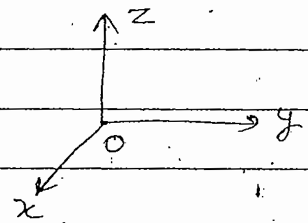

$$\vec{dl_1} = dy\,\hat{y} + dz\,\hat{z}, \qquad \vec{dl_2} = dy\,\hat{y} - dz\,\hat{z}$$

$$\text{Total } \vec{dl} = \vec{dl_1} + \vec{dl_2}$$

$$= 2\,dy\,\hat{y}$$

$$\text{Force } \vec{F} = I \int \vec{dl} \times \vec{B} \qquad B \to \text{into the page } (-\hat{x})$$

$$= I\,2B \int_0^{R/2} dy\,\hat{z} \qquad \hat{y} \times (-\hat{x}) = +\hat{z}$$

$$= I\,\cancel{2}B\,\frac{R}{\cancel{2}}\,\hat{z}$$

$$= IBR\,\hat{z}$$

<u>Note</u> :- for half circle $\vec{F} = 2IBR\,\hat{z}$

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Exercise-2

Q.1 :- $V(r,t) = 0$

$$A(r,t) = \frac{-1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r}$$

Find $\vec{E}$, $\vec{B}$, q & ρ = ?

$$\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t} = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}}$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$= \frac{1}{r\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ \frac{-1}{4\pi\varepsilon_0}\frac{qt}{r^2} & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{r\sin\theta}\left[\hat{r}(0) + \hat{\theta}(0) + \hat{\phi}(0)\right]$$

$$\boxed{\vec{B} = 0}$$

charge = q

charge distribution,

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\rho = \varepsilon_0(\vec{\nabla}\cdot\vec{E})$$

$$\rho = \varepsilon_0\frac{q}{4\pi\varepsilon_0}4\pi\delta^3(r)$$

$$\boxed{\rho = q\delta^3(r)}$$

Current distribution,

$\vec{J} = 0$ because there is no displacement current & no conduction current.

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Q.2:- $V = 0$, $\vec{A} = A_0 \sin(kx - \omega t)\,\hat{y}$

$\vec{E}$ & $\vec{B}$ = ?

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \quad \cancel{= 0 + A_0 \omega \cos(\omega t - x}$$

$$= 0 + A_0 \omega \cos(kx - \omega t)\,\hat{y}$$

$$\boxed{\vec{E} = A_0 \omega \cos(kx - \omega t)\,\hat{y}}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & A_0 \sin(kx - \omega t) & 0 \end{vmatrix}$$

$$\vec{B} = \hat{z}\,[A_0 \cos(kx - \omega t) \cdot k]$$

$$\boxed{\vec{B} = A_0 k \cos(kx - \omega t)\,\hat{z}}$$

Now, verify Maxwell's eqn,

(1) $\vec{\nabla} \cdot \vec{E} = \frac{\partial}{\partial y}[A_0 \omega \cos(kx - \omega t)] = 0$

$\nabla \cdot E = 0$

(2) $\vec{\nabla} \cdot \vec{B} = 0$

(3) $\vec{\nabla} \times \vec{E} = -A_0 \omega k \sin(kx - \omega t)\,\hat{z}$

$$-\frac{\partial B}{\partial t} = -A_0 \omega k \sin(kx - \omega t)\,\hat{z}$$

So $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(4) $\vec{\nabla} \times \vec{B} = A_0 k^2 \sin(kx - \omega t)\,\hat{y}$

$$\frac{\partial \vec{E}}{\partial t} = A_0 \omega^2 \sin(kx - \omega t)\,\hat{y}$$

$\because$ $\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$\Rightarrow A_0 k^2 \sin(kx - \omega t) = \frac{1}{c^2} A_0 \omega^2 \sin(kx - \omega t)$$

$$\Rightarrow \frac{\omega^2}{k^2} = c^2 \Rightarrow \frac{\omega}{k} = c \Rightarrow \boxed{\omega = ck}$$

This is the condn to be imposed on ω & k.

Q.3:- $\lambda = \frac{-1}{4\pi\varepsilon_0}\frac{qt}{r}$, $V(r,t) = 0$

$$\vec{A}(r,t) = \frac{-1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r}$$

$V' \ \& \ A' = ?$

$$V' = V - \frac{\partial\lambda}{\partial t}$$

$$V' = 0 - \frac{-1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\boxed{V' = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

$$\vec{A}' = \vec{A} + \vec{\nabla}\lambda$$

$$\vec{A}' = \frac{-1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r} + \frac{1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r}$$

$$\boxed{\vec{A}' = 0}$$

Q.4 (i) $V(r,t) = 0$

$$|\vec{A}(r,t) = \frac{-1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r}$$

Coulomb Gauge $\vec{\nabla}\cdot\vec{A} = 0$

Lorentz Gauge $\vec{\nabla}\cdot\vec{A} + \mu_0\varepsilon_0\frac{\partial V}{\partial t} = 0$

Here, $\vec{\nabla}\cdot A = -\frac{qt}{4\pi\varepsilon_0} 4\pi\delta^3(r)$

$$= -\frac{qt}{\varepsilon_0}\delta^3(r)$$

& $\vec{\nabla}\cdot\vec{A} + \mu_0\varepsilon_0\frac{\partial V}{\partial t} = \frac{-qt}{\varepsilon_0}\delta^3(r) \neq 0$

So Neither C.G. nor L.G. is followed.

(ii) $V = 0, \vec{A} = A_0 \sin(kx - \omega t)\hat{y}$

Here $\vec{\nabla}\cdot\vec{A} = 0$

$\vec{\nabla}\cdot A + \mu_0\varepsilon_0\frac{\partial V}{\partial t} = 0$ } both C.G. & L.G. followed.

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Exercise-3

Q.1 :- $\vec{B} = B\hat{y}$, $\vec{E} = 0$ in rest S frame.
find all components of $\vec{E}$ & $\vec{B}$ in S' frame
$\vec{v} = v\hat{x}$.

$$\boxed{\vec{B}_x' = B_x = 0}$$
$$\boxed{\vec{E}_x' = E_x = 0}$$

$\vec{E}_y' = \gamma(E_y - vB_z)$, $E_y = 0, B_z = 0$

$$\boxed{\vec{E}_y' = 0}$$

$\vec{B}_y' = \gamma\left(B_y + \frac{vE_z}{c^2}\right)$, $\vec{B}_y = B_y$, $E_z = 0$

$$\boxed{\vec{B}_y' = \gamma B_y = \frac{B_y}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$\vec{E}_z' = \gamma(E_z + vB_y)$ $E_z = 0$, $\vec{B}_y = B_y$.

$$\boxed{\vec{E}_z' = \gamma v B_y = \frac{vB_y}{\sqrt{1-v^2/c^2}}}$$

$\vec{B}_z' = \gamma\left(B_z - \frac{vE_y}{c^2}\right)$ $B_z = 0$, $E_y = 0$

$$\boxed{\vec{B}_z' = 0}$$

$(\vec{B}' = \vec{B}_x' + \vec{B}_y' + \vec{B}_z')$

So, $\boxed{\vec{B}' = \frac{B}{\sqrt{1-v^2/c^2}}\hat{y}}$

& $\vec{E}' = \vec{E}_x' + \vec{E}_y' + \vec{E}_z'$

$$\boxed{\vec{E}' = \frac{vB}{\sqrt{1-v^2/c^2}}\hat{z}}$$

$\vec{E}\cdot\vec{B} = 0$ (in rest)
Also $\vec{E}'\cdot\vec{B}' = 0$

Q.2 :- $\vec{E} = \alpha\hat{x}$ & $\vec{B} = \beta\hat{y}$
$\vec{E}'$ & $\vec{B}' = ?$
$\vec{v} = v\hat{y}$

$$\boxed{B_y' = B_y = \beta\hat{y}}$$
$$\boxed{E_y' = E_y = 0}$$

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$$\vec{B}_x' = \gamma\left(B_x + \frac{V}{c^2}E_z\right)$$

$$\vec{B}_x' = 0$$

$$E_x' = \gamma(E_x + vB_z) = \gamma E_x$$

$$\vec{E}_x' = \frac{\alpha}{\sqrt{1-v^2/c^2}}$$

$$\vec{B}_z' = \gamma\left(B_z + \frac{V}{c^2}E_x\right) = \gamma E_x \frac{V}{c^2}$$

$$= \frac{v\alpha}{c^2\sqrt{1-\frac{v^2}{c^2}}}$$

$$\vec{E}_z' = \gamma(E_z - vB_x) = 0$$

~~So~~ $\vec{B}_x' + \vec{B}_y' + \vec{B}_z' = \vec{B}'$

$$\Rightarrow \vec{B}' = \beta\hat{y} + \frac{v\alpha}{c^2\sqrt{1-v^2/c^2}}\hat{z}$$

& $\vec{E}' = \vec{E}_x' + \vec{E}_y' + \vec{E}_z'$

$$\vec{E}' = \frac{\alpha}{\sqrt{1-v^2/c^2}}\hat{x}$$

& $\boxed{\vec{E}\cdot\vec{B} = 0 = \vec{E}'\cdot\vec{B}'}$

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