Particle Physics

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Proton m_p +ve charge
Neutron $\simeq m_p$ zero charge

Initially p & n are same if charge is not considered.
As electromag. theory comes in nature nucleon split in two parts

$$\overbrace{p \quad\quad n}^{1}$$

Electromag. int. field is responsible for proton & n charge.
For masses Higg's Int. is responsible
The whole of the material world can be classified into 2 broad catagories :-

1) Matter 2) Radiation

The interaction of the radiation with matter is governed by four fundamental interactions of the nature.

1. Strong
2. Weak
3. EM
4. Gravitational

I EM is first discoved by Maxwell. It is governed by four Maxwell's eqn. (1915)

II II attemp made by Albert Einstein to (1920-1925) combine EM int with gravitation but failed.

III Attemp to comine weak int with EM called electroweak theory.
by Glasow, Abdus Salam, Weinberg

IV Grand unified theory to combine all 4.

Range of these forces :-

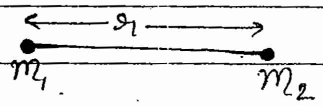
Gravitational

$$\vec{F} = G \frac{M_1 M_2}{r^2} \hat{r}$$

dir^n of force is along the line joining two masses.

when $r \to \infty$

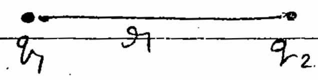
$F \to 0$



These are long range forces, having infinite range.

EM Interaction :- Governed by Coloumb's law.

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$



Range is ∞ for EM. int.

Strong Interaction :- Range of strong forces is 10^{-15} m

$= 1$ fm

It acts in nucleus. If 2 protons are at distance of the order of 1 fm then strong int. will dominate. It is strongest force in the nature.

Weak Interaction :- Range $= 10^{-18}$ m

These int. are responsible for radioactive decay. Their range is 1000 times smaller than strong int. Weak forces are also called Contact forces (almost toching surface).

Strength :- The order of strength of the four int. is as follows

Strong	Em	Weak	Gravitational
1	10^{-2}	10^{-13}	10^{-42}
			$\approx 10^{-39}$

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<u>Nature</u> :- The nature of the forces can be explained by Maxwell's ~~or~~ classical theory as well as by Quantum mechanical explanation.

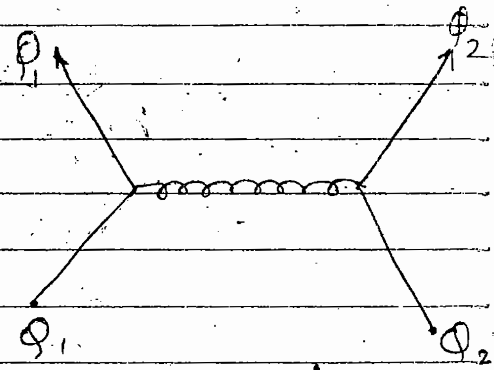
Acc. to Maxwell, a charge creats a field around it & intensity of field due to this charge

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

Another charge q

Q q

$$F = qE$$



The intraction b/w two charged particle takes place bcoz of the exchange of the particles called Virtual photons. It is consistant with Heisenberg's uncertainity Principle.

$$\Delta E\,\Delta t \geq \hbar$$

For time Δt, ΔE amount of energy can be created or can be destroy. We don't know what is happen in time Δt.

In Δt time interval we can create as well as we can destroy ΔE amount of energy consistant with the Heisenberg's Uncertainity relation ($\Delta E\,\Delta t \geq \hbar$)

$$\text{Range} = \text{speed} \times \text{time}$$
$$= c \times \Delta t$$
$$= c \times \frac{\hbar}{\Delta E} = c \times \frac{\hbar}{Mc^2}$$
$$= \frac{\hbar}{Mc}$$

$M \rightarrow$ mass of exchange particle.

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Range of the int. is inversly proportional to the mass of the particle being exchanged.

$$R \propto \frac{1}{M}$$

EM Int. , $R = \infty$

Mass of exchange particle will have rest mass $\simeq 0$

$\Rightarrow$ Photons.

$\rightarrow$ When electromag. field is quantised then get particle, & for EM Int. the particle which continuously exchange is photon.

Gravitantional Int. , $R = \infty$

exchange particle = Graviton

It has not been discovered yet. Whenever it will be discovered it has 0 rest mass & spin 2.

Strong Int. , $R \simeq 10^{-15}$ meters

say ~~$R = 2 \times 10^{-15}$ m~~

$$R = \frac{\hbar}{Mc}$$

$$M = \frac{\hbar}{Rc} = \frac{6.626 \times 10^{-34}}{2 \times 3.14 \times 2 \times 10^{-15} \times 3 \times 10^{8}}$$

$=$ Kg

$Mc^2 =$ J

$\because$ 1eV $= 1.6 \times 10^{-19}$ J

$M \simeq 100$ MeV

Note :- p ——— 2fm ——— p   Strong Int.

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Experimently it has been confirmed that the exchange particle in case of strong int. are π^+, π^0, π^-. Their mass are of the order of 100 MeV (140 MeV)

<u>Weak Int.</u> $R \simeq 10^{-18}$ m

$$M = \frac{\hbar}{Rc} = \frac{6.626 \times 10^{-34}}{2 \times 3.14 \times 2 \times 10^{-18} \times 3 \times 10^{8}}$$

$$= 100 \text{ MeV} \times 1000$$

$$= 10^5 \text{ MeV}$$

$$= 100 \text{ GeV}$$

Experimently it has been confirmed the mass of the exchange particle in weak int. is of the order of 100 GeV.

Exchange particles → $W^{\pm}$, Z^{0} bosons

2. <u>Summerize</u> :-

Int.	Strength	Range	exh. partich	Mass	Spin	Theory to explain
Strong	1	10^{-15}	π	100 MeV	1	Quantum Chromodynamics
EM	10^{-2}	~~10^{-18}~~ ∞	Photon	0	1	Quantum Electrodynamics
Weak	10^{-13}	~~∞~~ 10^{-18}	$W^{\pm}, Z^{0}$	100 GeV.	1	Quantum Flavourdynamics
Gravitational	10^{-42}	∞	Graviton	0	2	Quantum Geo-metrodynamics

Elementry Particles

1) It should be infinitely hard & structureless
2) It should be unbreakable.
3) Just like a pereodic table of the elements almost 200-300 elementry particles have been classified on the basis of 4 properties -
(i) Mass
(ii) Interaction
(iii) Lifetime
(iv) Spin

(i) On the basis of Mass, all the elementry particles are classified in 3 catagories -
→ LEPTONS 0-135 MeV (masses)
→ MESONS 135-938.3 MeV
→ HADRONS > 938.3 MeV

LEPTONS :- There are 6 Leptons. They are fermions having spin 1/2.

		charge	mass	
1- Electron	e^-	-1	0.511 MeV	
2- Electron neutrino	ν_e	0	0	
3- Muon	μ^-	-1	105.6 MeV	unstable
4- Muon neutrino	ν_μ	0	0	
5- Tau	τ^-	-1	1786 MeV	unstable
6- Tau neutrino	ν_τ	0	0	

exception

Lepton Number :- All those particles which are leptons their leptonic no. will be +1. & for antileptons the leptonic no. will be -1, & for others it will be zero.

L_e	+1	e^-, ν_e
	-1	$e^+, \bar{\nu}_e$
	0	others
L_μ	+1	μ^-, ν_μ
	-1	$\mu^+, \bar{\nu}_\mu$
	0	othrs
L_τ	+1	τ^-, ν_τ
	-1	$\tau^+, \bar{\nu}_\tau$
	0	others

Note :- For Any elementry partic reeⁿ to take place Leptonic no. must be conserved.

* Check the feasibility of reaⁿ.

1) $n \longrightarrow p + e^-$

Not feasible, L.No. $0 \not\to 0 + 1$ Not conserved

2) $n \longrightarrow p + e^- + \bar{\nu}_e$

L_e, L.No. $0 \longrightarrow 0 + 1 - 1$ Conserved.

(This is additive Quantum No.)

3) $\pi^+ \longrightarrow \mu^+ + \nu_\mu$

L_μ $0 \longrightarrow -1 + 1$ Conserved.

ON the basis of Interaction, we define the particle as:

Leptons :- leptons are those particles which do not participate in strong interaction.

Hadrons :- Hadrons are those particles which do participate in strong int.

Hadrons are further classified as

Hadrons

- Mesons (Bosons)
  - Pseudo scalar meson
  - Vector Meson
- Baryons (fermions)
  - Baryons
  - Baryon Resonance

Note :- Rank zero Tensor $\rightarrow$ Scalar , $3^n = 3^0 = 1$
Rank 1 " $\rightarrow$ Vector , $3^n = 3^1 = 3$

Pseudo $\rightarrow$ Virtual

$\Rightarrow$ We define the parity operator $\hat{P}$ as

$$\hat{P} : \vec{r} \longrightarrow -\vec{r}$$

$\hat{P}$: Scalar $\longrightarrow$ Scalar (True Scalar) [invariant]

$\hat{P}$: Vector $\longrightarrow$ $-$Vector (True Vector)

$\hat{P}$: Vector $\longrightarrow$ Vector (pseudo vector or axial vector)

$\hat{P} : \vec{r} \longrightarrow -\vec{r}$ (True Vector)

$\hat{P} : \vec{v} \longrightarrow -\vec{v}$ (")

$\hat{P} : \vec{p} \longrightarrow -p$ (")

$\hat{P} : \vec{F} \longrightarrow -F$ (")

$\hat{P} : \vec{\tau} \longrightarrow \tau$ (pseudo vector)

$\hat{P} : \vec{L} \longrightarrow L$ (")

(True Vector) $\cdot$ (pseudo vector) $\equiv$ Scalar
$\Downarrow$

$\hat{P}$: scalar $\longrightarrow$ $-$scalar [Pseudo Scalar]

Classification on the basis of life time :- Only few elementary particle such as e^-, ν_e, ν_μ, γ, ν_τ are stable.
On the basis of lifetime unstable particles may be devided in groups :
1) Metastable — $\Gamma << M$ $(10^{-8} s)$
2) Resonance — $\Gamma \simeq M$ $(10^{-23} s)$
If the width of the enegy level is much less than mass no. then particle will be of metastable group.

Baryon No. :- For all the particle which are baryon have baryon no. $B = +1$ for Baryons
$= -1$ for Anti Baryons
$= 0$ others

Conservation laws :- There are 2 laws :-

1) Exact Conservation law ↓	2) Approximate Conservation law
If it vilates, sec^n is not conserved	If it vilates it can be conserved or not.
(i) Energy Conservation :- Enegy of system is conserved bcoz time is homogeneous	(i) Isospin :-
(ii) Linear Momentum :- bcoz of homoginity of space.	(ii) Strangeness :-
(iii) Angular Mom :- It is con. bcoz of isotropy of space (same prop. in diff. dir^n)	(iii) Parity
(iv) charge (leptonic charge, baryonic charge, electronic charge) :- bcoz of Gauge invarience of EM field.	(iv) Charge confi.
	(v) Time reversal.

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(v) CPT :-

C ≡ charge configuration
P ≡ Parity
T ≡ time reversal

→ Isospin :- There is no physical reality, is purely abstract.

Protons	938.3 MeV	+ve charge
Neutrons	939.6 MeV	0 "

for a good approximation, we can ignore the mass difference.

The difference b/w these two is charge. Since the mass of the n & p is almost equal to each other & the difference b/w the two is protons have +ve charge & n are neutral. so If we consider a wall in which no electromag. interaction takes place then p & n can not be distinguish. These two can be regarded as the two charge states of same intity called nucleons:-

Nucleons can be differentiated by a property Isospin.

Isospin $I = \frac{1}{2}$

III-component of isospin, I_3 → $+\frac{1}{2}$ proton, $-\frac{1}{2}$ Neutron

It can be rep^n as :-

$|I \; I_3\rangle$

Proton $|\frac{1}{2} \; \frac{1}{2}\rangle$

Neutron $|\frac{1}{2} \; -\frac{1}{2}\rangle$

No. of Multiplets / degeneracy :- $= 2I + 1$

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Nucleons $= 2 \times \frac{1}{2} + 1 = 2$

for Triplet of pions π^+, π^0, π^-,

degenracy $\Rightarrow 2I+1 = 3$

$I = 1$

$I_3 \equiv$ from $+I$ to $-I$ with a diff. of 1

$I_3 = 1, 0, -1$

$\pi^+ \equiv |1, 1\rangle$

$\pi^0 \equiv |1\ 0\rangle$

$\pi^- \equiv |1\ -1\rangle$

for K^+, K^0

$2I+1 = 2$

$I = 1/2$

$I_3 = +\frac{1}{2}, -\frac{1}{2}$

$K^+ \equiv |\frac{1}{2}\ \frac{1}{2}\rangle$

$K^0 \equiv |\frac{1}{2}\ -\frac{1}{2}\rangle$

for $\Sigma^+, \Sigma^0, \Sigma^-$

$2I+1 = 3$

$I = 1$

$I_3 = 1, 0, -1$

$\Sigma^+ \equiv |1\ 1\rangle$

$\Sigma^0 \equiv |1\ 0\rangle$

$\Sigma^- \equiv |1\ -1\rangle$

for Cascade Ξ^0, Ξ^-

$2I+1 = 2$, $I = 1/2$

$I_3 = 1/2, -\frac{1}{2}$

$\Xi^0 \equiv |\frac{1}{2}\ \frac{1}{2}\rangle$

$\Xi^- \equiv |\frac{1}{2}\ -\frac{1}{2}\rangle$

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For $\Delta^{++}, \Delta^{+}, \Delta^{\circ}, \Delta^{-}$ Quadreplet :-

$$2I+1 = 4$$
$$I = 3/2$$
$$I_3 = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$$

$$\Delta^{++} \equiv \left|\frac{3}{2}\ \frac{3}{2}\right>$$

$$\Delta^{+} \equiv \left|\frac{3}{2}\ \frac{1}{2}\right>$$

$$\Delta^{\circ} \equiv \left|\frac{3}{2}\ -\frac{1}{2}\right>$$

$$\Delta^{-} \equiv \left|\frac{3}{2}\ -\frac{3}{2}\right>$$

Consider the following

		Q	I_3	
Baryons	p	$+1$	$+1/2$	$Q = I_3 + \frac{1}{2}$
	n	0	$-1/2$	$B = +1$
Mesons	π^{+}	$+1$	$+1$	$Q = I_3$
	π°	0	0	$B = 0$
	π^{-}	-1	-1	

Gellmen Nishizina Ralation,

$$\boxed{Q = I_3 + \frac{B}{2}}$$

For Λ° (Lamda) Baryon

$$B = 1$$
$$2I+1 = 1$$
$$\Rightarrow I = 0$$
$$\therefore I_3 = 0$$

$$Q = I_3 + \frac{B}{2}$$

$\Rightarrow 0 = 0 + \frac{1}{2}$. Here Gellmen & Nishizina relⁿ is not applicable.

Some property is missing from Gellmen Nishizina Relation.

Strangness :- A large no. of particle were discovered. And speciallity about them was their formation was governed by strong interaction while the decay was governed by weak interaction. This was a strange property (or behaviour). Therefore these particles were given the name Strange Particles.

$$\pi^- + p \longrightarrow \underbrace{\Lambda^0 + K^0}_{\text{strange particles}}$$

$$\Lambda^0 \longrightarrow p + \pi^- \quad \text{Weak Int.}$$

$$\Xi^- \longrightarrow \Lambda^0 + \pi^-$$

Gellmen & Nishizina modified their relation by assigning a strangness quantum no. S to these strange particles.

Modified Gellmen & Nishizing relⁿ is

$$Q = I_3 + \frac{(B+S)}{2}$$

where $\boxed{B + S = Y}$ = Hyper. charge

for p, $Q = I_3 + \frac{(B+S)}{2}$

$$1 = \frac{1}{2} + \frac{1+S}{2} \Rightarrow \frac{1}{2} = \frac{1+S}{2}$$

$$\boxed{S = 0}$$

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for π^+, $Q = I_3 + \frac{B+S}{2}$

$1 = 1 + \frac{\cancel{B}0+S}{2}$

$\boxed{S=0}$

Λ^0, $Q = I_3 + \frac{B+S}{2}$

$0 = 0 + \frac{1+S}{2} \Rightarrow S+1=2$

$\boxed{S=-1}$

Σ^- $-1 = -1 + \frac{1+S}{2} \Rightarrow \boxed{S=-1}$

Ξ^0 $0 = \frac{1}{2} + \frac{1+S}{2} \Rightarrow \boxed{S=-2}$

Ξ^- $-1 = -\frac{1}{2} + \frac{1+S}{2} \Rightarrow \cancel{S}\,-2 = -1+1+S$

$\Rightarrow \boxed{S=-2}$

Ω^- $-1 = 0 + \frac{1+S}{2} \Rightarrow -2 = 1+S$ $\qquad 2I+1=1$

$\boxed{S=-3}$ $\qquad I=0$, $I_3=0$

* Strangness for the particle & the antiparticle are opposite.
* Strangness is conserved in strong & EM int.

selection Rule
$\Delta S = 0$
change in strangness is zero then int. is strong.
$\Delta S = \pm 1$ Recn may also be possible

but $\Delta S \not> 2$ Recn is not allowed i.e. forbidden

Hadrons
- Mesons
  - pseudo scalar: $\cancel{J^P \neq 0}$, $J^P = 0^-$
  - vector: $J^P = 1^-$
- Baryons
  - Baryon: $J^P = \frac{1}{2}^+$
  - Baryon Resonance: $J^P = \frac{3}{2}^+$

$P \equiv$ parity
$J \equiv$ ang. mom. Q. No.

	Particle	I	I_3	B	S ($Q = I_3 + \frac{B+S}{2}$)	Y ($Y = B+S$)	Quark Content
$2I+1 = 3$, $I = 1$	π^+	1	+1	0	0	0	$u\bar{d}$
	π^0	$\cancel{0}1$	0	0	0	0	$u\bar{u}, d\bar{d}, s\bar{s}$
	π^-	+1	−1	0	0	0	$\bar{u}d$
$2I+1 = 2$, $I = \frac{1}{2}$	K^+	1/2	1/2	0	1	1	$u\bar{s}$
	K^0	1/2	−1/2	0	1	1	$d\bar{s}$
$2I+1 = 2$, $I = \frac{1}{2}$	$\bar{K}^0$	1/2	1/2	0	−1	−1	$\bar{d}s$
	K^-	1/2	−1/2	0	−1	−1	$\bar{u}s$
$2I+1 = 1$, $I = 0$	η^0	0	0	0	0	0	$u\bar{u}, d\bar{d}, s\bar{s}$

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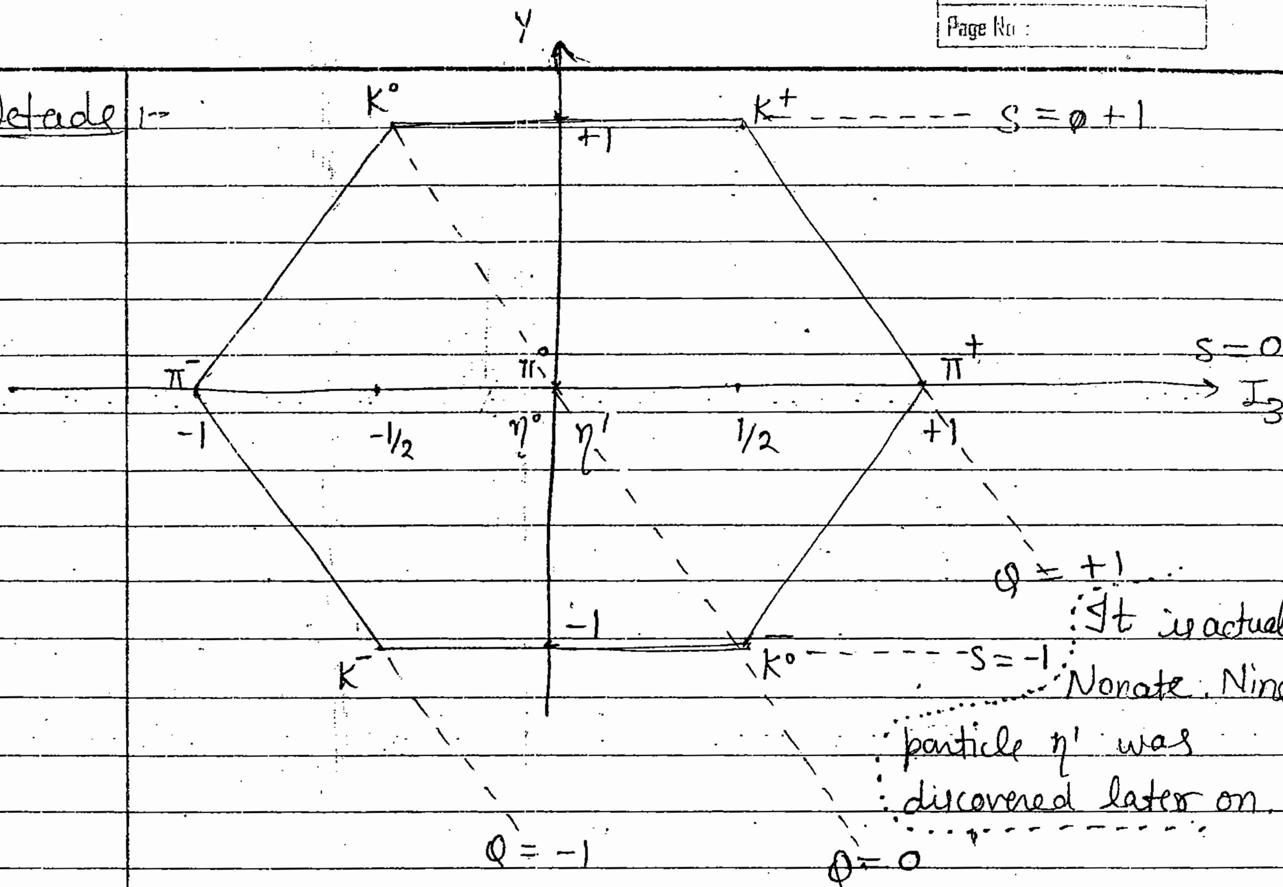
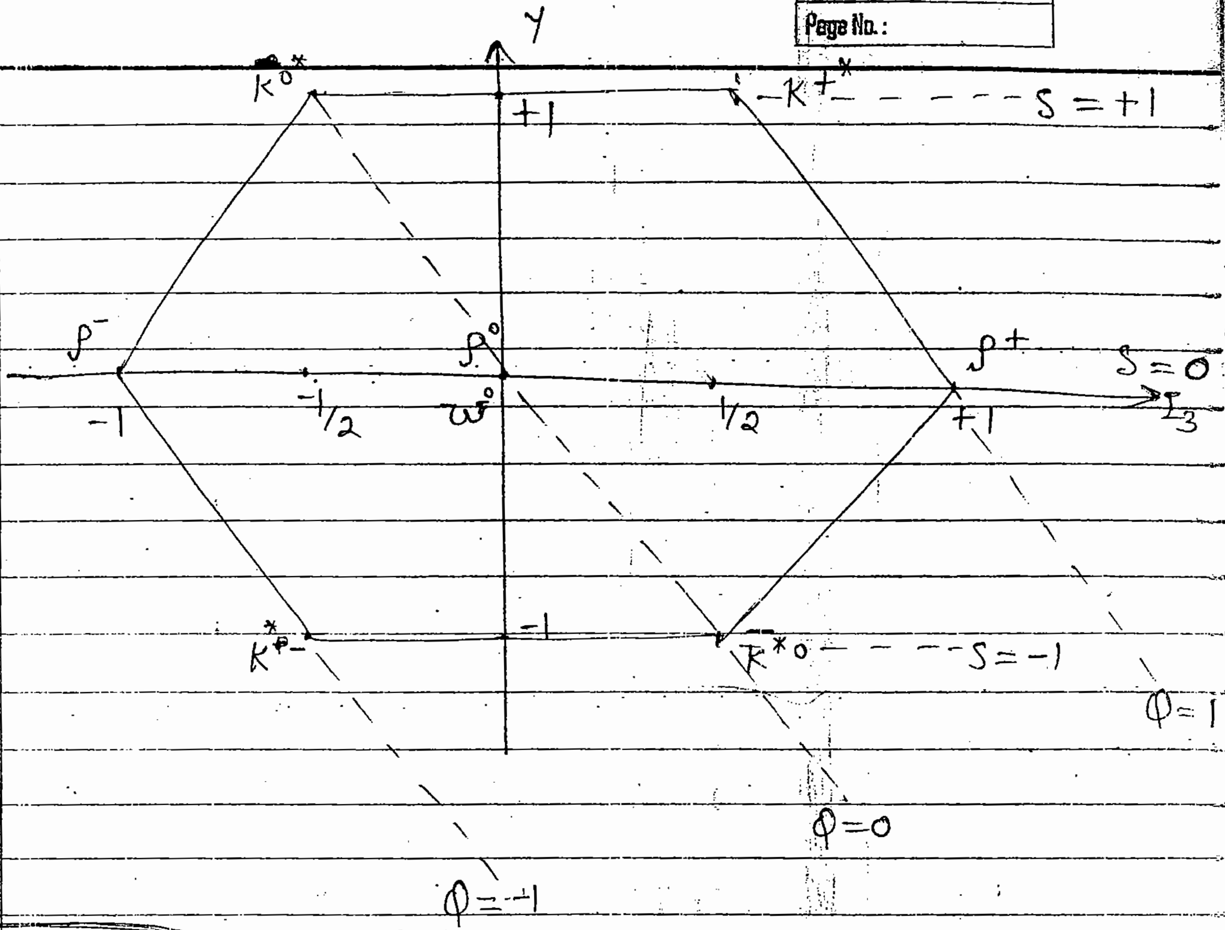


It is actually Nonate. Nine particle η' was discovered later on.

Vector Mesons :- $J^P = 1^-$

Particles	I	I_3	B	S	Y	Quark structure
ρ^+	1	+1	0	0	0	$u\bar{d}$
ρ^0	1	0	0	0	0	$u\bar{u}, d\bar{d}, s\bar{s}$
ρ^-	1	-1	0	0	0	$d\bar{u}$
K^{*+}	1/2	+1/2	0	+1	+1	$u\bar{s}$
K^{*0}	1/2	-1/2	0	+1	+1	$d\bar{s}$
$\bar{K}^{*0}$	1/2	+1/2	0	-1	-1	$s\bar{d}$
K^{-*}	1/2	-1/2	0	-1	-1	$s\bar{u}$
ω^0	0	0	0	0	0	$u\bar{u}, d\bar{d}, s\bar{s}$

$2I+1=3$
$2I=2$
$I=1$



Baryons :- $J_P = \frac{1}{2}^+$

$$3 \otimes 3 \otimes 3 \equiv 1 \oplus 8 \oplus 8 \oplus 10$$

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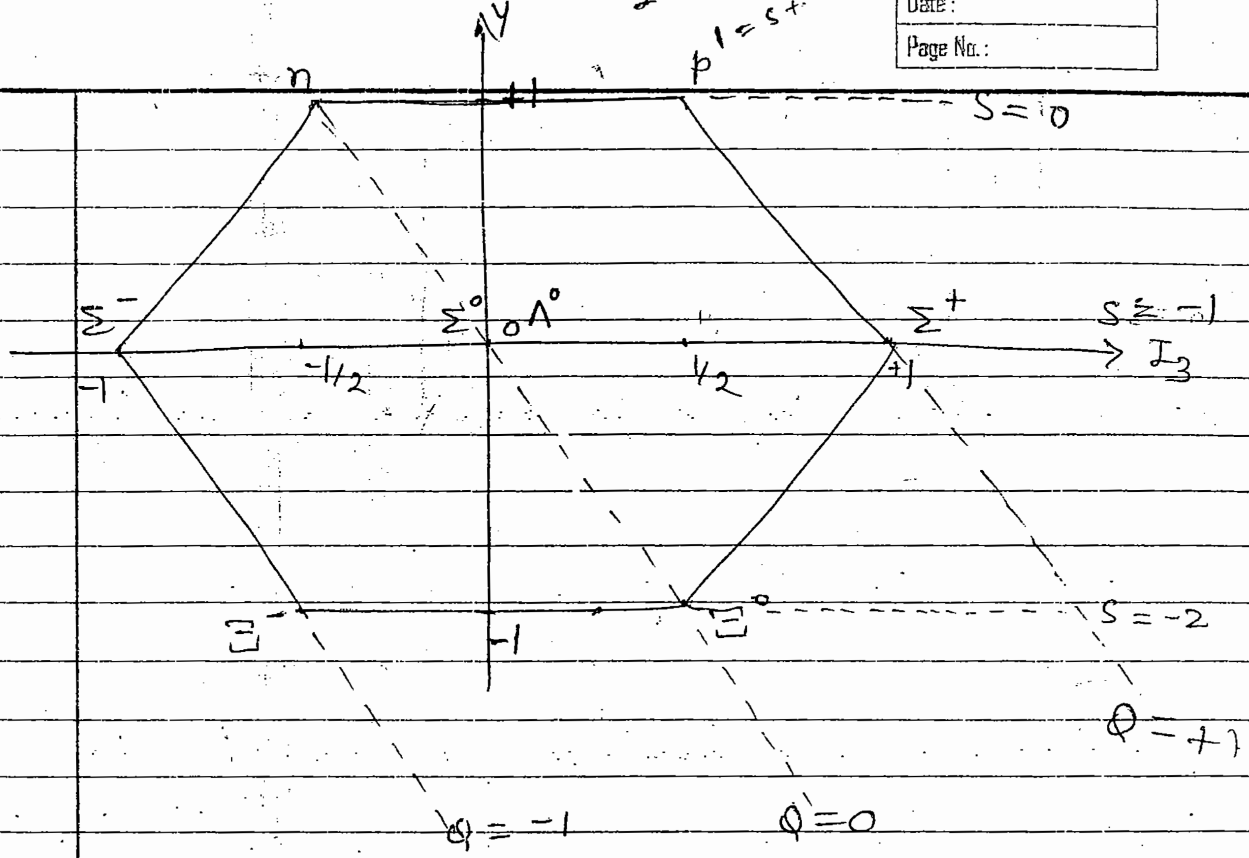
Particles	I	I_3	B	S	Y	Quark structure
p	$\frac{1}{2}$	$1/2$	1	0	1	uud
n	$1/2$	$-1/2$	1	0	1	udd
Λ^0	0	0	1	-1	0	uds
Σ^+	1	+1	1	-1	0	uus
Σ^0	1	0	1	-1	0	uds
Σ^-	1	-1	1	-1	0	dds
Ξ^0	1/2	1/2	1	-2	-1	uss
Ξ^-	1/2	-1/2	1	-2	-1	dss

$Q = I_3 + \frac{B+S}{2}$

$2 = \frac{3}{2} + \frac{1+S}{2}$

$1 = S +$



Baryon Rasonance :- $J^P = 3/2^+$

	Particles	I	I_3	B	S	Y	Quark Structure
2I+1 = 4, I = 3/2	Δ^{++}	3/2	+3/2	1	0	1	uuu [u(R) u(G) u(B)]
	Δ^{+}	3/2	+1/2	1	0	1	uud
	Δ^{0}	3/2	-1/2	1	0	1	udd
	Δ^{-}	3/2	-3/2	1	0	1	ddd
2I+1=3, I = 1	Σ^{+*}	1	+1	1	-1	0	uus
	Σ^{0*}	1	0	1	-1	0	uds
	Σ^{-*}	1	-1	1	-1	0	dds
2I+1=2, I = 1/2	Ξ^{0*}	1/2	+1/2	1	-2	-1	uss
	Ξ^{-*}	1/2	-1/2	1	-2	-1	dss
2I+1=1, I = 0	Ω^{-}	0	0	1	-3	-2	sss

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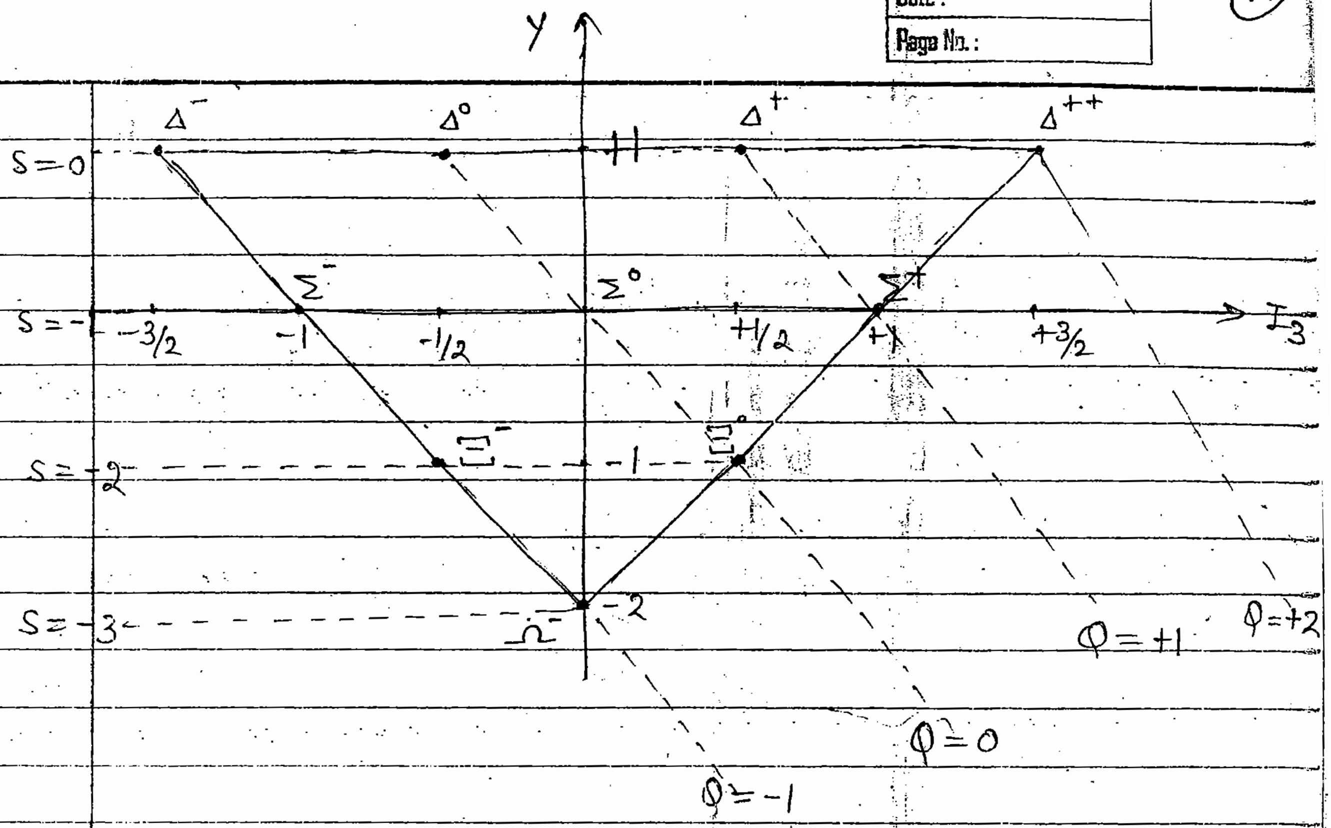

Conservation Laws :-

Quantity Conserved	Strong	EM	Weak
Energy	✓	✓	✓
Momentum	✓	✓	✓
charge → Electronic, → Leptonic, → Baryonic	✓	✓	✓
Isospin I	✓	✗	✗
I_3	✓	✓	✗
Strangness S	✓ $\Delta S = 0$	✓ $\Delta S = 0$	✗ $\Delta S = \pm 1$
Parity P	✓	✓	✗
Charge confi. C	✓	✓	✗
Time reversal	✓	✓	✓

$J^P = 0^-, \cancel{1}\ 1^-, \frac{1}{2}^+, \frac{3}{2}^{\pm}$. These signs are the parity assign to the particle or called Intrinsic parity.

Parity = $(-1)^{\Delta J}$

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Parity :– The intrinsic parity of all the mesons is ~~of~~ -1°. & the intrinsic parity of all the baryon is +1°.

→ For an antiparticle charge q will be opposite, spin s will be same and L_e, L_μ, L_τ & B will be opposite.
→ Isospin I will remain same.
→ 3rd component of isospin I_3 will be opposite.
→ Strangness will be opposite.
→ Rest mass & lifetime of the particle remain same.

Prob :– Identify weather the following reactions are allowed or forbidden and also find the interaction governed by them:

1) $\pi^- + p \longrightarrow \Sigma^+ + K^-$

charge Q	$-1 + 1$	$+1 - 1$	conserved
B. No. B	$0 + 1$	$+1 + 0$	"
I	$+1 + \frac{1}{2}$	$+1 + \frac{1}{2}$	"
I_3	$-1 + \frac{1}{2}$	$+1 - \frac{1}{2}$	x
S	$0 + 0$	$-1 + -1$	$\Delta S = 0 - (-2) = 2$

Recn is not allowed.

2) $\pi^+ \longrightarrow e^+ + \gamma$

Q	$+1$	$+1 + 0$	Conserved
L_e	0	$-1 + 0$	x

Recn is not allowed bcz L.No. is not con.

3) $\Omega^- \longrightarrow \Sigma^- + \pi^0$

Q	-1	$-1 + 0$	Conserved
B	1	$1 + 0$	"
I	0	$1 + 1$	x
I_3	0	$-1 + 0$	x
S	-3	$-1 + 0$	x $\quad \Delta S = -3 - (-1) = -2$

Not allowed.

4) $\Omega^- \longrightarrow \Lambda^0 + K^-$

Q	-1	0 - 1	Consvred
B	1	1 + 0	"
I	0	0 + 1/2	x
I_3	0	$0 - \frac{1}{2}$	x
S	-3	-1 - 1	$\Delta S = -3-(-2) = -1$

Reeⁿ is allowed but by weak int. bcz ΔS is changes by -1,

5) $\Lambda^0 \longrightarrow \pi^- + \pi^+$

Q	0	-1 + 1	✓
B	1	0 + 0	x

Not allowed bcz B.No. is Not conserved.

6) $\mu^- \longrightarrow e^- + \gamma$

Q	-1	-1 + 0	✓
L_e	0	1 + 0	x
L_μ	1	0 + 0	x

Not allowed.

7) $K^- + p \longrightarrow \Omega^- + K^+ + K^0$

Q	-1 + 1	-1 + 1 + 0	✓	
B	0 + 1	1 + 0 + 0	✓	
I	$\frac{1}{2} + \frac{1}{2}$	$0 + \frac{1}{2} + \frac{1}{2}$	✓	
I_3	$-\frac{1}{2} + \frac{1}{2}$	$0 + \frac{1}{2} - \frac{1}{2}$	✓	
S	-1 + 0	-3 + 1 + 1	✓	$\Delta S = 0$

Allowed. By Strong Int.

Parity : -1 +1   +1 -1 -1 $\pi_i = \pi_f (-1)^{\Delta J}$

$\pi_i = -1 \times +1 = -1$ $\pi_i = +1 \times -1 \times -1 = +1$

$J \rightarrow 0 + \frac{1}{2} \longrightarrow \frac{3}{2} + 0 + 0$ ✓ $\Delta J = 3/2 - 1/2 = 1$

$\pi_i = -1$, $\pi_f = +1 \times (-1)^1 = -1$ ✓

I → same particle & antip.
Parity → opposite
B.No → "

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8) $\Omega^- \longrightarrow \Xi^0 + \pi^-$

Q	-1	0 − 1	✓
B	1	1 + 0	✓
I	0	½ + 1	×
I_3	0	+½ − 1	×
S	-3	-2 + 0	ΔS = −3 −(−2) = −1

Allowed by Weak Int: ΔS changes by −1.

9) $\pi^+ + p \longrightarrow p + p + \bar{n}$

Q	1 + 1	1 + 1 + 0	✓
B	0 + 1	1 + 1 − 1	✓
I	$1 + \frac{1}{2}$	$\frac{1}{2} + \frac{1}{2} \mp \frac{1}{2}$	✓
I_3	$1 + \frac{1}{2}$	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$	✓
S	0 + 0	0 + 0 + 0	✓

Allowed by Strong Int.

10) $\bar{n} \longrightarrow \bar{p} + e^+ + \nu_e$

Q	0	-1 + 1 + 0	✓
L_e	0	0 ∓ 1 + 1	✓
B	-1	-1 + 0 + 0	✓
I	1/2	1/2 — —	✓
I_3	1/2	-1/2 — —	×
S	0	0 + 0 + 0	✓

Allowed by Weak Int.
This is strangeness conserved weak interaction.

Note:- If neutrinos are involved in a reaction then the interaction governed will have a greater probability of being weak, if a reaction is allowed.

If sources finite → Interference
infinite → ~~diffraction~~

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11) $\Lambda^0 \longrightarrow n + \gamma$

Q	0	0 + 0	✓
B	1	1 + 0	✓
I	0	$\frac{1}{2}$ + 0	×
I_3	0	$-\frac{1}{2}$ + 0	×
S	~~0~~ −1	0 + 0	$\Delta S = -1 - 0 = -1$

~~Allowed by weak Int.~~ Not Allowed (forbidden).

Note:- Whenever γ-photons are involved in a reeⁿ, interaction will be electromag subject to the fulfilment of all other conservation laws, for the EM Int to take place

12) $\mu^+ \longrightarrow e^+ + e^- + e^-$

Q	1	1	−1	−1	×	
L_μ	−1	0	0	0	×	forbidden

13) $\Lambda^0 \longrightarrow p + \pi^-$

Q	0	1 − 1	✓
B	1	1 + 0	✓
I	0	$\frac{1}{2}$ + 1	×
I_3	0	$\frac{1}{2}$ − 1	×
S	−1	0 + 0	$\Delta S = -1$

Allowed by weak int.

14) $K^0 \longrightarrow \pi^+ + \pi^-$

Q	0	1 − 1	✓
I	$\frac{1}{2}$	1 + 1	×
I_3	$-\frac{1}{2}$	+1 − 1	×
S	1	0 + 0	$\Delta S = 1$

weak int.

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15) $\Xi^- \longrightarrow \Lambda^0 + \pi^-$

Q	-1	$0 - 1$	✓
B	1	$1 + 0$	✓
I	$\frac{1}{2}$	$0 + 1$	x
I_3	$-\frac{1}{2}$	$0 - 1$	x
S	-2	$-1 + 0$	$\Delta S = -2+1 = -1$

Allowed. Weak int.

16) $p + p \longrightarrow p + p + p + \bar{p}$

Q	$1 + 1$	$1 + 1 + 1 - 1$	✓
B	$1 + 1$	$1 + 1 + 1 - 1$	✓
I	$\frac{1}{2} + \frac{1}{2}$	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$	x
I_3	$\frac{1}{2} + \frac{1}{2}$	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} - \frac{1}{2}$	✓
S	$0 + 0$	$0 + 0 + 0 + 0$	✓

Allowed. EM. int.

Tau Theta Puzzle ($\tau - \theta$ Puzzle) :- In 1950, two particles were discovered in cosmic rays which have almost same mass, charge, half life & spin. These two particles were same in all the aspects ~~a~~ except their decay mode

$$\tau^+ \longrightarrow \pi^+ + \pi^0$$
$$\theta^+ \longrightarrow \pi^+ + \pi^0 + \pi^0$$

Intrinsic Parity of pseudo scalar meson is -1. It is a multiplicative Q. No. So

for (i) R.H.S. $-1 \times -1 = +1$

(ii) R.H.S. $-1 \times -1 \times -1 = -1$

This was the 1st observation of parity violation.
Both particles were same if we assign the parity $+1$ to the particle then its decay into

three pions gives a parity -1 which means that there is a violation of parity conservation. And later on it was confirmed experimently that parity is violated in weak int.

17-7-12

- Relative Strength of a force ~~greater than~~ determines the time scale over which it acts.
- Larger will be the strength, quickly the process will happen.

- If we bring two particles close enough together for any of these forces to act then a longer time is required for a weak int. to take place as compared to that of strong as well as EM int.

The characterstic time for the interaction are as follows:

		(If these present then Most probably reaⁿ will be)
Strong	$< 10^{-22}$ sec	K, n, p, π
Electromagnetic	10^{-14} to 10^{-20} sec	e, μ, π, K, n, p
Weak	10^{-8} to 10^{-13} sec	All

→ Leptons will never participate in strong reaⁿ.

→ If neutrino & antineutrino are involve in a reaⁿ then it will be weak.

Que 1- ~~Write~~ A relativistic particle travells a length of 3×10^{-3} m in air before decaying is governed by

1) strong
2) EM
3) Weak

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the
into

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$$t = \frac{3 \times 10^{-3}\ m}{3 \times 10^{8}\ m/sec}$$

$\simeq 10^{-11}$ sec

Weak Int.

Que:- The mean life time for the decay of pions are

(i) $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ $\quad \bar{T} = 2.6 \times 10^{-8}$ sec

(ii) $\pi^0 \rightarrow \gamma + \gamma$ $\quad \bar{T} = 8.4 \times 10^{-17}$ sec

(i) Weak $\quad$ (ii) EM.

Que:- Fill in the blanks:

(i) $K^- + p \longrightarrow \Omega^- + \underline{K^+} + \underline{K^0}$

$S = -1 + 0 \qquad -3 \quad +1 \quad +1$

$-1 \qquad -1$

$\Delta S = 0$

(ii) $\gamma + n \longrightarrow \pi^- + p$

Q	$0 + 0$	$-1 + 1$	✓
B	$0 + 1$	$0 + 1$	✓
I	$0 + \frac{1}{2}$	$+1 + \frac{1}{2}$	✗
I_3	$0 - \frac{1}{2}$	$-1 + \frac{1}{2}$	✓
S	$0 + 0$	$0 + 0$	✓ $\Delta S = 0$

Isospin is Not conserved & γ-photons are involved.
Reacⁿ will be Electromagnetic.

* Charateristic secⁿ cross-section for an EM interaction is 10^{-4} mbarn. for weak interaction $\simeq 10^{-12}$ mB

Strong " $\simeq 10$ mB

Quark Model :- All the hadrons follow ~~sh~~ ~~such~~ multiplet or supermultiplet structure then it was proposed that ~~some~~ they are composed of some more fundamental particles called Quarks.

Since, Hadrons → Integer spin (pseudo scalar or vector meson)
→ Half integer : (Baryon & baryon resonance)

- Quarks ~~be~~ are having spin 1/2.

✓ Mesons are composed of 2 Quarks. (One quark, one anti ")
✓ Baryons " " " 3 " .

Baryon no. (of ~~Baryons~~ Quarks) = 1/3 $\left[B = \frac{1}{3}\right]$

Since there are strange particle as well therefore at least one of the quark should be strange.
They have fractional charge.

$$Q = +\frac{2}{3}e \;,\; -\frac{1}{3}e$$

Since Quarks are not free so $\boxed{Q = ne}$ is still valid.
Since the quarks are not observed as a free particle therefore having fractional charges will not ~~valid~~ violates $Q = ne$.

Quarks	Charge	I	I_3	B	S	Y
up 'u'	+2/3	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{1}{3}$
down 'd'	−1/3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{1}{3}$
Strange 's'	−1/3	0	0	$\frac{1}{3}$	−1	$-\frac{2}{3}$

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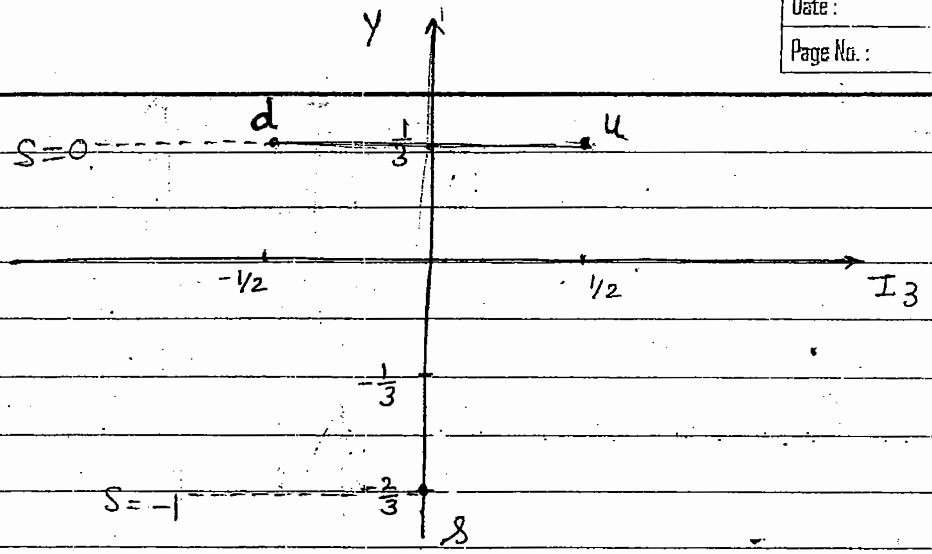

To Each quark q there corresponds an antiquark $\bar{q}$ with charge & strangness opposite.

Quark	Anti Quark
u	$\bar{u}$
d	$\bar{d}$
s	$\bar{s}$

* Since Δ^{++} is composed of 3 identical up quarks which is the violation of Pauli Exclusion Principle. Therefore it was proposed that there must be some more quarks with different Quantum no's.

charm quark

Beauty

Top or Truth

Note :- If we consider I as a vector quantity then I will be vectorially added.

e.g. $\mu^- + \mu^+ \longrightarrow K^- + K^+$

$I \quad - \quad - \quad \frac{1}{2} + \frac{1}{2}$

So 2 posibilities ~~i.e.~~ (i) $0 \longrightarrow \frac{1}{2} - \frac{1}{2} = 0$ ✓ (Anti II spin)

(ii) $0 \longrightarrow \frac{1}{2} + \frac{1}{2} = 1$ ✗ (II spin)

In 1 probability Reac^n is conserved but it can not be strong int. becoz leptons (μ^+ & μ^-) are involved.

Quantum No.	u	d	s	c	b	t
Q	$+2/3$	$-1/3$	$-1/3$	$+2/3$	$-1/3$	$+2/3$
I	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_3	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0
B	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
S	0	0	-1	0	0	0
charm quark C	0	0	0	$+1$	0	0
Beauty B	0	0	0	0	-1	0
Top or Truth T	0	0	0	0	0	$+1$

Modified Gellmen & Nishizima Relation

$$\boxed{Q = I_3 + \frac{(B + S + C + B + T)}{2}}$$

Ex :- (1) $\mu^- + \mu^+ \longrightarrow K^- + K^+$

Q	-1	$+1$	$-1 + 1$	✓
L	1	-1	$0 + 0$	✓
I	—	—	$\frac{1}{2} + \frac{1}{2}$	✗
I_3	—	—	$-\frac{1}{2} + \frac{1}{2}$	✓
S	0	0	$-1 + 1$	✓

EM ~~Weak~~ Int.

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2) $\pi^+ + \eta^0 \longrightarrow K^0 + K^+$

3) $\bar{\nu}_e + p^+ \longrightarrow n^0 + \mu^+$

4) $\bar{\nu}_e + p^+ \longrightarrow n^0 + e^+$

5) $\Sigma^+ \longrightarrow p + \pi^0$.

Que:- What will be the Quark Analoug of the rec''

$$\pi^- + p \longrightarrow \Lambda^0 + K^0$$

$$\bar{u}d + uud \longrightarrow uds + d\bar{s}$$

$$\Rightarrow \bar{u} + \cancel{d} + u + \cancel{u} + \cancel{d} \longrightarrow \cancel{u} + \cancel{d} + s + \cancel{d} + \bar{s}$$

$$\bar{u}u \longrightarrow s\bar{s}$$

* → For Pseudo Scalar Mesons, charge confi. Q.No. $C = +1$

→ for Vector Meson $C = -1$

→ for Gamma Photon, $C = -1$

Imp Que The Neutral pion decays into 2 γ- photons & not in 3, why?

$$\pi^0 \longrightarrow \gamma + \gamma$$

$$\pi^0 \longrightarrow \gamma + \gamma + \gamma$$

(i) $\pi^0 \longrightarrow \gamma + \gamma$

C : $+1$ $\quad -1 \times -1$ ✓ allowed

(ii) $\pi^0 \longrightarrow \gamma + \gamma + \gamma$

C : $+1$ $\quad -1 \times -1 \times -1$ ✗ Not allowed

So because of charge conifiguration Q. No. π^0 decay in 2 γ-photon not in 3.

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(iii) $\omega^0 \longrightarrow \pi^0 + \gamma$:

$-1 \quad\quad 1 \times -1$ ✓ allowed & E.M. int.

* Note :- Remember above 3 reeⁿs, In these reeⁿ only ~~corr~~ cheak the charge confi. Q.No. (in which all zero charged particle participate)

Que :- Find out the no. of down quarks in oxygen atom.

$^{16}_{8}O$

No. of protons = 8 (uud)
neutrons = 8 (udd)

$d \equiv 3 \times 8 = 24$

$u \equiv 3 \times 8 = 24$

(2) $\pi^+ + n^0 \longrightarrow K^0 + K^+$

Q	$+1 + 0$	$0 + 1$	✓
B	$0 + 1$	$0 + 0$	X

Not allowed. $\Delta B \neq 0$ & $\Delta S = 2$.

(3) * $\bar{\nu}_e + p^+ \longrightarrow n^0 + \mu^+$

Q	$0 + 1$	$0 + 1$	✓
L_e	$-1 + 0$	$0 + 0$	X
B	$0 + 1$	$1 + 0$	✓

Not allowed

(4) $\bar{\nu}_e + p^+ \longrightarrow n^0 + e^+$

Q	$0 + 1$	$0 + 1$	✓
L_e	$-1 + 0$	$0 - 1$	✓
B	$0 + 1$	$1 + 0$	✓
I	$- + \frac{1}{2}$	$\frac{1}{2} + -$	✓
I_3	$- + \frac{1}{2}$	$-\frac{1}{2} + -$	X
S	$0 + 0$	$0 + 0$	✓

✓ ~~Weak int.~~

~~Not allowed~~.

Strange particle always decay by weak int.

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(5) $\Sigma^{+} \longrightarrow p + \pi^{0}$

	Σ^+	$p + \pi^0$	
Q	+1	1 + 0	✓
B	1	1 + 0	✓
I	1	$\frac{1}{2}$ + 1	×
I_3	+1	$+\frac{1}{2}$ + 0	×
S	−1	0 + 0	× $\Delta S = -1-0 = -1$.

Weak int.

18-7-2012

Note :- There are only 2 multiplicative Q.No. → parity & charge configuration & other Q-No.s are additive.

(i) $K^{-} + p \longrightarrow \Sigma^{-} + \pi^{+}$

	$K^- + p$	$\Sigma^- + \pi^+$	
Q	−1 +1	−1 + 1	✓
B	0 + 1	1 + 0	✓
I	1/2 + 1/2, $\frac{1}{2}-\frac{1}{2}$	+1 + 1, 1−1	Add vectorially $\left[\begin{array}{l}\frac{1}{2}+\frac{1}{2}, \frac{1}{2}-\frac{1}{2} \rightarrow 1+1, 1-1 \\ 1, 0 \rightarrow 2, 0\end{array}\right.$
I_3	−1/2 + 1/2	−1 + 1	✓
S	−1 + 0	−1 + 0	✓ $\Delta S = 0$

strong Reac^n (Reac^n is allowed if we add isospin I vectorially).

* If $\Delta S = 0$ the process will be strong, EM or it may also be a weak interaction

	ΔS	ΔI	ΔI_3
Strong	0	0	0
EM	0	×	0
Weak	0, ±1	×	×

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(ii) $\pi^+ + n^0 \longrightarrow \Lambda^0 + K^+$

Q	$1+0$	$0+1$	✓
B	$0+1$	$1+0$	✓
I	$1+\frac{1}{2}, 1-\frac{1}{2}$	$0+\frac{1}{2}, 0-\frac{1}{2}$	
	$\frac{3}{2}, \frac{1}{2}$	$\frac{1}{2}, -\frac{1}{2}$	✓
I_3	$+1-\frac{1}{2}$	$0+\frac{1}{2}$	✓
S	$0+0$	$-1+1$	✓ $\Delta S=0$

Allowed. Strong Int.

(iii) $n \longrightarrow p + e^- + \bar{\nu}_e$

Q	0	$1-1+0$	✓
B	1	$1+0+0$	✓
L_e	0	$0+1-1$	✓
I	$\frac{1}{2}$	$\frac{1}{2}$ — —	✓
I_3	$-\frac{1}{2}$	$\frac{1}{2}$ — —	×
S	0	$0+0+0$	$\Delta S=0$

1+1, 1-1
2, 0

Allowed by Weak int.

(iv) $\pi^0 + n \longrightarrow \bar{K}^0 + \Sigma^0$

Q	$0+0$	$0+0$	✓
B	$0+1$	$0+1$	✓
I	$1+\frac{1}{2}$	$\frac{1}{2}+1$	✓
I_3	$0-\frac{1}{2}$	$\frac{1}{2}+0$	×
S	$0+0$	$-1-1$	$\Delta S = +2$

Not allowed since $\Delta S = +2$.

(v) $p^+ \longrightarrow e^+ + \gamma$

Q	1	$1+0$	✓
L_e	0	$-1+0$	×
B	1	$0+0$	×

Not allowed, B & L_e are not conserved.

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(vi) $\pi^+ + n \longrightarrow \pi^- + p$

Q	1 + 0	-1 + 1	X

Not allowed, charge is not conserved.

(vii) $\Sigma^+ \longrightarrow p + \pi^0$

Q	+1	1 + 0	✓
B	1	1 + 0	✓
I	1	$\frac{1}{2}$ + 1	X
I_3	+1	$\frac{1}{2}$ + 0	X
S	-1	0 + 0	$\Delta S = -1$

Weak int.

(viii) $\Xi^- \longrightarrow n + \pi^- + \pi^0$

Q	-1	0 - 1 + 0	✓
B	1	1 + 0 + 0	✓
S	-2	0 + 0 + 0	$\Delta S = -2$

Not allowed

(ix) $p + p \longrightarrow p + \Lambda^0 + \Sigma^+$

Q	1 + 1	1 + 0 + 1	✓
B	1 + 1	1 + 1 + 1	X

Not allowed.

(x) $e^+ + e^+ \longrightarrow \mu^+ + \pi^-$

Q	1 + 1	1 - 1	X

Not allowed

Note :- If Mass is given in kilogram then

e.g. 2 Kg then 2 Kg c^2 Jules

$1 eV = 1.6 \times 10^{-19}$ J

$1 MeV = 10^6$ eV

$1 GeV = 10^9$ eV

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amu $\rightarrow$ 'u' unified mass unit.

$$amu = \frac{1}{16} \times \text{mass of 1 atom of } {}^{16}_{8}O \quad \text{(before 1961)}$$

Now,

$$1\,u = \frac{1}{12} \times \text{mass of 1 atom of } {}^{12}_{6}C$$

1 mole of ^{12}C ~~and~~ has the mass = 12 gm

i.e. 6.02205×10^{23} atoms of ^{12}C " = 12 gm

$$\text{Mass of 1 atom of } {}^{12}C = \frac{12 \times 10^{-3}}{6.02205 \times 10^{23}} \text{ kg}$$

$$1\,u = \frac{1}{12} \times \frac{12 \times 10^{-3}}{6.02205 \times 10^{23}} \text{ kg}$$

$$\boxed{1\,u = 1.660566 \times 10^{-27} \text{ kg}}$$

In Energy units,

$$1\,u = 1.660566 \times 10^{-27} \times c^2 \text{ J}$$

$$= 1.660566 \times 10^{-27} \times (3 \times 10^8)^2 \text{ J}$$

$$= \frac{\quad}{1.6 \times 10^{-19}} \text{ eV}$$

$$\boxed{1\,u = 931.502 \text{ MeV}}$$

Rest Mass of Electron,

$$\boxed{m_e = 9.10953 \times 10^{-31} \text{ kg}}$$

$$= \frac{9.10953 \times 10^{-31}}{1.660566 \times 10^{-27}} \text{ u}$$

$$\boxed{m_e = 5.4858 \times 10^{-4} \text{ u}}$$

$$= 5.4858 \times 10^{-4} \times 931.502 \text{ MeV}$$

$$= 5110 \times 10^{4} \text{ MeV}$$

$$\boxed{m_e = 0.511003 \text{ MeV}}$$

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Mass of proton :-

$$\boxed{m_p = 1.67265 \times 10^{-27}\ Kg}$$

$$= \frac{1.67265 \times 10^{-27}}{1.660566 \times 10^{-27}} u$$

$$\boxed{m_p = 1.0072765\ u}$$

$$m_p = 1.0072765 \times 931.502\ MeV$$

$$\boxed{\begin{array}{l} m_p = 938.2805\ MeV \\ m_p \simeq 938.3\ MeV \end{array}}$$

Mass of Neutron :-

$$\boxed{m_n = 1.67495 \times 10^{-27}\ Kg}$$

$$= \frac{1.67495 \times 10^{-27}}{1.660566 \times 10^{-27}}\ u$$

$$\boxed{m_n = 1.008665\ u}$$

$$m_n = 1.008665 \times 931.502\ MeV$$

$$\boxed{m_n = 939.57077\ MeV}$$

Ground State Properties Of Nucleus :-

are also called STATIC PROPERTIES :-

Just like a Atom, nucleus is a bound Quantum mechanical system & hence can exist in different Quantum state. The lowest energy state is called the Ground State & the properties of the nucleus in this state are called ground

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state properties or static properties.

Static properties are as follows:

1) Electric charge
2) Mass
3) Binding Energy
4) Size
5) Shape
6) Angular Momentum
7) Magnetic Dipole Moment
8) Electric Quadroupole moment
9) Statistics
10) Parity & Isospin

If Mass of Nucleons (combind mass of p & n) = M_1
Mass of Nucleus = M_2

then $M_1 - M_2 = \Delta M$

$= (\Delta M) c^2$

This is the amount of energy liberated in the formation of Nucleus.

The mass of the atom can be written as

$M(Z, A)$

$Z \rightarrow$ electrons
$Z \rightarrow$ protons
$(A-Z) \rightarrow$ Neutrons

Mass of the atom is

$$M(Z,A) = {}^{A}_{Z}M_{Nucleus} + ZM_e - B_e(Z)$$

$$\cancel{= ZM_p + (A-Z)}$$

All the masses are written in terms of energy units.

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$$M(Z,A) = ZM_p + (A-Z)M_n - B(Z,A) + ZB_H - ZB_H + ZM_e - B_e(Z)$$

$B_H \rightarrow$ B.E. of hydrogen atom

$$M(Z,A) = Z(M_p + M_e - B_H) + (A-Z)M_n + ZB_H - B(Z,A) - B_e(Z)$$

→ Hydrogen atom

^1_1H has 1p & 1e.

$$\boxed{M_H = (M_p + M_e) - B_H}$$

Now, Use this in above relation,

$$M(Z,A) = ZM_H + (A-Z)M_n + ZB_H - B(Z,A) - B_e(Z)$$

where,

$B(Z,A) \rightarrow$ B.E. of the nucleus $\cong$ MeV

$B_H \rightarrow$ " " hydrogen atom $\cong$ KeV

$B_e \rightarrow$ " " electrons $\cong$ eV

For a Good Approximation, we can ignore B_H + B_e

$$\boxed{M(Z,A) = ZM_H + (A-Z)M_n - B(Z,A)}$$

Binding energy of Nucleus,

$$\boxed{B(Z,A) = ZM_H + (A-Z)M_n - M(Z,A)}$$

$M_H \longrightarrow$ Mass of H_2 atom $\quad M_H \cong M_p$

$M(Z,A) \rightarrow$ Mass of the atom

→ B.E.

→ Seperation Energy

→ Q-value

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Average Binding Energy :- Binding Energy per nucleon is the average B.E. & it is defined as

$$\text{Avg. B.E.} = \frac{\text{Binding Energy}}{\text{Total No. of Nucleons}}$$

$$\boxed{\bar{B}(Z,A) = \frac{B(Z,A)}{A}}$$

Aston Rule :- The accurate determination of the atomic masses show that, they are very close to the whole numbers, which are actually the mass no.s of the atoms when expressed in ^{12}C-12 energy ~~levels~~ units.

for
$A < 20$
$A > 180$ } Atomic Masses are slighthly greater than A (whole no.)

But for the Intermediate nuclei, masses are slightely smaller than A.

This departure is called Mass Defect.

$$\boxed{\Delta M = M(Z,A) - A}$$

Packing fraction :-

$$f = \frac{\Delta M}{A}$$

$$f = \frac{M(Z,A)}{A} - \frac{A}{A}$$

$$\boxed{A(1+f) = M(Z,A)}$$

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Q.1:- Calculte the B.E. per nucleon for the deutron ^2_1H.

If $M_H = 1.007825$ u

$M_n = 1.008665$ u

$M_d = 2.014102$ u

$\left\{ {}^2_1H \rightarrow p=1, \; n=1 \right.$

$$B(Z,A) = ZM_H + (A-Z)M_n - M(Z,A)$$

$$= 1 \times 1.007825 + 1 \times 1.008665 - 2.014102$$

$$= 1.007825 + 1.008665 - 2.014102$$

$$= 2.016490 - 2.014102$$

$$B(Z,A) = 0.002388 \text{ u}$$

$$= 0.002388 \times 931.502 \text{ MeV}$$

$$= 2.22442678 \text{ MeV}$$

$$\frac{B(Z,A)}{A} = \frac{2.22442678}{2} \qquad [A=2]$$

$$\bar{B} = \boxed{B(Z,A)/A = 1.11221339 \text{ MeV/nucleon}}$$ Ae

Q.2:- Calculate the B.E/A for α-particle (^4_2He) $M_\alpha = 4.002603$ u

$$B(Z,A) = ZM_H + (A-Z)M_n - M(Z,A)$$

$$= 2 \times 1.007825 + 2 \times 1.008665 - 4.002603$$

$$= 2.015650 + 2.017330 - 4.002603$$

$$= 0.030377$$

$$\frac{B.E.}{\cancel{A}} = \frac{0.030377 \times 931.502}{\cancel{4}} \text{ MeV}$$

$$= 28.3 \text{ MeV}$$

$$\bar{B} = \frac{B.E.}{A} = \frac{28.3}{4}$$

$$= 7.075 \text{ MeV/nucleon}$$

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Q.3 :- $^{56}_{26}Fe$, $M_{Fe} = 55.934939\ u$

$$B.E. = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$$
$$= 26.20345 + 30.25995 - 55.934939$$
$$= 56.4634 - 55.934939$$
$$= 0.5285$$

$$B.E. = 0.5285 \times 931.502\ MeV$$
$$= 492.9775\ MeV$$

$$\bar{B} = B.E./A = \frac{492.9775}{56}$$
$$= 8.79103125\ MeV/nucleon.$$

Q.4 :- $^{238}_{92}U$; $M_U = 238.0507854\ u$

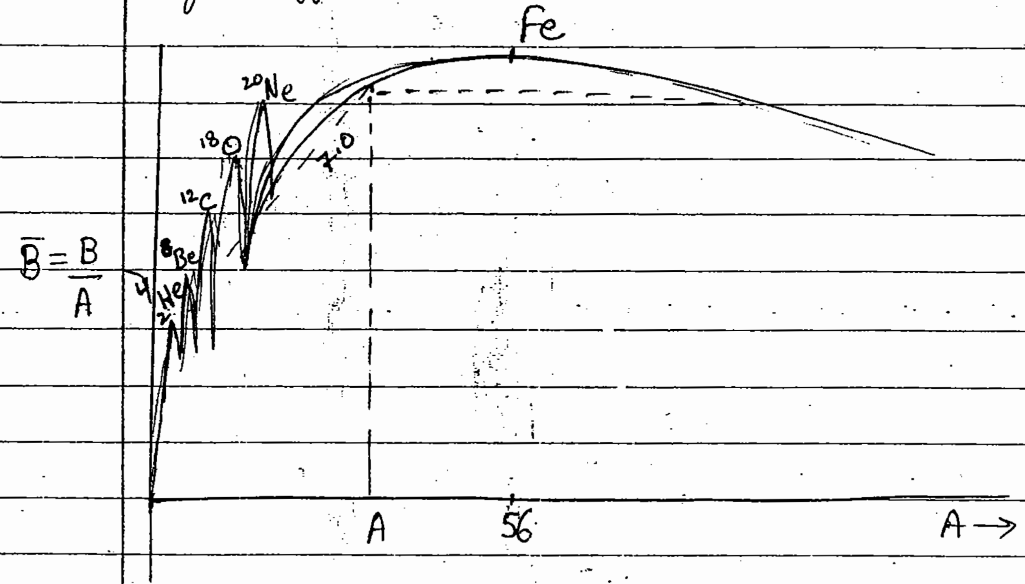
$$B.E. = 92 \times 1.007825 + 146 \times 1.008665 - 238.0507854$$
$$= 92.7199 + 147.26509 - 238.0507854$$
$$= 239.98499 - 238.0507854$$
$$= 1.9342$$

$$= 1.9342 \times 931.502\ MeV$$
$$= 1801.7204\ MeV$$

$$\frac{B.E.}{A} = \frac{1801.7204}{\cancel{9}238}$$
$$= 7.57\ MeV/nucleon$$

$$\bar{B} = \boxed{B.E./A = 7.57\ MeV/N}$$

Binding Energy Curve



Separation Energy :- "Energy required to make an entity free".

It is just like latent heat.

Consider the decay reaction

$${}^{A}_{Z}X \longrightarrow {}^{a}_{z}x + {}^{A-a}_{Z-z}Y$$

We define the separation energy as, it is the sum of the masses of the products minus sum of the masses of the reactants.

$$S_x = M(Z-z, A-a) + M(z,a) - M(Z,A)$$

Note → { Q-Value :- Masses of reactants − Masses of products }

$$S_x = \cancel{(Z-z)M_H} + (A-\cancel{a-Z+z})M_n - B(Z-z, A-a)$$
$$+ \cancel{zM_H} + \cancel{(a-z)}M_n - B(z,a)$$
$$- \cancel{ZM_H} - \cancel{(A-Z)}M_n + B(Z,A)$$

$$\boxed{S_x = B(Z,A) - B(Z-z, A-a) - B(z,a)}$$

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Q- Value :- $-S_x = Q$

$$\boxed{Q = B(z,a) + B(Z-z, A-a) - B(Z,A)}$$

(i) If S_x is +ve i.e. $\boxed{S_x > 0}$

S_x is +ve means system is going from more bound state to lesser bound state.

S_x can never be +ve naturally. The process will be artificial.

(ii) If S_x is -ve i.e. $\boxed{S_x < 0}$

system is going from lesser bount state to more bound state.

It is natural. So It is Spontenuous process.

for Q-Value,

If $Q > 0$ then process is Spontenuous.

& If $Q < 0$ then proess is artificial.

Que :- Calculate the seperation Energy for proton.

$${}^{A}_{Z}X \longrightarrow p + {}^{A-1}_{Z-1}Y$$

Separation energy for p,

$$S_p(A) = B(Z,A) - B(Z-1, A-1)$$

proton is a free particle. It does not have B.E.

$$S_p(A) = A\bar{B}(Z,A) - (A-1)\bar{B}(Z-1, A-1) + \bar{B}(Z,A) - \bar{B}(Z,A)$$

$$= \bar{B}(Z,A) + (A-1)\underbrace{\left[\bar{B}(Z,A) - \bar{B}(Z-1, A-1)\right]}_{1}$$

$$= \bar{B}(Z,A) + (A-1)\frac{\partial}{\partial A}\bar{B}(Z,A)$$

(from the defination of derivative)

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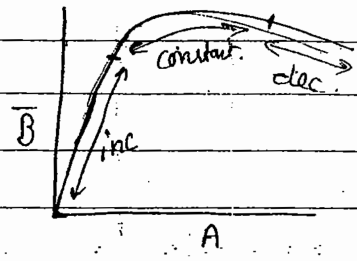
$$\left\{ f' = \frac{f(x) - f(x - \Delta x)}{\Delta x} \right\}$$



→ For light Nuclei

$$\frac{\partial \bar{B}}{\partial A} = +ve$$

hence $\boxed{S_p(A) > \bar{B}(Z, A)}$

→ for heavy Nuclei

$$\frac{\partial \bar{B}}{\partial A} = -ve$$

hence $\boxed{S_p(A) < \bar{B}(Z, A)}$

→ for Moderate nuclei

$$\frac{\partial \bar{B}}{\partial A} = 0$$

$$\boxed{S_p(A) = \bar{B}(Z, A)}$$

Que :- Calculate the separation energy for α-decay.

$${}^{A}_{Z}X \longrightarrow {}^{4}_{2}He + {}^{A-4}_{Z-2}Y$$

$$S_\alpha(A) = B(Z, A) - B(2, 4) - B(Z-2, A-4)$$
$$= A\bar{B}(Z, A) - 4\bar{B}(2, 4) - (A-4)\bar{B}(Z-2, A-4) + 4\bar{B}(Z, A) - 4\bar{B}(Z, A)$$
$$= 4\bar{B}(Z, A) + (A-4)\left[\bar{B}(Z, A) - \bar{B}(Z-2, A-4)\right] - 4\bar{B}(2, 4)$$

Multiply & divide by 4 in IInd term,

$$= 4\bar{B}(Z, A) + 4(A-4)\frac{\left[\bar{B}(Z, A) - \bar{B}(Z-2, A-4)\right]}{4} - 4\bar{B}(2, 4)$$

$$= 4\bar{B}(Z,A) + 4(A-4)\frac{\partial \bar{B}}{\partial A}(Z,A) - 4\bar{B}(2,4)$$

$$= 4\left[\bar{B}(Z,A) - \bar{B}(2,4)\right] + 4(A-4)\frac{\partial \bar{B}}{\partial A}(Z,A)$$

For heavy Nuclei,

$$\bar{B}(Z,A) \simeq 7.5 \text{ MeV}$$

for α-particle, $\bar{B}(2,4) \simeq 7.0$ MeV

$$S_\alpha(A) \simeq \underbrace{2}_{\substack{4(7.5-7.0)\\ 4\times 0.5}} + 4(A-4)\frac{\partial \bar{B}}{\partial A}(Z,A)$$

$$\frac{\partial \bar{B}}{\partial A} = -ve$$

so $\boxed{S_\alpha(A) < 0}$ spontaneous.

Natural radioactivity occur only for heavy nucleus.

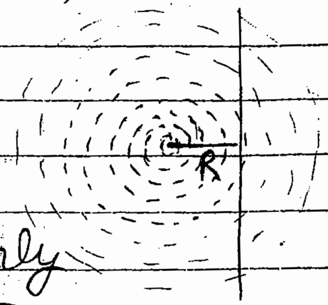
For light Nuclei, $\bar{B}(Z,A) \simeq$

$\boxed{S_\alpha(A) > 0}$ Artificial (we have to supply Energy).

Artificial radioactivity occur only for light nuclei.

<u>Density of Nucleus</u> :- Since there are no sharp boundries for the nucleus therefore to determine the radius, we define

$$\rho(r) = \text{Very High} \quad \text{if } r < R$$
$$= \text{Very Low} \quad \text{if } r > R$$



Considering the spherical shape only it is experimently observed that

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the density inside the sphere is constant.

$$\rho = \text{constant}$$

$$\frac{\text{Mass}}{\text{Volume}} = \text{constant}$$

$$\text{Volume} \propto \text{Mass}$$

$$\frac{4}{3}\pi R^3 \propto \text{mass of one nucleon} \times \underbrace{\text{no. of nucleons}}_{A}$$

$$R^3 \propto \text{No. of Nucleons}$$

$$R^3 \propto A$$

$$R \propto A^{1/3}$$

$$\boxed{R = R_0 A^{1/3}}$$

$R_0 \rightarrow$ Constant

$R_0 \simeq 1.2$ fm

Que:- Radius of $^{64}_{29}Cu$ nucleus is measured to be 4.8×10^{-13} cm. Find the radius of $^{27}_{12}Mg$ Nucleus.

Radius of $^{64}_{29}Cu = 4.8 \times 10^{-13}$

$$R = R_0 A^{1/3}$$

$$4.8 \times 10^{-13} = R_0 (64)^{1/3}$$

$$4.8 \times 10^{-13} = R_0 \times 4$$

$$R_0 = 1.2 \times 10^{-13} \text{ cm}$$

for $^{27}_{12}Mg$, $R = R_0 A^{1/3}$

$$= 1.2 \times 10^{-13} \times (27)^{1/3}$$

$$= 1.2 \times 10^{-13} \times 3$$

$$R = 3.6 \times 10^{-13} \text{ cm}$$

$$\boxed{R = 3.6 \text{ fm}}$$ Ans

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Nucleur Density :-

$$\rho = \frac{Mass}{Volume}$$

$$\text{Number Density} = \frac{\text{No. of Nucleons}}{\text{Volume}}$$

$$= \frac{A}{\frac{4}{3}\pi R^3} = \frac{A}{\frac{4}{3}\pi R_0^3 A} \quad [R = R_0 A^{1/3}]$$

$$= \frac{3}{4\pi R_0^3} = \frac{3}{4 \times 3.14 \times (1.2 \times 10^{-15})^3}$$

$$= 10^{44} \text{ Nucleons}/mt^3$$

Density of Nucleus = No. Density x Mass of one nucleon

$$\rho = 10^{44} \times 1.6 \times 10^{-27} \text{ Kg}$$

$$\boxed{\rho \simeq 10^{17} \text{ Kg}/mt^3}$$

(highly dense)

Qu 1:- If earth would consist of only nuclear matter. What will be its radius if the mass of the earth is 6×10^{24} Kg.

$$\text{Volume} = \frac{\text{mass}}{\text{density (Nucleus)}}$$

$$\frac{4}{3}\pi R^3 = \frac{6 \times 10^{24}}{10^{17}}$$

$$R \simeq 243 \text{ Meters}$$

(Actual $R = 6400000 = 6400 \times 10^3$ meters)

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Que 1- A beam of pions is incident on a proton target

$\pi^+ p \longrightarrow n + \pi^+ + \pi^+$

(i) Assuming that the decay process is strong int. the total isospin I and III component of isospin I_3 are

(a) $I = \frac{3}{2}, I_3 = \frac{3}{2}$ (b) $I = \frac{5}{2}, I_3 = \frac{5}{2}$

(c) $I = \frac{5}{2}, I_3 = \frac{3}{2}$ (d) $I = \frac{1}{2}, I_3 = \frac{-1}{2}$

$$\pi^+ + p \longrightarrow n + \pi^+ + \pi^+$$

$$I \quad 1 \quad \frac{1}{2} \qquad \frac{1}{2} \quad 1 \quad 1$$

$$I_3 \quad \underbrace{+1 \quad +\frac{1}{2}}_{3/2} \qquad \underbrace{-\frac{1}{2} \quad +1 \quad +1}_{3/2}$$

for I, $\underbrace{\frac{1}{2}, \frac{3}{2}} \rightarrow \underbrace{\frac{1}{2} \quad 1}_{\frac{1}{2}, \frac{3}{2}} \quad 1$

$$\left[\frac{1}{2} + 1 = \frac{3}{2}\right]$$

$I = \frac{3}{2}$

(a) is correct.

Que 1- Using Isospin symmetry the cross-section for the above process can be related to that of

(a) $\pi^- n \longrightarrow p \pi^- \pi^-$

(b) $\pi^- \bar{p} \longrightarrow \bar{n} \pi^- \pi^-$

(c) $\pi^+ n \longrightarrow p \pi^+ \pi^-$

(d) $\pi^+ \bar{p} \longrightarrow n \pi^+ \pi^-$

Replace all the particle with their antiparticle as they have same I.

$\pi^+ p \longrightarrow n + \pi^+ + \pi^+$

(b) $\pi^- \bar{p} \longrightarrow \bar{n} \pi^- \pi^-$ correct

Clebsch gordon cofficiat :-

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Mean Square Radius :-

$$\langle r^2 \rangle = \frac{\int r^2 \rho(r)\, d\tau}{\int \rho(r)\, d\tau}$$

$$= \frac{\int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} r^2 \rho(r)\, r^2 \sin\theta\, dr\, d\theta\, d\phi}{\int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} \rho(r)\, r^2 \sin\theta\, dr\, d\theta\, d\phi}$$

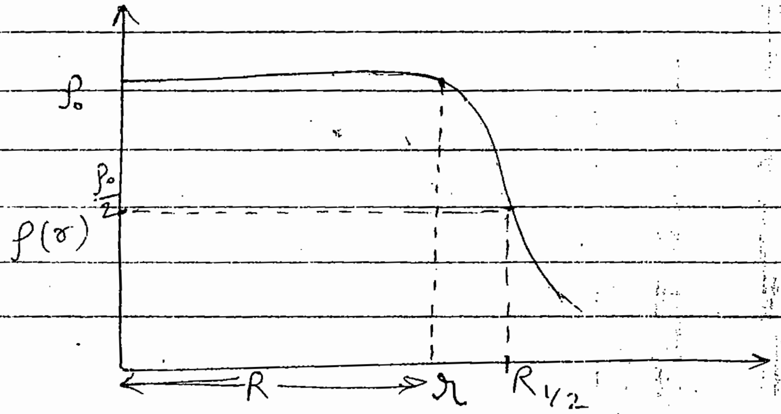
$$= \frac{\int_0^{\infty} r^4\, dr}{\int_0^{\infty} r^2\, dr}$$

Assuming the density of the nucleus to be fairly constant.

$$\langle r^2 \rangle = \frac{\left[\frac{r^5}{5}\right]_0^{\infty}}{\left[\frac{r^3}{3}\right]_0^{\infty}} \qquad \rho(r) = \text{const.},\ r < R$$
$$\qquad\qquad\qquad\qquad = 0 \qquad r > R$$

$$= \frac{\int_0^{R} r^4\, dr + \int_R^{\infty} r^4\, dr}{\int_0^{R} r^2\, dr + \int_R^{\infty} r^2\, dr} = \frac{\frac{R^5}{5}}{\frac{R^3}{3}}$$

$$\boxed{\langle r^2 \rangle = \frac{3}{5} R^2}$$



Experimental curve

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The density funⁿ corresponding to this experimental curve will be given by

$$\rho(r) = \frac{\rho_0}{1 + \exp\left[\frac{r - R_{1/2}}{a}\right]}$$

This is called Fermi Distribution.

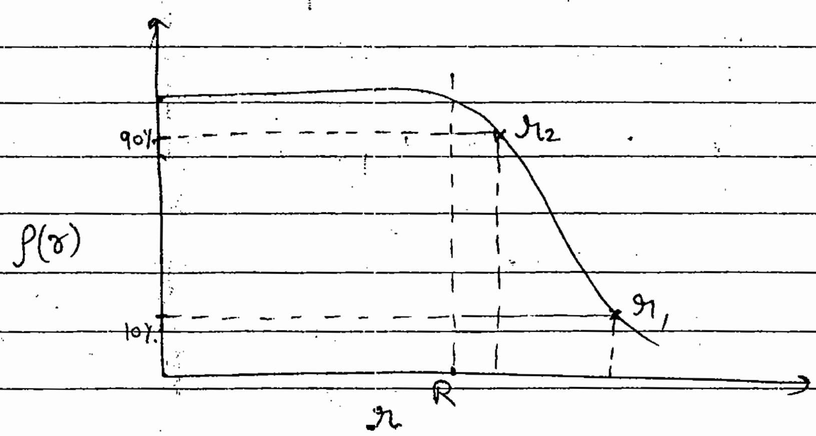
$\Rightarrow$ $R_{1/2} \rightarrow$ That value of distance at which density fall (or remains) $1/2$ of its value.

Ques :- The radial dependence of the density of the nucleus may be described by

$$\rho(r) = \frac{\rho_0}{1 + \exp\left\{\frac{r - C}{Z}\right\}}$$

Show that the parameter C is the radius of the nucleus measured from the centre to a point where density falls to roughly half of its maximum value and the parameter, $Z = \frac{t}{4.4}$ where t is the radial distance b/w two points of the nucleus whose density are 10 % & 90 % of its maximum value.

Solⁿ :-



$r_1 - r_2 = Z$

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$$\rho(r) = \frac{\rho_0}{2}$$

$$\therefore \quad \frac{\rho_0}{2} = \frac{\rho_0}{1+\exp\left(\frac{r-c}{z}\right)}$$

$$1+\exp\left(\frac{r-c}{z}\right) = 2 \Rightarrow \exp\left(\frac{r-c}{z}\right) = 1$$

$$\frac{r-c}{z} = \ln 1 \Rightarrow \frac{r-c}{z} = 0$$

$$\Rightarrow r-c = 0$$

$$\Rightarrow \boxed{r = c}$$

Say,

at $r = r_1$ $\quad \rho(r) = 10\%$ of ρ_0

$= 0.1\,\rho_0$

at $r = r_2$ $\quad \rho(r) = 90\%$ of ρ_0

$= 0.9\,\rho_0$

Now $\quad 0.1\,\rho_0 = \dfrac{\rho_0}{1+\exp\left(\frac{r-c}{z}\right)}$

$$1+\exp\left(\frac{r_1-c}{z}\right) = 1/0.1 \quad \text{or} \quad 1+\exp\left(\frac{r_1-c}{z}\right) = 10$$

$$\exp\left(\frac{r_1-c}{z}\right) = 9 \quad \text{——— (1)}$$

IIly, $\quad 1+\exp\left(\frac{r_2-c}{z}\right) = \cancel{10}\ 1/0.9 \quad \text{or} \quad 1+\exp\left(\frac{r_2-c}{z}\right) = \frac{10}{9}$

$$\exp\left(\frac{r_2-c}{z}\right) = \frac{1}{9} \quad \text{——— (2)}$$

Devide (1) by (2) $\Rightarrow$

$$\exp\left\{\frac{r_1-r_2}{z}\right\} = 9^2$$

Taking Natural log of both sides,

$$\frac{r_1-r_2}{z} = 2\ln 9 \Rightarrow \frac{r_1-r_2}{z} = 4 \times \ln 3$$

$$\frac{r_1-r_2}{z} = 4.39 \Rightarrow \boxed{z = \frac{r_1-r_2}{4.39} = \frac{t}{4.39} \simeq \frac{t}{4.4}}$$

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Que :- For the charge density $\rho(r) = \rho_0 e^{-r/a}$ show that the R.M.S. radius of the nucleus is $a\sqrt{12}$.

Avg. value of square of radius i.e.

R.M.S. radius

$$\langle r^2 \rangle = \frac{\int r^2 \rho(r)\, d\tau}{\int \rho(r)\, d\tau}$$

$$\langle r^2 \rangle = \frac{\int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} r^2 \rho_0 e^{-r/a}\, r^2 \sin\theta\, dr\, d\theta\, d\phi}{\int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} \rho_0 e^{-r/a} r^2 \sin\theta\, dr\, d\theta\, d\phi}$$

$$= \frac{\int_0^{\infty} r^2 \rho_0 e^{-r/a}\, \cancel{4\pi} r^2\, dr}{\int_0^{\infty} \rho_0 e^{-r/a} \cancel{4\pi} r^2\, dr} = \frac{\int_0^{\infty} r^4 e^{-r/a}\, dr}{\int_0^{\infty} r^2 e^{-r/a}\, dr}$$

$\left\{ \Gamma\text{-func}^n,\ \Gamma n = \int_0^{\infty} e^{-x} x^{n-1}\, dx \right\}$

$\frac{r}{a} = t$

$dr = a\,dt$

$$\langle r^2 \rangle = \frac{\int_0^{\infty} e^{-t} (t)^{5-1} \cdot a^5\, dt}{\int_0^{\infty} e^{-t} (t)^{3-1} a^3\, dt}$$

$$= \frac{a^5 \Gamma 5}{a^3 \Gamma 3} = \frac{a^2 \cdot \Gamma 4+1}{\Gamma 3} = \frac{a^2\, 4\, \Gamma 4}{\Gamma 3}$$

$$= \frac{a^2\, 4 \times 3 \times \cancel{\Gamma 3}}{\cancel{\Gamma 3}} = 12 a^2 \qquad \left\{ \Gamma n+1 = n \Gamma n \right\}$$

$$\boxed{\langle r \rangle = a\sqrt{12}}$$ Ans

Que :- Assuming that the nuclear radius is given by $R = R_0 A^{1/3}$. Find the no. of neutron density in the nucleus with no. of neutrons = no. of protons.

$Z = N$

$A = Z + N \Rightarrow A = N + N \Rightarrow N = \frac{A}{2}$

$$\text{No. of Neutron density} = \frac{\text{No. of Neutrons}}{\text{volume of nucleus}}$$

$$= \frac{A/2}{\frac{4}{3}\pi R^3} = \frac{\cancel{A}}{2\times\frac{4}{3}\pi R_0^3 \cancel{A}}$$

$$= \frac{3}{8\pi R_0^3}$$

$$= \frac{3}{8(3.14)(1.2\times10^{-15})^3}$$

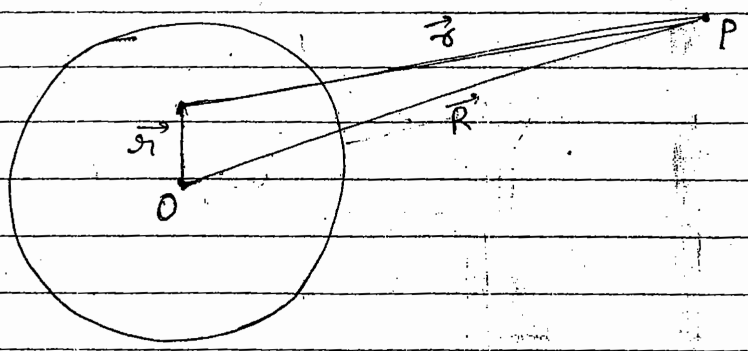
$$= \frac{3\times10^{45}}{8\times3.14\times(1.2)^3}$$

$$= 0.069 \text{ no. of neutrons}/fm^3$$

Quadrupole Moment :- Q.M. define the shape of nucleus. By defination

$Q = 0$	spherical Shape	
$Q > 0$	prolate "	
$Q < 0$	Oblate "	

Consider a charge distribution of the nucleus



$$\vec{r'} + \vec{r} = \vec{R}$$
$$\vec{r} = \vec{R} - \vec{r'}$$
$$\& \quad \vec{R} \gg \vec{r'}$$

Pot^n at P, $\phi(\vec{R}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')\,d\tau}{|\vec{R}-\vec{r'}|}$

$$|\vec{R}-\vec{r'}| = (R^2 + r'^2 - 2r'R\cos\theta)^{1/2}$$

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$$\frac{1}{|R-r|} = \frac{1}{R}\left[1+\frac{r^2}{R^2}-\frac{2r\cos\theta}{R}\right]^{-1/2}$$

Expand binoimally

$$= \frac{1}{R}\left[1-\frac{r^2}{2R^2}+\frac{r\cos\theta}{R}+\frac{3}{2}\frac{r^2\cos^2\theta}{R^2}+\cdots\right]$$ higher order term $+ O\left(\frac{r^3}{R^3}\right)$

$$= \frac{1}{R}\left[1+\frac{r\cos\theta}{R}+\frac{(3\cos^2\theta-1)r^2}{2R^2}+\cdots\cdots\right]$$

So
$$\phi(\vec{R}) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{R}+\frac{1}{R^2}\int \rho(r)\, r\cos\theta\, d\tau + \frac{1}{2R^2}\int r^2\rho(r)(3\cos^2\theta-1)\, d\tau + \cdots\cdots\right]$$

(integration only for source charge)

Here, I, II, III terms denote the monopole, dipole & Qudrupole terms.

Let $\vec{r} \equiv x_i$

$\vec{R} \equiv X_i$

$$r\cos\theta = \frac{\vec{r}\cdot\vec{R}}{R} = \frac{\sum_i x_i X_i}{R}$$

Monopole term, $= \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

Dipole term, $= \frac{1}{4\pi\epsilon_0}\frac{1}{R^2}\int \rho(r)\frac{\vec{r}\cdot\vec{R}}{R}\, d\tau$

$$= \frac{1}{4\pi\epsilon_0}\frac{1}{R^3}\,\vec{R}\cdot\int \rho(\vec{r})\,\vec{r}\, d\tau$$

$$= \frac{1}{4\pi\epsilon_0}\frac{1}{R^3}\,\vec{R}\cdot\vec{p}$$

where $\underbrace{\vec{p}}_{\text{dipole Mom.}} \equiv \int \rho(\vec{r})\,\vec{r}\, d\tau$

In discrete system, $= \frac{1}{4\pi\epsilon_0}\frac{1}{R^3}\sum_i p_i R_i$

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higher order term of $\left(\frac{r^3}{R^3}\right)$

$d\tau$

le

⇒ If we expand the potⁿ of nucleus then we find monopole term, dipole term & also qudrupole term — this qudropole term will be deciding term for the shape of nucleus.

Similarly the Quadrupole term,

$$= \sum_{ij} \frac{1}{2} \frac{1}{4\pi\epsilon_0} Q_{ij} \frac{x_i X_j}{R^4}$$

where

$$Q_{ij} = \int \rho(\vec{r})(3x_i x_j - r^2 \delta_{ij})\, dz$$

$\downarrow$ Qudrupole Mom.

$\delta_{ij} = 1, \; i=j$
$= 0 \;;\; i \neq j$

Qudrupole Mom. is a ~~ø~~ Tensor of rank 2 having 9 components.

So we can write Q in form of a Matrix.

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{bmatrix}_{3\times 3}$$

$$Q = \begin{bmatrix} 3x^2 - r^2 & 3xy & 3xz \\ 3yx & 3y^2 - r^2 & 3yz \\ 3zx & 3zy & 3z^2 - r^2 \end{bmatrix}$$

If we diogonalize the matrix by changing the frame of reference by x, y, z to $\bar{x}, \bar{y}, \bar{z}$ then all the digonal term will servive & rest terms will be zero.

$$Q = \begin{bmatrix} 3\bar{x}^2 - r^2 & 0 & 0 \\ 0 & 3\bar{y}^2 - r^2 & 0 \\ 0 & 0 & 3\bar{z}^2 - r^2 \end{bmatrix}$$

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So the z-component of the Quadrupole momento,

$$Q_{zz} = \int \rho(r)\,(3\bar{z}^2 - r^2)\,d\tau$$

selection of Quardinate system $(\bar{x}, \bar{y}, \bar{z})$ are arbitrary so we can write,

$$\boxed{Q_{zz} = \int \rho(r)(3z^2 - r^2)\,d\tau}$$

for the complete spherical shape,

$$x^2 = y^2 = z^2$$

$$Q_{zz} = \int \rho(r)\,(3z^2 - (z^2 + z^2 + z^2))\,d\tau$$

$$\boxed{Q_{zz} = 0}$$

for Prolate Shape $\quad 3z^2 > r^2 \quad , \quad Q_{zz} > 0$

for Oblate shape $\quad 3z^2 < r^2 \quad , \quad Q_{zz} < 0$

Nuclear Reactions :-

$$\underset{\text{Projectile}}{x} + \underset{\text{Target}}{X} \longrightarrow Y + y$$

$$X\,(x\;y)\,Y$$

for Relativistic Motion,

total Energy of particle,

$$E = K.E. + \text{Rest mass energy}$$

$$E = \sqrt{p^2c^2 + m_0^2 c^4}$$

If particle at rest, $(p=0)$ $E = \sqrt{m_0^2 c^4} = m_0 c^2$

If particle moving with c, $E = \sqrt{p^2 c^2} = pc$

$m_0 = 0$

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If Total Energy & K.E. is conserved - Elastic Collision
If K.E. is Not conserved - Inelastic
→ In Inelastic rec^n - The mass of product is more that the reactants bcoz K.E. of x is converted into mass.

Q- Value :-

$$Q = (\Sigma m_i - \Sigma m_f) c^2$$

$Q \equiv +ve$ Exoergic Rec^n (elastic collision)
$Q \equiv -ve$ Endo~~rgic~~ thermic Rec^n (Inelastic collision)

Consider a rec^n,

$${}^{b}_{a}x + {}^{A}_{Z}X \longrightarrow {}^{A'}_{Z'}Y + {}^{b'}_{a'}y$$

by Conservation of mass and proton No.

$$b + A = A' + b'$$
$$a + Z = Z' + a'$$

$$Q = [m_x + m(X) - m(Y) - m(y)] c^2 \quad \text{——— (1)}$$

Now,
from the conservation of Energy
{ for ${}^{b}_{a}x$, Total Energy = K.E. + Rest enrgy $= T_x + m_x c^2$
& 11ly for all }

$$m_x c^2 + T_x + m(X) c^2 + T_X = m(Y) c^2 + T_Y + m(y) c^2 + T_y$$

$$[m_x - m(X) - m(Y) - m(Z)] c^2 = T_Y + T_y - T_X - T_x$$

$$\boxed{Q = T_Y + T_y - T_X - T_x} \quad \text{——— (2)}$$

$Q =$ K.E. of products $-$ K.E. of reactants

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If Target is at rest, $T_X = 0$

$$Q = T_Y + T_y - T_x$$

Consider a reac^n,

$$ {}^{b}_{a}x + {}^{A}_{Z}X \longrightarrow {}^{b+A}_{a+Z}Y $$

Q-value of this reac^n,

$$Q = m_x + m(X) - M(Y)$$

$$Q = a M_H + (b-a) M_n - B(x) + Z M_H + (A-Z) M_n - B(X)$$
$$- (a+Z) M_H - (A+b-a-Z) M_n + B(Y)$$

$$\boxed{Q = B(Y) - B(X) - B(x)}$$

Q value in terms of B.E.

Ques :- (a) Compute the Q-value of the reaction

$$ {}^{2}_{1}H + {}^{63}Cu \longrightarrow n + {}^{64}Zn $$

(b) Deutrons of energy 12 MeV are incident on a ^{63}Cu target, neutrons are observed with 16.85 MeV of K.E. Find the K.E. of ^{64}Zn if mass of $^{2}_{1}H \equiv 2.014102\ u$, $M(n) = 1.008665\ u$, $M(^{63}Cu) = 62.929599\ u$, $M(^{64}Zn) = 63.929145\ u$

(a) $$Q = [M(^{2}_{1}H) + M(^{63}Cu) - M(n) - M(^{64}Zn)]\, c^2$$

$$= [2.014102 + 62.929599 - 1.008665 - 63.929145]$$

$$= [64.943701 - 64.937810]\, c^2 \times 931.502\ \text{MeV}$$

$$= 0.005891 \times 931.502\ \text{MeV}$$

$$\boxed{Q = 5.487\ \text{MeV}}$$ Ans

Q → +ve so Reac^n is Exothermic.

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(b) $Q = T_y + T_Y - T_x - \cancel{x}$ [Target at rest]

$\cancel{Q = T_y + T_Y - T_x}$

$= K.E.(Zn) + K(n) - K({}^2_1H)$

$5.487 = K(Zn) + 16.85 - 12$

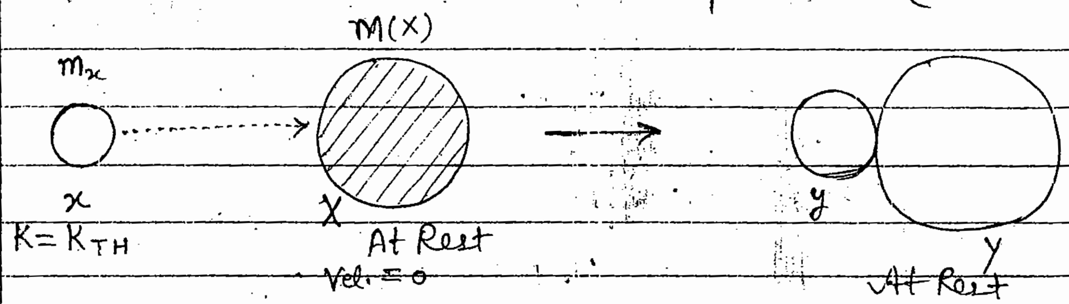
$K(Zn) = 5.487 - 4.85$

$\boxed{K(Zn) = 0.637 \text{ MeV}}$ Ans

<u>Note</u> :- Reaction for which $Q > 0$ the nuclear energy will be converted into the K.E. of y & Y and are called <u>Exothermic Reaction</u>. Reaction with <u>$Q < 0$</u> required the input energy in the form of K.E. of x to be converted into nuclear energy. These are called <u>Endothermic reaction</u>.

<u>Thresold Value of K.E. for Endothermic Reaction :- (K_{TH})</u>

There is a minimum value of the K.E. called the thresold K.E. (K_{TH}) below which the endothermic reaction will not proceed (And K_{TH} of x)



This problem can be solved in centre of mass frame of reference.

(1) In the Lab frame of reference the velocity of the centre of mass will be ~~zero~~.

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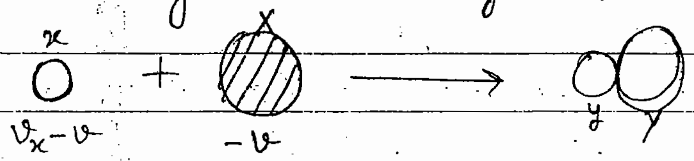
$$r_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$\frac{d}{dt}(r_{cm}) = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \qquad V_X = 0$$

$$= \frac{m_x v_x}{m(x) + m(X)}$$

(2) In C.M. frame of ref. we would see the projectile x moving with velocity $v_x - v$. And the target will be moving with velocity $-v$.



Therefore,
from Conservation of Energy

$$E_x + E_X = E_y + E_Y$$

$$m_x c^2 + \frac{1}{2} m_x (v_x - v)^2 + m(X)c^2 + \frac{1}{2} m(X) v^2 = m_y c^2 + m(Y) c^2$$

$$\cancel{m_x} + \frac{1}{2} m_x \left\{ v_x - \frac{m_x v_x}{m(x) + m(X)} \right\}^2 + \frac{1}{2} m(X) \left\{ \frac{m_x v_x}{m_x + m(X)} \right\}^2 = m_y c^2 + m(Y)c^2 - m_x c^2 - m(X)c^2$$

$$\Rightarrow \frac{1}{2} m_x \left\{ \frac{M(X) v_x}{m_x + M(X)} \right\}^2 + \frac{1}{2} M(X) \left\{ \frac{m_x v_x}{m_x + m(X)} \right\}^2 = -Q$$

$$\Rightarrow \frac{1}{2} m_x M(X) \left\{ \frac{M(X) + m_x}{(m_x + M(X))^2} \right\} v_x^2 = -Q$$

$$\Rightarrow \frac{1}{2} \frac{m_x M(X)}{m_x + M(X)} v_x^2 = -Q$$

$$\Rightarrow K_{TH} = \frac{-Q \cdot \{ m_x + M(X) \}}{m(X)}$$

$\left\{ \frac{1}{2} m_x v_x^2 = K_{TH} \right.$ min. K.E. of x $\}$

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$$\Rightarrow \boxed{K_{TH} = -Q\left[1 + \frac{m_x}{m(X)}\right]}$$

K.E. can't be negative. This is a special case of Endothermic reⁿ for which Q is -ve.

<u>Que</u> 1:- Calculate the thresold kinetic energy for the reaction

projectile
target

$$p + {}^3_1H_2 \longrightarrow {}^2_1H_1 + {}^2_1H_1$$

a) If protons are incident on ${}^3_1H$ at rest
b) If ${}^3_1H$ i.e. tritons are incident on proton.

Mass of $p = 1.007825\ u$
${}^3H = 3.016049\ u$
${}^2H = 2.014102\ u$

(a)
$$Q = [M(p) + M({}^3_1H_2) - M({}^2H) - M({}^2H)] \times 931.502\ MeV$$
$$= 1.007825 + 3.016049 - 2.014102 - 2.014102$$
$$= 0.00433\ u \times 931.502\ MeV$$
$$= -4.0334037\ MeV$$

2

$$K_{TH} = -Q\left[1 + \frac{m_x}{M(X)}\right]$$

$$= 4.0334037\left[1 + \frac{1.007825}{3.016049}\right]$$

$$= 5.38118188 \simeq \underline{5.381\ MeV\ Ans}$$

(b)
$$Q = [M({}^3H) + M(p) - M({}^2H) - M({}^2H)] \times 931.502\ MeV$$
$$Q = -4.0334037\ MeV$$

$$K_{TH} = 4.0334037\left[1 + \frac{3.016049}{1.007825}\right]$$

$$K_{TH} = 16.1038953 \simeq \underline{16.10\ MeV\ Ans}$$

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p is lighter particle then ^3_1H. Thus less energy (thresold energy) is required in the case where p is incident on ^3_1H to proceed the reeⁿ

* <u>Energetics of the particle decay:-</u> If a particle x is at rest & it is being decay into several other particles

$$x \longrightarrow a + b + c$$

i) → The decay will occur only if $Q > 0$.
ii) The available Q energy is shared as a kinetic energy of the decay products in order to conserve the linear Momentum.

<u>Que</u> :- Compute the energies of proton & π mesons that result from the decay of Λ°.

$$\Lambda^\circ \longrightarrow p + \pi^-$$

$m(\Lambda^\circ) = 1116$ MeV
$m(p) = 938$ MeV
$m(\pi^-) = 140$ MeV

$Q = m(\Lambda^\circ) - m(p) - m(\pi^-)$
$Q = 1116 - 938 - 140 = 1116 - 1078$ MeV
$Q = 38$ MeV ——— (1)

~~In terms of K.E., The Q-value will be~~
~~$Q = K_p + K_{\pi^-} = 38$~~ (∵ Λ° is at rest) ——— (2)

Now using the relativistic formula

$E = K.E + $ Rest mass energy

$E = \sqrt{p^2c^2 + m_0^2c^4}$

Eqⁿ (2) ⇒ $\sqrt{p_p^2c^2 + m_p^2c^4} - m_pc^2 + \sqrt{p_\pi^2c^2 + m_\pi^2c^4} - m_\pi c^2 = 38$ ——— (3)

K.E of p = Total energy of p − rest mass energy of p

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Now, from the conservation of linear Mom.,

$$0 = \vec{p}_p + \vec{p}_{\pi^-}$$

$$\vec{p}_p = -\vec{p}_{\pi^-}$$

or $|\vec{p}_p| = |\vec{p}_{\pi^-}|$

generally; $p_p = p_{\pi^-}$

So eq^n (3) becomes,

$$\sqrt{p_p^2c^2 + m_p^2c^4} - m_p c^2 + \sqrt{p_p^2c^2 + m_\pi^2 c^4} - m_\pi c^2 = 38$$

$$\Rightarrow \sqrt{(m_p c^2)^2 + p_p^2c^2} + \sqrt{(m_\pi c^2)^2 + p_p^2 c^2} = 38 + 938 + 140 = 1116$$

$$\Rightarrow \sqrt{(938)^2 + p_p^2c^2} = 1116 - \sqrt{(140)^2 + p_p^2c^2}$$

$$\Rightarrow (938)^2 + \cancel{p_p^2c^2} = (1116)^2 - (140)^2 + \cancel{p_p^2c^2} - 2 \times 1116\sqrt{(140)^2 + p_p^2c^2}$$

$$\Rightarrow 2 \times 1116 \times \sqrt{(140)^2 + p_p^2c^2} = (1116)^2 + (140)^2 - (938)^2$$

$$\Rightarrow p_p^2c^2 = 10185.93 \text{ MeV}$$

$$\left.\begin{aligned} p_p &\cong \frac{101}{c} \quad \frac{\text{MeV}}{\text{velocity unit}} \\ p_\pi &\cong \frac{101}{c} \quad \text{"} \end{aligned}\right\}$$

K.E. of p $= \sqrt{p_p^2c^2 + (m_p c^2)^2} - m_p c^2$

$= \sqrt{10185.93 + (938)^2} - 938$

$\cong 5$ MeV

Similarly, K.E. of π^- $= \sqrt{p_\pi^2 c^2 + (m_{\pi^-} c^2)^2} - m_{\pi^-} c^2$

$= \sqrt{10185.93 + (140)^2} - 140$

$\cong 32.58$ MeV

- γ, ν_e, $\bar{\nu}_e$, ν_μ, $\bar{\nu}_\mu$, ν_τ, $\bar{\nu}_\tau$ all have rest mass $= 0$ & only have K.E. & move with speed of light c.

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Que :- What is the maximum K.E. of the electron emitted in the decay mode is $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

Given that $m_{\mu^-} = 105.7$ MeV

$m_{e^-} = 0.511$ MeV

Prove that, the K.E. of e^-'s $= \dfrac{Q^2}{2 m_\mu c^2}$

$$Q = m(\mu^-) - m(e^-) - m(\bar{\nu}_e) - m(\nu_\mu)$$

$$Q = 105.7 - 0.511 - 0 - 0$$

$$Q = 105.189 \text{ MeV}$$

In terms of K.E.,

$$Q = KE_{e^-} + KE_{\bar{\nu}_e} + KE_{\nu_\mu} - 0$$

{K.E. of $\mu^- = 0$ (at rest)}

$$KE_{e^-} + KE_{\bar{\nu}_e} + KE_{\nu_\mu} = 105.189 \quad \text{——— (1)}$$

The energy of the e^- will be maximum when minimum amount of energy is given to the neutrino.

There may be 2 possibilities :-

⇒ The little amount of energy is equally shared by both neutrino.

⇒ The min. amount of energy is given to one of the neutrino & the little amount of energy is not equally shared by both neutrino.

If particle moving with c then energy → $E = pc$

$$= |p| c$$

~~The~~ Let E_e & p_e be the energy & momentum of the e^- & E_ν & p_ν be the energy & mom. of neutrino.

By Mom Con. $\quad 0 = \vec{p}_e + \vec{p}_\nu$

$$\vec{p}_e = -\vec{p}_\nu$$

K.E.

$$|\vec{p}_e| = |\vec{p}_\nu|$$

$$p_e = p_\nu$$

Therefore $Q = (m_\mu - m_{e^-})c^2$ —(2)

& $Q = KE(e^-) + KE(\nu)$

$$Q = E_e - m_e c^2 + p_\nu c$$ {bcoz rest energy of $\nu = 0$

$$Q = E_e - m_e c^2 + p_e c$$

$\because$ $E_e = \sqrt{p_e^2 c^2 + m_e^2 c^4}$

$$E_e^2 = p_e^2 c^2 + m_e^2 c^4$$

$$E_e^2 - m_e^2 c^4 = p_e^2 c^2$$

$$\therefore \quad Q = E_e - m_e c^2 + \sqrt{E_e^2 - m_e^2 c^4}$$

$$m_\mu c^2 - \cancel{m_e c^2} = E_e - \cancel{m_e c^2} + \sqrt{E_e^2 - m_e^2 c^4}$$ {from (2)

$$(m_\mu c^2) - E_e = \sqrt{E_e^2 - m_e^2 c^4}$$

$$(m_\mu c^2)^2 + \cancel{E_e^2} - 2 m_\mu c^2 E_e = \cancel{E_e^2} - m_e^2 c^4$$

$$2 m_\mu c^2 E_e = (m_\mu c^2)^2 + (m_e c^2)^2$$

$$2 m_\mu c^2 E_e = \{(m_\mu c^2) - (m_e c^2)\}^2 + 2 m_\mu m_e c^4$$

$$2 m_\mu c^2 E_e = Q^2 + 2 m_\mu m_e c^4$$

$$E_e = \frac{Q^2}{2 m_\mu c^2} + \frac{2 m_\mu m_e c^4}{2 m_\mu c^2}$$

$$\boxed{E_e = \frac{Q^2}{2 m_\mu c^2} + m_e c^2}$$

K.E. of e $= E_e - m_e c^2$

$$\boxed{KE = \frac{Q^2}{2 m_\mu c^2}}$$

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Energetics of the Particle Reactions :-

If we have 2 particles of mass m_1 & m_2. m_1 collides with m_2 which is at rest. In that case

$$m_1 + \underset{\substack{(\text{Target})\\ \text{at rest}}}{m_2} \longrightarrow m_3 + m_4 + m_5 + \cdots$$

The thresold energy (thresold K.E.) will be

$$K_{TH} = -Q\ \frac{m_1 + m_2 + m_3 + m_4 + \cdots}{2 m_2}$$

$$\boxed{K_{TH} = -Q\ \frac{\text{Sum of the masses of all the particles}}{2 \times \text{Mass of the target}}}$$

Total Energy = K.E. + Rest Mass Energy

$$\boxed{E_{TH} = K_{TH} + \text{Rest Mass Energy}}$$

In this case, the rest mass energy $= m_1 c^2$

So $\boxed{E_{TH} = K_{TH} + m_1 c^2}$

Que :- Calculate the thresold K.E. p to produce π-meson from the reᶜⁿ

$$p + p \longrightarrow p + p + \pi^0$$

$m(p) = 938$ MeV
$m(\pi^0) = 135$ MeV

$$Q = [2 \times m(p) - 2 \times m(p) - m(\pi^0)]\ c^2$$

$$Q = [-135 \text{ MeV}]$$

$$K_{TH} = -Q \times \frac{m(p) + m(p) + m(p) + m(p) + m(\pi^0)}{2 \times m(p)}$$

$$= 135 \times \frac{4 \times 938 + 135}{2 \times 938} = 135 \times \frac{3752 + 135}{1876}$$

$$= 135 \times \frac{3887}{1876} = 279.714819 \simeq \underline{280 \text{ MeV}}$$

Qus In 1956, An experiment was performed at Workley to search for the antiproton which could be produced in the reer

$$p + p \longrightarrow p + p + p + \bar{p}$$

what is the thresold energy of the reen,

$$Q = [2 \times m(p) - 3\, m(p) - m(\bar{p})]c^2$$
$$= -2 \times m(p)c^2 = -2 \times 938 \text{ MeV} = -1876 \text{ MeV}$$

$$K_{TH} = -Q \times \frac{\cancel{m(p) + m(p) + m(p) + m(p) + m(p) + m(\bar{p})}}{\cancel{2 \times m(p)}}$$

$$= 1876 \times \frac{6 \times 938}{2 \times 938} = \underbrace{1876}_{+2m_pc^2} \times 3$$

$$= \cancel{5628 \text{ MeV}} \simeq 6 m_p c^2$$

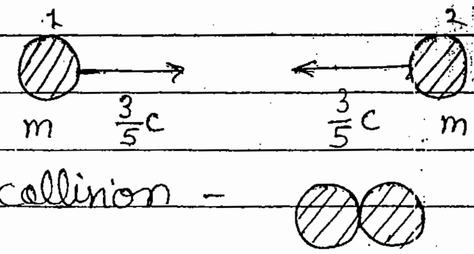
$$E_{TH} = K_{TH} + m_p c^2 = 5678 + m_p c^2$$
$$= 6 m_p c^2 + m_p c^2$$

$$\boxed{E_{TH} = 7 m_p c^2}$$

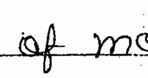
Collisions :-

Qus 1- 2 lumps of clay each of mass m collide head-on at $\frac{3}{5}c$. They stick together. What is the mass M of the final ~~component~~ composite lump?

before collision -



After collision -



from Conservation of mom.

$$\vec{p_1} + \vec{p_2} = \vec{p_M}$$
$$\Rightarrow \vec{p_1} + \vec{p_2} = 0$$

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Con. of energy, $E_1 + E_2 = E_M$

$$mc^2 + mc^2 = Mc^2$$

$$\frac{2m_0 \cancel{c^2}}{\sqrt{1-V^2/c^2}} = M\cancel{c^2}$$

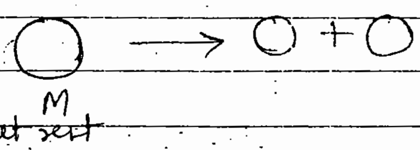
$$M = \frac{2m_0}{\sqrt{1-\frac{9}{25}}} = \frac{2m_0}{\sqrt{\frac{16}{25}}}$$

$$\boxed{M = \frac{5}{2}m_0} \text{ Ans.} \simeq \boxed{M = 2.5m_0}$$

⇒ Here K.E. of M is being converted into mass. bcoz $m+m = \boxed{2m = M}$ ideally but here $M = \frac{5}{2}m_0$

Que: A particle of mass M initially at rest decays into 2 pices each of mass m. What is the speed of each piece as it flies off?

$$M \longrightarrow m + m$$



By Energy Conservation,

$$E = E_1 + E_2$$

$$Mc^2 = \frac{mc^2}{\sqrt{1-V^2/c^2}} + \frac{mc^2}{\sqrt{1-V^2/c^2}}$$

$$\sqrt{1-V^2/c^2} = \frac{2m}{M}$$

$$1 - \frac{V^2}{c^2} = \left(\frac{2m}{M}\right)^2 \Rightarrow \frac{V^2}{c^2} = 1 - \left(\frac{2m}{M}\right)^2$$

$$\boxed{v = c\sqrt{1-\left(\frac{2m}{M}\right)^2}}$$

Que: A Tau particle is at rest & decays into

$$\tau^- \longrightarrow \pi^- + \nu_\tau$$

The masses of τ^- & π^- & ν_τ are m_τ, m_π & 0 respectively.

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find the energy & velocity of π^-.

By con. of mom.

$$\cancel{p_\tau} = \cancel{p_\pi - \nu} \qquad 0 = \vec{p}_{\pi^-} + \vec{p}_{\nu_e}$$

$$\vec{p}_{\pi^-} = -\vec{p}_{\nu_e}$$

$$\Rightarrow |\vec{p}_{\pi^-}| = |\vec{p}_{\nu_e}|$$

$$\Rightarrow p_{\pi^-} = p_{\nu_e}$$

from the con. of energy

$$E_{\tau^-} = E_{\pi^-} + E_\nu$$

bcoz τ is at rest so it'll have only rest mass energy

$$m_\tau c^2 = \sqrt{p_{\pi^-}^2 c^2 + m_\pi^2 c^4} + p_\nu c \qquad (p_\nu = p_\pi)$$

$$m_\tau c^2 = \sqrt{p_\pi^2 c^2 + m^2 c^4} + p_\pi c$$

$$(m_\tau c^2 - p_\pi c) = \sqrt{p_\pi^2 c^2 + m^2 c^4}$$

$$(m_\tau c^2 - p_\pi c)^2 = p_\pi^2 c^2 + m_\pi^2 c^4$$

$$(m_\tau c - p_\pi)^2 = p_\pi^2 + m_\pi^2 c^2$$

$$m_\tau^2 c^2 + \cancel{p_\pi^2} - 2 m_\tau c p_\pi = \cancel{p_\pi^2} + m_\pi^2 c^2$$

$$2 m_\tau c p_\pi = (m_\tau^2 - m_\pi^2) c^2$$

$$\boxed{p_\pi = \frac{(m_\tau^2 - m_\pi^2) c}{2 m_\tau}}$$

$$E_\pi = \sqrt{p_\pi^2 c^2 + m_\pi^2 c^4}$$

$$= \sqrt{\frac{(m_\tau^2 - m_\pi^2)^2 c^4}{4 m_\tau^2} + m_\pi^2 c^4}$$

$$= c^2 \sqrt{\frac{(m_\tau^2 - m_\pi^2)^2 + 4 m_\tau^2 m_\pi^2}{4 m_\tau^2}}$$

$$= c^2 \sqrt{\frac{(m_\tau^2 + m_\pi^2)^2}{4 m_\tau^2}}$$

$$\boxed{E_\pi = \frac{(m_\tau^2 + m_\pi^2) c^2}{2 m_\tau}}$$

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Velocity :-

for the particle moving relativistically

$$E = \frac{m}{\sqrt{1-v^2/c^2}}c^2 \quad \& \quad p = \frac{m}{\sqrt{1-v^2/c^2}}v$$

So $$\frac{E}{p} = \frac{mc^2}{mv}$$

$$\Rightarrow \quad v = \frac{pc^2}{E}$$

$$v = \frac{\frac{(m_\tau^2 - m_\pi^2)c \times \cancel{c^2}}{\cancel{2m_\tau}}}{\frac{(m_\tau^2 - m_\pi^2)\cancel{c^4}}{\cancel{2m_\tau}}}$$

$$\boxed{v = \left\{\frac{m_\tau^2 - m_\pi^2}{m_\tau^2 + m_\pi^2}\right\}c}$$

Que :- An atom of mass M can be excited to a state of mass $M+\Delta$ by photon capture. The freq. of photon which can cause this transition is

(a) $\Delta c^2/2h$ (b) $\Delta c^2/h$ (c) $\frac{\Delta^2 c^2}{2mh}$ (d) $\frac{\Delta(\Delta+2m)c^2}{2mh}$

$$\underset{\substack{\text{Photon}\\\text{projectile}}}{\gamma} + \underset{\substack{M\\\text{target}}}{m_i} \longrightarrow \underset{(M+\Delta)}{m_f}$$

We need to calculate the thresold ~~the~~ K.E. or energy to initiate this reac^n..

$$Q = [m(\gamma) + \cancel{m(x)}\ M - (M+\Delta)]c^2$$

$$Q = 0\ [+M - M - \Delta]c^2 = -\Delta c^2$$

$$K_{TH} = -Q \times \frac{M + M + \Delta}{2M} = \Delta c^2 \frac{(2M+\Delta)}{2M}$$

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$$\therefore \quad h\nu = \Delta c^2 \frac{(2M+\Delta)}{2M}$$

$$\boxed{\nu = \frac{\Delta(2M+\Delta)c^2}{2Mh}}$$ Ans.

(d) is correct.

- If a particle A of energy E hits a particle B which is at rest producing the particles $C_1 + C_2 + C_3 + \cdots\cdots$

$$\underset{m_1}{A} + \underset{m_2}{B} \longrightarrow \underset{m_3}{C_1} + \underset{m_4}{C_2} + \underset{m_5}{C_3} + \cdots\cdots$$

then the thresold energy for the reaction

$$E_{th} = \text{Rest mass energy} + K_{th}$$

$$= \boxed{\frac{(m_3+m_4+m_5+\cdots\cdots)^2 - (m_1^2 - m_2^2)}{2m_2}}$$

This is the direct formula to calculate the thresold energy E_{th}.

e.g :- $\gamma + M \longrightarrow M + \Delta$

$$E_{th} = \frac{(M+\Delta)^2 - M^2(\gamma) - M^2}{2M} = \frac{(M+\Delta)^2 - M^2}{2M}$$

$$E_{th} = \frac{\Delta^2 + 2M\Delta}{2M}$$

$$\boxed{E_{th} = \frac{\Delta(\Delta + 2M)}{2M}}$$

- If a particle A at rest decays into 2 particles B & C.

$$\underset{m_1}{A} \longrightarrow \underset{m_2}{B} + \underset{m_3}{C}$$

Then ~~Calculate~~ the energy of the outgoing particles in terms of various masses.

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Energy can be written as

$$\boxed{E_2 = \frac{m_1^2 + m_2^2 - m_3^3}{2m_1} c^2}$$

$$\boxed{E_3 = \frac{m_1^2 + m_3^2 - m_2^2}{2m_1} c^2}$$

$$\boxed{p_B = p_C = \frac{\sqrt{\lambda(m_1^2, m_2^2, m_3^2)}}{2m_1} c}$$

Where λ is triangle fun^n

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$$

e.g. $\tau^- \longrightarrow \pi^- + \nu_\tau$

Calculate the energy & momentum & velocity of π.

$M(\tau^-) = m_\tau$, $M(\pi^-) = m_\pi$

$$E_\pi = \frac{m_\pi^2 + \cancel{0}\, m_\tau^2 - 0}{2m_\tau} c^2 = \boxed{\frac{m_\pi^2 + m_\tau^2}{2m_\tau} c^2} \text{ Ans}$$

$$p_\pi = \frac{\sqrt{\lambda(m_\tau^2, m_\pi^2, 0)}}{2m_\tau} = \frac{\sqrt{m_\tau^4 + m_\pi^4 - 2m_\tau^2 m_\pi^2}}{2m_\tau}$$

$$\boxed{p_\pi = \frac{(m_\tau^2 - m_\pi^2)}{2m_\tau} c} \text{ Ans}$$

$$E = \frac{m}{\sqrt{1 - v^2/c^2}} c^2 \quad \& \quad p = \frac{m}{\sqrt{1 - v^2/c^2}} v$$

$$\frac{E}{p} = \frac{mc^2}{mv} = \frac{c^2}{v} \Rightarrow v = \frac{pc^2}{E}$$

$$v = \frac{(m_\tau^2 - m_\pi^2)c}{\cancel{2m_\tau}} \times \frac{\cancel{2m_\tau}}{(m_\pi^2 + m_\tau^2)\cancel{c^2}} \times \cancel{c^2}$$

$$\boxed{v = \frac{(m_\tau^2 - m_\pi^2)c}{(m_\tau^2 + m_\pi^2)}} \text{ Ans}$$

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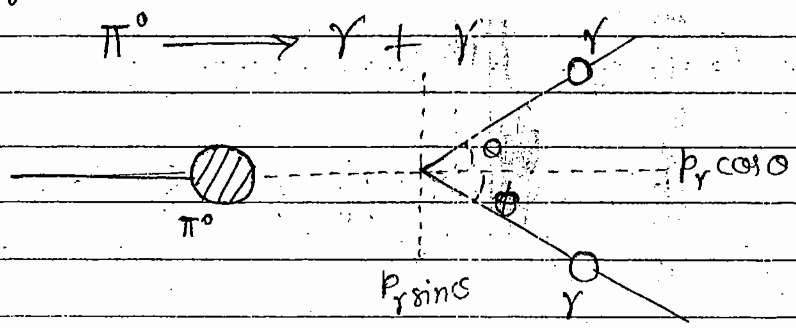
Que 1- A neutral pion whose K.E. is equal to its rest mass energy decays in flight. Find the angle b/w 2 γ-ray photons that are produced if their energies are same.

$$\pi^0 \longrightarrow \gamma + \gamma$$



$2\theta = ?$

γ- photon moving with velocity c. The energy & mom. will be same for both photons.

$$E = pc \Rightarrow p_1 = p_2$$

from the conservation of linear momentum

$$p_\pi = 2p_\gamma \cos\theta \quad \text{———} (1)$$

We must take both angles are same bcoz if θ is not same the vertical comp. will be different & energy & mom. conservation violates. So θ must be same.

given, K.E. of π^0 = Rest mass energy

$$E_\pi - m_\pi c^2 = m_\pi c^2$$

$$\sqrt{p_\pi^2 c^2 - m_\pi^2 c^4} = 2m_\pi c^2$$

squaring,

$$p_\pi^2 c^2 - m_\pi^2 c^4 = 4m_\pi^2 c^4$$

$$p_\pi^2 c^2 = 3m_\pi^2 c^4$$

$$p_\pi = \sqrt{3}\, m_\pi c$$

Now, from the conservation of energy,

$$E_\pi = 2E_\gamma$$

$$E_\pi = p_\gamma c + p_\gamma c$$

$$K.E. + \text{Rest mass energy} = 2p_\gamma c$$

(bcoz K.E. = R.M.E.)

$$2m_\pi c^2 = 2p_\gamma c$$

$$\boxed{p_\gamma = m_\pi c}$$

$M_{\pi^0} \rightarrow 135$ MeV

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eqn (1) $\Rightarrow$ $p_\pi = 2p_r \cos\theta$

$$\sqrt{3}\, \cancel{m_\pi} \cancel{c} = 2\cancel{m_\pi}\cancel{c} \cdot \cos\theta$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\boxed{\theta = 30^\circ}$$

Que :- Find the threshold energy of a following reaction

$$p + p \longrightarrow p + p + \pi^+ + \pi^-$$

Assuming the target proton is stationary

$M(p) \rightarrow 938$ MeV

$M(\pi^+)$ & $M(\pi^-) \longrightarrow 140$ MeV

$$E_{th} = \frac{[M(p) + M(p) + M(\pi^+) + M(\pi^-)]^2 - m_p^2 - m_p^2}{2m_p}$$

$(-2m_p^2)$

$$E_{th} = \frac{(938 + 938 + 140 + 140)^2 - (938)^2 - (938)^2}{2 \times 938}$$

$$E_{th} = \frac{(2156)^2 - (938)^2 - (938)^2}{1876} = \frac{4648336 - 1759688}{1876}$$

$$= \frac{\cancel{1128960}}{\cancel{1876}} = \cancel{601.791045}$$

$$= \frac{2888648}{1876} = 1539.79104$$

$$\boxed{E_{th} = 1539.79 \text{ MeV}}$$

II method

Also by $E_{th} = K_{th} +$ Rest mass energy

$$E_{th} = K_{th} + m_p c^2 \,, \quad K_{th} = -Q \times \frac{4m_p + m_{\pi^+} + m_{\pi^-}}{2 \times m_p}$$

N no. of nuclei

$$Q = -(m_{\pi^+} + m_{\pi^-})c^2$$

$$Q = -280 \text{ MeV}$$

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Qu. Find the thresold energy of rec?

$$\pi^- + p \rightarrow p + \bar{p} + n$$

$$E_{th} = \frac{[m(p) + m(\bar{p}) + m(n)]^2 - m_{\pi^-}^2 - m_p^2}{2m_p}$$

$$= \frac{[938 + 938 + 939]^2 - (140)^2 - (938)^2}{2 \times 938}$$

$$= \frac{(2815)^2 - (140)^2 - (938)^2}{1876}$$

$$= \frac{7924225 - 19600 - 879844}{1876}$$

$$= \frac{7024781}{1876}$$

$$= 3744.55277 \text{ MeV}$$

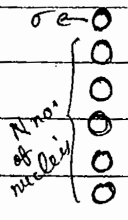
Cross - Section :- The probability of occurence of a nuclear reaction is measured by The Reaction Cross section.

It is designated by the symbol σ. ~~(x,y)~~

Its unit is Barn.

$$\boxed{1 \text{ barn} = 10^{-28} \, m^2} = 10^{-24} \, cm^2$$

$$x \longrightarrow X \longrightarrow y + Y \quad \Rightarrow \sigma(x, y)$$



Suppose a beam of particles in incident on a target having N no. of nuclei. The effective area of cross-section of each nuclei is σ.

Total effective cross-section (of N no. of nuclei)

$$= \sigma N$$

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If S is the target area then the fraction of the area which the effective cross section represents will be

$$\boxed{\text{Effective fraction} = \frac{\sigma N}{S}}$$

$$\boxed{\text{Probability} = \frac{\sigma N}{S}}$$

Let I_x be the no. of particles incident per second and I_y be " " " outgoing " "
Then the probability to find y per incident particle x will be

$$x + X \longrightarrow Y + y$$

$$\boxed{\text{Probability} = \frac{I_y}{I_x}}$$

Equating the two probabilities,

$$\frac{I_y}{I_x} = \frac{\sigma N}{S}$$

Therefore $\sigma = \frac{I_y}{I_x} \cdot \frac{1}{(N/S)}$

$N \rightarrow$ No. of nuclei in target
$S \rightarrow$ Area of target
$\frac{N}{S} \rightarrow$ No. of nuclii per unit area.

$$\boxed{\sigma(x,y) = \frac{I_y}{I_x \times \text{Number of Nucleii per unit area}}}$$

Larger will be the C.S., greater will be the probability of the occurance of reac^n.

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Ques:- A 16 μA beam of α-particles having cross-sectional area $10^{-4}\ m^2$ is incident on a rhodium Rh target, of thickness $1\mu m$.

$$\alpha + {}^{100}Rh \longrightarrow {}^{101}Pd + 3n$$

This produces n through this reaction.

(1.) The no. of α-particles hitting the target per sec. will be

(a) 0.5×10^{14} (b) 1.0×10^{14}
(c) 2.0×10^{20} (d) 4.0×10^{20}

(2.) The nutrons are observed at the rate of 1.806×10^{8} per sec. If the density of the Rh is approximated as $10^{4}\ kg/m^3$. Then the cross-section for this reaction will be

(a) 0.01 (b) 0.2
(c) 0.4 (d) 0.8

$$\alpha + Rh \longrightarrow Pd + 3n$$

(1) Current $= 16\ \mu A = 16 \times 10^{-6}\ C/sec$

This charge flow in 1 sec.

${}^{4}_{2}He$, charge $= 2 \times 1.6 \times 10^{-19}\ C$

No. of α-particles incident on target per sec

$$= \frac{\cancel{16}^{8} \times 10^{-6}}{\cancel{2} \times 1.6 \times 10^{-19}}$$

$$I_x = 5 \times 10^{13} \simeq 0.5 \times 10^{14} \text{ particle/sec.}$$

(a) $\longrightarrow$ Correct

(2) $$\sigma(x, y) = \frac{I_y}{I_x \cdot N}$$ This formula is valid for 1 to 1 correspondance.

* for 1 α-particle only 3 n are emitted.

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Rate at which no. of α-particles are hitting the target

$$I_x = 0.5 \times 10^{14} /sec.$$

" " " nutrons are produced $= 1.806 \times 10^{8}/sec$

$$I_y = \frac{1}{3} \times 1.806 \times 10^{8}/sec$$

$$I_y = 0.602 \times 10^{8}/sec.$$

{ for 1 n, the rate of production will be ~~ox~~ 1/3 bcoz for 1 α-particle there emits 3 nutrons }.

100 gm of Rh will ~~produce~~ posses $= 6.02 \times 10^{23}$

1 " " " $= 6.02 \times 10^{21}$

10^{7} gm " " " $= 6.02 \times 10^{28}$

$$\rho = 10^{4}\ kg/m^3 = 10^{7}\ gm/m^3$$

1 m^3 " " " $= 6.02 \times 10^{28}$

1 area X 1 thickness $= 6.02 \times 10^{28}$

1 area X 1 μm $= 6.02 \times 10^{28} \times 10^{-6}$

$$N = 6.02 \times 10^{22}$$

$$\sigma(x,y) = \frac{I_y}{I_x N} = \frac{0.602 \times 10^{8}}{0.5 \times 10^{14} \times 6.02 \times 10^{22}}$$

$$= \frac{0.602}{\cancel{0}3.01} \times 10^{-28}$$

$$= 0.2 \times 10^{-28}$$

$\boxed{\sigma = 0.2 \text{ Barn}}$ Ans (b) $\rightarrow$ Correct

Que :- for a certain incident ~~proton~~ energy the rec^n

$$p + {}^{56}Fe \longrightarrow n + {}^{56}Co$$

have a cross section of 0.60 barn. If we bombard a target in the form of 1 cm^2, 1 μm thick

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iron foil with a beam of protons equivalent to a current of 3 μA & if the beam is spread uniformly over the entire surface of the target. At what rate neutrons are being produced?

Given that, density of Iron = 7.9 gm/cm^3

$I_y = ?$

Current = 3 μA = 3×10^{-6} C/sec

charg on one proton = 1.6×10^{-19} C

No. of protons being incident per sec on target

$$I_x = \frac{3 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.875 \times 10^{+13} /\text{sec}.$$

$$\sigma = 0.6 \times 10^{-28}\ m^2 = 0.6 \times 10^{-24}\ cm^2$$

56 gm of Fe = 6.02×10^{23}

7.9 " " = $\frac{6.02 \times 10^{23}}{56} \times 7.9$

1 cm^3 " " = "

1 cm^2 × 1 cm " " = "

1 cm^2 × 10^{-4} cm " " = $\frac{6.02 \times 7.9 \times 10^{23}}{56} \times 10^{-4}$ /cm^2

$(1\ \mu m = 10^{-4}\ cm)$

$$N = \frac{6.02 \times 7.9 \times 10^{23} \times 10^{-4}}{56} /cm^2$$

$$N = 0.84925 \times 10^{+19} /cm^2$$

$$\boxed{\sigma(x, y) = \frac{I_y}{I_x N}}$$

$\Rightarrow$ $I_y = \sigma \times I_x \times N$

$I_y = 0.6 \times 10^{-24} \times 1.875 \times 10^{13} \times 0.84925 \times 10^{19}$

$I_y = 0.95541 \times 10^{8} = 9.55 \times 10^{7}$ /sec. Ans

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Que 1:- A beam of 20 μA of protons is incident on $2 cm^2$ of a target of ^{107}Ag of thickness 4.5 μm.

$$p + {}^{107}Ag \longrightarrow {}^{105}Cd + 3n$$

Producing this rec^n. Nutrons are observed at a rate of 8.5×10^6 /sec. What is the cross section for this rec^n at this proton energy: given that density of silver is 10.5 gm/cm^3.

$\sigma = ?$

$$\rho = 10.5 \ gm/cm^3$$

Current = 20 μA = 20×10^{-6} C/sec

Charge on 1 p = 1.6×10^{-19} C

No. of p being incident per sec = $\frac{20 \times 10^{-6}}{1.6 \times 10^{-19}}$

$$I_x = 12.5 \times 10^{13} \ /sec$$

~~No.~~ Rate at which neutrons are produced

$$I_y = \frac{1}{3} \times 8.5 \times 10^6$$

$$I_y = 2.833 \times 10^6 \ /sec$$

107 gm of Ag = 6.023×10^{23}

10.5 " " = $\frac{6.02 \times 10^{23}}{107} \times 10.5$

1 cm^3 = "

$1 cm^2 \times 1 cm$ = "

$2 cm^2 \times 4.5 \times 10^{-4}$. $= \frac{2 \times 6.02 \times 10.5 \times 4.5 \times 10^{23} \times 10^{-4}}{107}$

$= \frac{568.89}{107} \times 10^{19}$

$N = 5.31673 \times 10^{19}$ /cm^3

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cm²

date

for

$\times 10^{-4}$

$$\sigma(x,y) = \frac{I_y}{I_x N}$$

$$= \frac{2.833 \times 10^{6}}{12.5 \times 10^{13} \times 5.31673 \times 10^{19}}$$

$$= \frac{2.833}{66.45913} \times 10^{-26}$$

$$= 0.04263 \times 10^{-26}$$

$$\boxed{\sigma = 8.562 \text{ barn}}$$

Qu :- A beam of α-particles is incident on a target of ^{63}Cu resulting in the reaction

$$\alpha + {}^{63}Cu \longrightarrow {}^{66}Ga + n$$

$\rho = 8.94$ gm/cm³

Assume the cross-section for the particular α-energy to be 1.25 barn. The target is in the form of a foil 2.5 um thick. The beam has a circular cross-section of diameter 0.50 cm. And a current of 7.5 uA. Find the rate of neutron emission.

Current $= 7.5\ \mu A = 7.5 \times 10^{-6}$ C/sec.

Charg of $^{4}_{2}He = 2 \times 1.6 \times 10^{-19}$ C

No. of α-particle incident per sec $= \frac{7.5 \times 10^{-6}}{2 \times 1.6 \times 10^{-19}}$

$I_x = 2.34375 \times 10^{13}$ /sec

$\sigma = 1.25$ barn $= 1.25 \times 10^{-24}$ cm²

63 gm of Cu $= 6.02 \times 10^{23}$

8.94 gm " $= \frac{6.02 \times 10^{23} \times 8.94}{\cancel{8.94}\ 63}$.

1 cm³ "

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$1\,cm^2 \times 1\,cm$ gm of Cu $= \frac{6.02 \times 8.94 \times 10^{23}}{63}$

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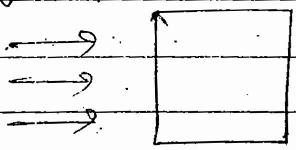
Qu:- A beam of neutrons of intensity I is incident on a thin slab of thickness dx, area A, density ρ, atomic rate M. The neutron absorption cross-section is σ.

(a) What is the loss in the Intensity dI of this beam in passing through the material.

(b) A beam of original intensity I_0 passes through a thickness x of the material. Show that the intensity of the emerging beam is $I = I_0 e^{-n\sigma x}$ where n is the no. of target nuclei per unit volume.



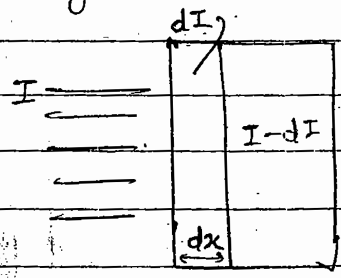
Intensity (no. of particle/sec) decreases.

Probability of occurence of a nuclear reaction is

$$\text{Probability} = \frac{\text{Interacting particles per second}}{\text{Incident particle per second}} = \frac{\text{Aggregate C.S.}}{\text{Target area}}$$

If I be the incident particle per second & dI be the loss in intensity (no. of particles) i.e. interacting particle per second. So

$$\text{Probability} = \frac{dI}{I}$$



$\sigma \equiv$ effective area of one nuclea

Aggregate Cross sectional area $= \sigma \times$ no. of nuclei present in the target

$$= \sigma \times nA\,dx$$

$$\text{Prob.} = \frac{dI}{I} = \frac{\sigma n A\,dx}{A}$$

{ $n \to$ no. of target nuclei per unit volume

$$\boxed{dI = nI\sigma\,dx}$$

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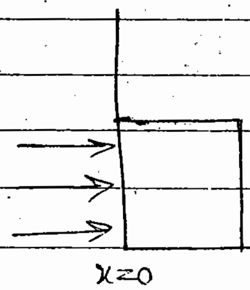
$$\frac{dI}{I} = -n\sigma dx$$

Because, there is the intensity of incident beam gradually decreasing (i.e. loss in no. of incident particle) so −ve sign occur.

On Integrating,



$$\int_{I_0}^{I} \frac{dI}{I} = \int_{x=0}^{x} -n\sigma dx$$

$$\ln I \Big|_{I_0}^{I} = -n\sigma x$$

$$\ln \frac{I}{I_0} = -n\sigma x$$

Taking, Antilog, $\frac{I}{I_0} = e^{-n\sigma x}$

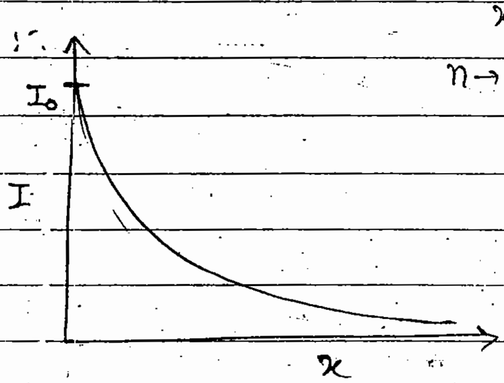
This is the intensity of particles left without reaching ⇒ $\boxed{I = I_0 e^{-n\sigma x}}$

I_0 → no. of particles incident per sec

x → thickness of material

n → no. of target nuclei per unit volume.

Plot of I v/s x :-



Prob :- The cross-sectional area of ^{113}Cd for capturing thermal neutrons is 2×10^4 barn. The mean atomic mass of natural cadmium is 112 u and its density is 8.64 gm/cm^3.

(a) What fraction of an incident beam of thermal neutrons is absorb by a ~~cal~~ cadmium sheet of 0.1 mm thick.

(b) What thickness of cadmium sheet is needed to absorb 99 % of an incident beam of thermal neutrons.

$\sigma = 2 \times 10^4$ barn

$$I = I_0 e^{-n\sigma x}$$

If I_0 be the no. of particle incident/sec & I be the no. of particles left without reaching then no. of particles absorb $= I_0 - I$

so, fraction of particle which are being absorbed $=$

$$\frac{I_0 - I}{I_0} = \frac{I_0 - I_0 e^{-nx\sigma}}{I_0}$$

Natural Cadmium is mixture of ^{112}C & ^{113}C. only ^{113}C absorb n.

^{113}C constitutes 12 % of the natural cadmium.

Average atomic mass $= 112$ u
Average density $= 8.64$ gm/cc

112 gm of natural Cd process $= 6.023 \times 10^{23}$ Nuclei

1 " " " $= \frac{6.023 \times 10^{23}}{112}$

8.64 gm " " " $= \frac{6.02 \times 10^{23}}{112} \times 8.64$

$1 cm^3$ " " $=$ "

{ If we need N then we write $1 cm^3$ = area x thickness
but for n, we need volume

$$n = 12 \% \text{ of } \frac{6.02 \times 10^{23} \times 8.64}{112}$$

$$= 12 \% \text{ of } \frac{52.0128 \times 10^{23}}{112}$$

$$= 12 \% \text{ of } 0.4644 \times 10^{23} = \frac{12 \times 0.4644 \times 10^{23}}{100}$$

$$= \frac{5.5728 \times 10^{23}}{100} = 5.5728 \times 10^{21}$$

(a) $\text{fraction} = 1 - e^{-n\sigma x} = 1 - e^{-(5.5728 \times 10^{21} \times 2 \times 10^4 \times 10^{-24} \times 0.1 \times 10^{-1})}$

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(b) $I = \cancel{99\%}\ 1\% \text{ of } I_0$

$I = I_0 e^{-n\sigma x}$

$0.01 I_0 = I_0 e^{-n\sigma x}$

$\frac{1}{100} = \frac{1}{e^{n\sigma x}} \Rightarrow e^{n\sigma x} = 10^2$

$n\sigma x = 2\ln 10$

$\Rightarrow x = \frac{2\ln 10}{n\sigma}$

$= \frac{2 \times 2.30259}{2 \times 10^4 \times 10^{-24} \times n}$

$x = 0.41$ mm Au

Mean free Path :-

Since $I = I_0 e^{-n\sigma x}$

$e^{-n\sigma x} dx$ represents the probability that a particle interact in the interval dx at the distance x so if we define the mean free path λ ~~within bracket~~ (average distance a particle can travel in a material medium before interacting) i.e.

$$\lambda = \frac{\int_0^\infty x e^{-n\sigma x} dx}{\int_0^\infty e^{-n\sigma x} dx}$$

$\left\{ \frac{\text{Sum of distance travel by the particle} \times \text{Prob.}}{\text{Total prob.}} \right.$

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$$\lambda = \frac{\left[x e^{-n\sigma x}/(-n\sigma)\right]_0^{\infty} - \int_0^{\infty} e^{-n\sigma x}\,dx}{\left[\frac{e^{-n\sigma x}}{-n\sigma}\right]_0^{\infty}}$$

$$= \frac{\left[x \frac{e^{-n\sigma x}}{-n\sigma}\right]_0^{\infty} - \left[\frac{e^{-n\sigma x}}{-n\sigma}\right]_0^{\infty}}{\left[\frac{e^{-n\sigma x}}{-n\sigma}\right]_0^{\infty}}$$

$$\boxed{\lambda = \frac{1}{n\sigma}}$$

Que :- A slab of absorber is 1 mean free path thick for a beam of certain incident particles. What % of particles will emerge from the slab.

$$\lambda = 1$$

$$x = \lambda = \frac{1}{\sigma n} = 1$$

$$I = I_0 e^{-n\sigma x}$$

$$= I_0 e^{-\cancel{n\sigma} \times \frac{1}{\cancel{n\sigma}}}$$

$$I = \frac{I_0}{e} \Rightarrow \frac{I}{I_0} = \frac{1}{e} = \frac{1}{2.71}$$

$$\frac{I}{I_0} = 37\%$$ Ans

% of particles which are undergoing the sca?

$= 63\%$

tract
f we
age

2a Band
ble X
b.

Que :- A relativistic particle of mass m_0 is moving with speed v. The value of v at which its K.E. is equal to rest mass energy.

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K.E = rest mass energy

$$\cancel{E - m_0c^2} = \cancel{m_0c^2}$$

$$E = 2m_0c^2$$

$$\frac{m_0\cancel{c^2}}{\sqrt{1-v^2/c^2}} = 2m_0\cancel{c^2}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{4} \Rightarrow \frac{v^2}{c^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\boxed{v = \frac{\sqrt{3}}{2}c}$$

Radioactivity

The phenomenone of spontaneous transformation of a nucleus is known as radioactivity.

→ The first nuclear phenomenone discovered

→ Henry Becquerel 1896.

These are of 3 kinds :-

1) α-decay

2) β-decay → e^- decay, e^+ decay, e^--capture

3) γ-decay

Due to the conversion of p into n or n into p, electrons are created inside the nucleus in decay secn.

At the time of decay within no time e^- liberated from nucleus

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

Which n is going to decay - Nobody can say
It is purely statistical phenomenon.

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* α-decay → To reduce size of nucleus α-particles liberates from nucleus.
i.e. α-decay takes place bcoz of Heavy size of nucleus.

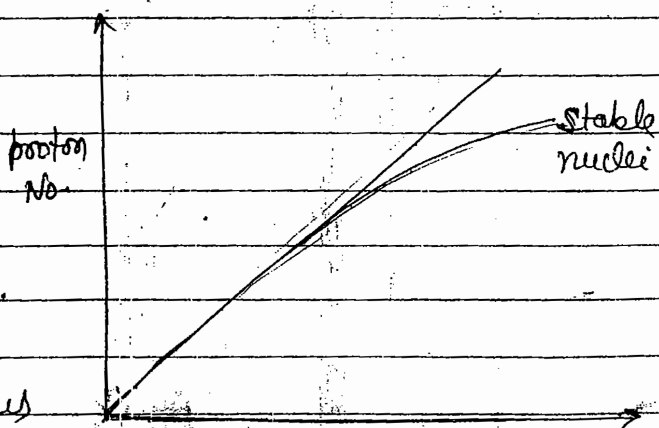
* β-decay → In nucleus n & p both exist. B/w n-p attraction & b/w n-n & P-P there will be repulsion.
If No. of p ↑ then ~~rep~~ columb force will occur.

if No. of n ↑ then attractive force ↑ & repulsive forces ↓.

e^- decay → occure bcoz of large no. of n
e^+ decay → " " " " p
e^- capture → " " " " p

γ-decay → Whenever an α, β or positron particle emits from an nucleus then remaining nucleus is in exited state.
γ-decay is always accompained by α & β decay.

As atomic no. ↑,
no. of n ↑
& no. of p ↓
i.e.
large Columnbian Repulsion b/w n & p.



The straight line shows no. of n = no. of p

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William Crook's Exp.:- In natural uranium, he observed that there are 2 kind of uranium - One part of Uraniunium participate in radioactivity & other not so he seperate them.

$U \rightarrow$ Not show radioactivity
$U_X \rightarrow$ Show radioactivity

} after some time U show radioactivity & radioactivity of $U_X \downarrow$.

He showed that the growth decay of the activity of U & U_X,

Activity of unknown U, $\boxed{A_X = A_0 \exp(-\lambda t)}$

decay continously

$$\boxed{A_U = A_0 \{1 - \exp(-\lambda t)\}}$$

Crook observed that there was some unknown substance present in the uranium salt which when chemically precipitated carried away entire radioactivity of the uranium salt. Uranium salt left after the chemical seperation lost its radio-activity. He then left the inactive uranium and the active precipitate as it is for few weeks. And then found that the uranium salt has regained its radioactivity.

$U \uparrow$ $U_X \downarrow$

activity exponentialy $\uparrow$ exponentialy $\downarrow$

U emits α- particle & becoms radioactive

$$U \longrightarrow \alpha + U_X$$

Consider the radioactive disintegration,

$$\underset{\text{Radioactive}}{P} \longrightarrow \underset{\text{stable}}{Q}$$

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$\therefore$ No. of atoms of P will be decreasing with time &
" " " Q " increasing " ".

Let at any instant t, N be the no. of atoms of P then rate of decay of N is directly proportional to No. of atoms.

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N$$

$\lambda \rightarrow$ decay constant / or disintegration constant or prob. of decay in 1 sec.

$\frac{dN}{dt} \rightarrow$ ~~de~~ Activity $(-\lambda N)$

$$\int_{N_0}^{N} \frac{dN}{N} = -\int_{t=0}^{t} \lambda \, dt$$

$$\ln[N]_{N_0}^{N} = -\lambda[t]$$

$$\ln \frac{N}{N_0} = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\boxed{N = N_0 e^{-\lambda t}}$$

Activity A depends upon the no. of nuclei of the parent and the probability λ (prob. of decay for 1 particular nuclei). so activity

$$A = \lambda N$$

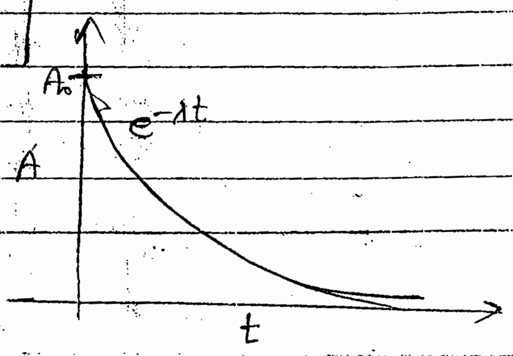
$$A = \underbrace{\lambda N_0}e^{-\lambda t} \qquad (\lambda N_0 = A_0)$$

$$\boxed{A = A_0 e^{-\lambda t}}$$

$$\underbrace{\log A}_{\text{variable}} = \underbrace{\log A_0}_{\text{const}} - \underbrace{\lambda}_{\text{c.}} \underbrace{t}_{\text{v.}}$$

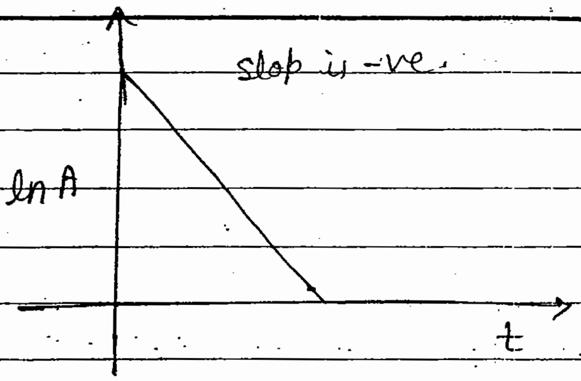
$$\boxed{y = mx + c}$$

Straight line



$C = \ln A_0$

slope $= \lambda$



Suppose the no. of radioactive atoms is reduced to half of its initial value in time $\boxed{t = t_{1/2}}$ This is called half life.

$A \rightarrow \frac{A_0}{2}$ or $N \rightarrow \frac{N_0}{2}$ at $t = t_{1/2}$

$$N = N_0 e^{-\lambda t}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$e^{\lambda t_{1/2}} = 2$$

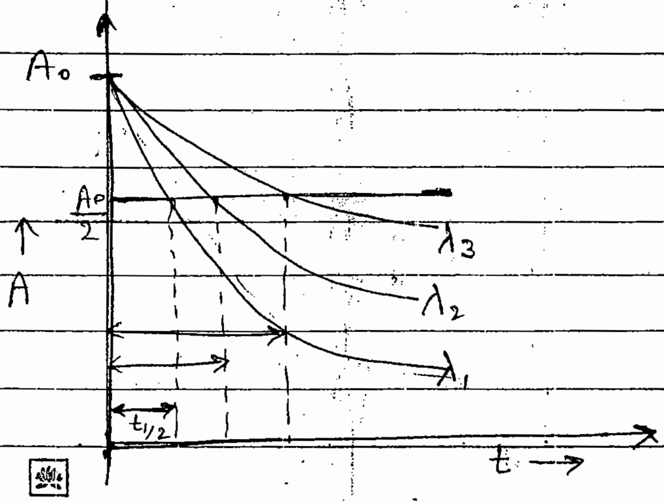
$$\lambda t_{1/2} = \ln 2$$

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$

19/1

The half life & decay constant are inversely proportional.

Half Life $\propto \frac{1}{\text{Decay Const,}}$



$\lambda_1 > \lambda_2 > \lambda_3$

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In One $t_{1/2}$, N becomes $= \frac{N_0}{2}$

two $t_{1/2}$, $= \frac{1}{2}\frac{N_0}{2} = \frac{N_0}{4} = \cancel{\frac{N_0}{2}} \left(\frac{1}{2}\right)^2 N_0$

three $t_{1/2}$, $= \left(\frac{1}{2}\right)^3 N_0$

In n $t_{1/2}$, $= \left(\frac{1}{2}\right)^n N_0$

$$\boxed{N = N_0 \left(\frac{1}{2}\right)^n}$$

$N = N_0 \left(\frac{1}{2}\right)^n$

~~Note~~

$n \rightarrow$ no. of half lives
$N \rightarrow$ No. of atoms left

$\boxed{n \times t_{1/2} = T}$

$T = t_{1/2}$

$$\boxed{n \times t_{1/2} = T}$$

19/Aug/2012

The average life time of a radioactive substance

$$\tau = \frac{1}{\lambda} = \frac{1}{\frac{0.693}{t_{1/2}}}$$

$$\boxed{\tau = 1.44 \times t_{1/2}}$$

So $A = A_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda \tau}$

$A = A_0 e^{-\lambda \times \frac{1}{\lambda}} = A_0 e^{-1}$

$$\boxed{\frac{A}{A_0} = \frac{1}{e} = 0.37}$$

The average life time of a radioactive substance is defined as ~~during~~ the time during which the radioactivity of the substance remains 37 % of its initial value.

The basic unit for measuring ~~radioactivity~~ activity is

Unit of activity :-

$1 \text{ Bq} = 1 \text{ Becquerel} = 1 \text{ disintegration/sec}$

Another unit is Curie.

$1 \text{ Ci} = 3.7 \times 10^{10} \frac{\text{disintegration}}{\text{sec}}$

$$\boxed{1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}}$$

Que :- The half life of ^{198}Au is 2.7 days.

(a) What is the decay constant of Au

(b) What is the prob. that ^{198}Au nuclei will decay in 1 sec.

(c) Suppose we had 1 μgm sample of ^{198}Au. What is the activity.

(d) How many decays per sec occur when sample is 1 week old.

(a) $t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$, $t_{1/2} = 2.7$ days

$= 2.7 \times 24 \times 60 \times 60$

$= 233280$ sec

$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{233680}$

$\lambda = 2.97 \times 10^{-6} / \text{sec}$

(b) $\lambda =$ Prob. to decay in 1 sec.

So $= 2.97 \times 10^{-6} / \text{sec}$.

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(c) Activity; $A = \lambda N_0$

~~$A = 2.97 \times 10^{-6} \times 1 \times 10^{-6}$ gm~~

To calculate N:

198 gm Au posses = 6.02×10^{23} Nuclei

1 " " " = $6.02 \times 10^{23}/198$

1×10^{-6} gm " " = $\frac{6.02 \times 10^{23}}{198} \times 10^{-6}$

$$N_0 = \frac{6.02}{198} \times 10^{17}$$

$A = \lambda N_0$

$= 2.97 \times 10^{-6} \times \frac{6.02}{198} \times 10^{17}$

$= 0.090344 \times 10^{11}$

$\boxed{A = 9.03 \times 10^9 \text{ decay/sec}}$ (or Bq)

$= \frac{9.03 \times 10^9}{3.7 \times 10^{10}}$ ($1\ Ci = 3.7 \times 10^{10} Bq$)

$= 2.44 \times 10^{-1}$

$\boxed{A = 0.244\ Ci}$

(d) ~~$N = N_0 e^{-\lambda t}$~~

After 1 week, what will be the activity = ?

$A = A_0 e^{-\lambda t}$

$= A_0 e^{-\frac{0.693}{2.7 d} \times 7 d}$ A_0 = initial activity

A = activity after certain time

$= 9.03 \times 10^9 \times e^{-1.788}$

$= 1.5 \times 10^9$ disintegration/sec

$= 1.5 \times 10^9$ Bq

30x60

sec

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Que Find the prob that a particular nucleus of ^{38}Cl will decay in 1 sec period. $t_{1/2}$ of ^{38}Cl is 37.2 minutes

$t_{1/2} = 37.2$ min

$= 37.2 \times 60 = 2232$ sec

$\lambda = \frac{0.6932}{t_{1/2}} = \frac{0.6932}{2232}$

$= 0.00031048$

$\boxed{\lambda = 3.1 \times 10^{-4} / sec}$

Que :- Half life of a radioactive isotope is 4×10^8 years. If there were 10^3 radioactive nuclei in the sample today, the no. of such nuclei in the sample 4×10^9 years back (ago).

(a) 128×10^3 (b) 256×10^3

(c) 512×10^3 (d) 1024×10^3 ✓

$t_{1/2} = 4 \times 10^8$ years

$N = 10^3$

$\cancel{\lambda = \frac{0.6932}{t_{1/2}}}$

$\cancel{N = N_0 e^{-\lambda t}}$

$n \times t_{1/2} = T$

$n \times 4 \times 10^8 = 4 \times 10^9$

$\Rightarrow n = 10$

$N = N_0 \left(\frac{1}{2}\right)^n$

$10^3 = N_0 \left(\frac{1}{2}\right)^{10} \Rightarrow 10^3 = N_0 \times \frac{1}{1024}$

$\boxed{N_0 = 1024 \times 10^3}$ (d) ✓

Another method, $N = N_0 e^{-\lambda t}$

$10^3 = N_0 e^{-\frac{0.693}{4 \times 10^8} \times 4 \times 10^9}$

$N_0 = 1024 \times 10^3$

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will
minuts

Que :- Half life of ^{235}U is 7.04×10^8 years. A sample of rock which solidified with the earth 4.55×10^9 years ago contain N atoms of ^{235}U. How many uranium 235 atoms did the same rock had at the time it solidified.

$$t_{1/2} = 7.04 \times 10^8\ y$$
$$T = 4.55 \times 10^9\ y$$
$$N = N$$
$$n \times t_{1/2} = T$$
$$n \times 7.04 \times 10^8 = 4.55 \times 10^9$$
$$n = \frac{4.55 \times 10^9}{7.04 \times 10^8} \Rightarrow n = 6.46$$

$$N = N_0 \left(\frac{1}{2}\right)^n \Rightarrow N_0 = 2^n N$$
$$N_0 = 2^{6.46} N$$
$$\boxed{N_0 = 88.02\ N}$$ Ans

Que :- If the decay constant of the radioactive sample is λ per min. then the fraction that decays in 5th min will be ?

(a) $e^{-4\lambda}$ (b) $e^{-5\lambda}$ (c) $e^{-4\lambda} - e^{-5\lambda}$
(d) $e^{-4\lambda} + e^{-5\lambda}$

No. of nuclei remain after 4 min $N_4 = N_0 e^{-4\lambda}$
" " " $N_5 = N_0 e^{-5\lambda}$

No. of nuclei decayed in 5th min. $= N_4 - N_5$
$$\text{Number} = N_0 e^{-4\lambda} - N_0 e^{-5\lambda}$$
$$\frac{\text{Number}}{N_0} = e^{-4\lambda} - e^{-5\lambda}$$

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Que :- A certain sample of radioactive material decays at the rate of 548 / sec at t=0. At t = 48 min the counting rate has fallen to 213 / sec.

(a) What is the half life of the sample.

(b) What is the decay constant.

(c) What will be the decay rate at t = 125 min.

(b) $A = A_0 e^{-\lambda t}$

decay rate = activity

at t = 0, A = 548 /sec

$\therefore \quad 548 = A_0 \times 1 \Rightarrow A_0 = 548$ decay/sec.

at t = 48 min, A = 213 /sec.

$$A = A_0 e^{-\lambda t}$$

$$213 = 548 \times e^{-\lambda \times 48 \times 60}$$

$$\frac{213}{548} = e^{-2880\lambda}$$

$$0.388 = e^{-2880\lambda} \Rightarrow -2880\lambda = \ln(0.388)$$

$$-2880\lambda = -0.96$$

$$\lambda = 3.28 \times 10^{-4} /sec.$$

(a) $t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{3.28 \times 10^{-4}} = 3.52 \times 10^{-3} \times 10^{4}$ min

$t_{1/2} = 35$ min.

(c) $\boxed{A = 46.7992}$

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Que :- How long does it take for 60 % of a sample of radon to decay. Given that half life is 3.82 days.

$t = ?$

$t_{1/2} = 3.82 \text{ days} = 3.82 \times 24 \times 60 \times 60$

$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{3.82 \times 24 \times 3600} = 2.099 \times 10^{-6}$

$N = N_0 e^{-\lambda t}$

$N \rightarrow$ no. of atoms left.

If 60% decays then 40% atoms left.

So $N = 40\% \, N_0$

$\Rightarrow \cancel{N_0} \times \frac{40}{100} = \cancel{N_0} e^{-\lambda t}$

$e^{\lambda t} = \frac{100}{40} = \frac{10}{4}$

$\lambda t = \ln\left(\frac{10}{4}\right) = \ln 10 - \ln 4$

$t = \frac{1}{\lambda}(\ln 10 - \ln 4)$

$t = \frac{t_{1/2}}{0.693}(\ln 10 - \ln 4)$

$t = 4.3 \times 10^5 / \text{sec}$

$t = 5.05$ days Ans.

Imp

Que :- The activity of a radioactive substance is defined as ~~X~~ $A = \lambda N$ where λ is the decay const. & ~~A~~ N be the no. of atoms. In an experiment the activity of a sample of ${}^{55}_{24}Cr$ was found to change as

(min) After	0	5	10	15	20
λN (in mCi)	19.2	7.13	2.65	0.99	0.37

The half life of Cr is

(a) 5.08 min (b) 3.52 min (c) 3.57 min (d) 5.16 min

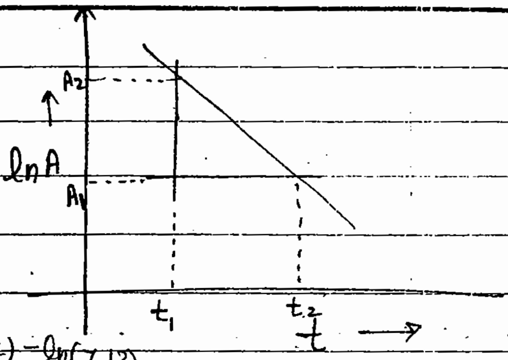
$A = \lambda N$

$A = A_0 e^{-\lambda t}$

$\ln A = \ln A_0 - \lambda t$

$\lambda t = \ln A_0 - \ln A$

$\lambda = \dfrac{\ln A_0 - \ln A}{t}$



$= \dfrac{\ln(19.2) - \ln(7.13)}{5-0}$

$\lambda = \dfrac{\ln A_1 - \ln A_2}{t_2 - t_1}$

$\lambda = \dfrac{\ln(7.13) - \ln(2.65)}{10-5} = \dfrac{1.9643 - 0.9745}{5}$

$\lambda = \dfrac{0.9898}{5} = 0.19795$

$t_{1/2} = \dfrac{0.6932}{0.19795} = 3.52 \text{ min}$

Chain Radioactivity :-

$P \xrightarrow{decay} Q \xrightarrow{decay} R$

Radioactive stable.

The process of decay continue upto stable atom - chain R.

Let decay constant & half life of P & Q are λ_1, $t^1_{1/2}$ and λ_2, $t^2_{1/2}$ respectively.

The rate with which P is being decayed is the rate with which Q form.

i.e. No. of Q form per sec = No. of P decay per sec.

Let at any instant of time t,

N_1 = Number of particles of P

then the rate at with P decay ⇒

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \qquad \text{——— (1)}$$

This is linear differential eq^n, on solving this, we get sol^n of (1) ; $N_1 = N_{10} e^{-\lambda_1 t}$ ——— (2)

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Rate with which Q is being formed $\Rightarrow$ $+\lambda_1 N_1$

Since Q is also radioactive so let at the same instant of time t, N_2 be the no. of atoms of Q then Rate of change of Q = rate of formation of R.

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad \text{——} (3)$$

~~Let the soln of this eqn~~

$$N_2(t) = f(t)\, e^{-\lambda_2 t} \quad \text{——} (4)$$

On differentiating, $\frac{dN_2}{dt} = \frac{df}{dt} e^{-\lambda t} - \lambda_2 f(t) e^{-\lambda_2 t}$

Substituting in eqn no. (3),

$$\frac{df}{dt} e^{-\lambda t} - \cancel{\lambda_2 f(t) e^{-\lambda_2 t}} = \lambda_1 N_1 - \lambda_2 N_2$$

$$= \lambda_1 N_1 - \cancel{\lambda_2 f(t) e^{-\lambda_2 t}}$$

$$\Rightarrow \frac{df}{dt} = \lambda_1 N_{10}\, e^{-\lambda_1 t}\, e^{+\lambda_2 t}$$ (putting the value of N_1)

$$\int df = \int \lambda_1 N_{10}\, e^{-(\lambda_1 - \lambda_2)t}\, dt$$

$$f(t) = \frac{\lambda_1 N_{10}\, e^{-(\lambda_1-\lambda_2)t}}{\lambda_2 - \lambda_1} + C$$

At time $t = 0$, $N_2 = 0$

i.e. at $t = 0$, Q is not start to decay. So there is no atom of Q. Only atoms of P are there.

$$N_2 = 0 \Rightarrow f(t) = 0$$

$$\Rightarrow C = \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2}$$

$$\therefore f(t) = \frac{\lambda_1 N_{10}\, e^{(\lambda_2-\lambda_1)t}}{\lambda_2 - \lambda_1} + \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2}$$

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$$f(t) = \frac{\lambda_1 N_{10}\, e^{(\lambda_2-\lambda_1)t}}{\lambda_2-\lambda_1} - \frac{\lambda_1 N_{10}}{\lambda_2-\lambda_1}$$

$$f(t) = \frac{\lambda_1 N_{10}}{\lambda_1-\lambda_2}\left[1 - e^{(\lambda_2-\lambda_1)t}\right]$$

Substitute the value of $f(t)$ in eqn (4),

$$N_2(t) = f(t)\, e^{-\lambda_2 t}$$

$$N_2(t) = \frac{\lambda_1 N_{10}}{\lambda_1-\lambda_2}\left[1 - e^{(\lambda_2-\lambda_1)t}\right] e^{-\lambda_2 t}$$

$$\boxed{N_2(t) = \frac{\lambda_1 N_{10}}{\lambda_1-\lambda_2}\left[e^{-\lambda_2 t} - e^{-\lambda_1 t}\right]}$$

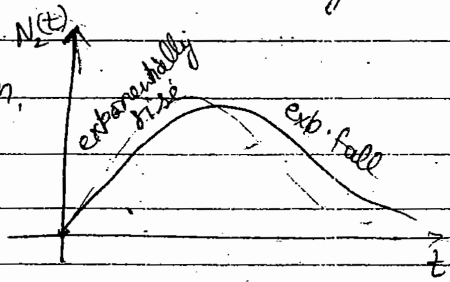
$N_2(t) \rightarrow$ no. of nuclei varies with time depend on half life of its own and also on ~~half life~~ decay const. of radioactive atoms (parent & daughter)

If $e^{-\lambda_2 t}$ dominates then $N_2(t) \uparrow$ with time

If $e^{-\lambda_1 t}$ " " $N_2(t) \downarrow$ " "

But generally first it increase & then decay.

The time t_m at which N_2 is maximum,



$$\frac{dN_2(t)}{dt} = 0$$

$$\Rightarrow \frac{\lambda_1 N_{10}}{\lambda_1-\lambda_2}\left[-\lambda_2 e^{-\lambda_2 t} + \lambda_1 e^{-\lambda_1 t}\right] = 0$$

$$\Rightarrow \lambda_2 e^{-\lambda_2 t_m} = \lambda_1 e^{-\lambda_1 t_m}$$

$$\Rightarrow \frac{\lambda_2}{\lambda_1} = e^{-\lambda_1 t_m + \lambda_2 t_m} = e^{(-\lambda_1+\lambda_2)t_m}$$

$$\Rightarrow (\lambda_2-\lambda_1)t_m = \ln\left(\frac{\lambda_2}{\lambda_1}\right) \quad \text{or} \quad (\lambda_1-\lambda_2)t_m = \ln\left(\frac{\lambda_1}{\lambda_2}\right)$$

$$\Rightarrow \boxed{t_m = \frac{1}{\lambda_1-\lambda_2} \ln \frac{\lambda_1}{\lambda_2}}$$

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Prob :- Consider the reaction $P \longrightarrow Q \longrightarrow R$. If the half life of P is $t_{1/2} = 10$ Hr. & half life of Q, $t_{1/2} = 1.3$ Hr. After what time Q will be maximum.

P, $t_{1/2} = 10$ Hr Q; $t_{1/2} = 1.3$ Hr

$$\lambda_1 = \frac{0.693}{t_{1/2}} = \frac{0.693}{10} \qquad \lambda_2 = \frac{0.693}{1.3}$$

$$= 0.06930 \qquad = 0.5330$$

$$t_m = \frac{1}{\lambda_1 - \lambda_2} \ln \frac{\lambda_1}{\lambda_2}$$

$$= \frac{1}{(0.06930 - 0.5330)} \ln \left[\frac{0.06930}{0.5330}\right]$$

$$= \frac{1}{-0.46377692} \ln(0.13001876)$$

$$= \frac{1}{-0.46377692} \times -2.040076\,53$$

$$= 4.39883151$$

$$\simeq 4.39 \quad \text{Ans}$$

Radioactive Equilibrium :-

If $P \longrightarrow Q \longrightarrow R$

The ratio of no. of P & Q atoms is constant then it is called the cond^n of radioactive equilibrium

Case. 1 :- $\lambda_1 > \lambda_2$

then $t^1_{1/2} < t^2_{1/2}$

half life of Parent < half life of daughter nucle

Case. 2 :- $\lambda_1 < \lambda_2$

$t^1_{1/2} > t^2_{1/2}$ Transient eq^m.

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Case-3 :- $\lambda_1 << \lambda_2$

$t_{1/2}^{(1)} >> t_{1/2}^{(2)}$ Secular eq^m

In Transient eq^m.
no. of nuclei of parent ↑ & no. of neucli of dauter ↑ ↓ but their ratio is constant.
But in Secular eq^m
They remain constant.

Transient Equilibrium :-

$\lambda_1 < \lambda_2$

$t_{1/2}^{(1)} > t_{1/2}^{(2)}$

After a long time compare to the half life of the product,

$t >> t_{1/2}^{(2)}$

t is much more larger than half life of the daughter atom.

$\lambda_2 t >> 1$

$e^{-\lambda_2 t} \longrightarrow 0$, Maxi. value of λ_2 will be ∞.

$$N_2(t) = \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2}\left[\underbrace{e^{-\lambda_2 t}}_{\text{approaching to } 0} - e^{-\lambda_1 t}\right]$$

$$N_2(t) = \frac{\lambda_1 N_{10}}{-\lambda_1 + \lambda_2} e^{-\lambda_1 t}$$

& $\lambda_2 > \lambda_1$ so $-\lambda_1 + \lambda_2 = +ve$

$N_2(t) \rightarrow$ no. of nuclei left at any instant of time t.

$N_2(t)$, is decaying with the half life of parent

& N_1 varies as $N_1 = N_{10} e^{-\lambda_1 t}$

N_1 & N_2 both varies with time but their

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ratio is constant.

$$\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1}$$

This is called Transient eqm.
i.e. At different value of time, N_1 & N_2 will be different (varies with time) b

Secular Equilibrium :-

$$\lambda_1 << \lambda_2$$

$$t_{1/2}^{(1)} >> t_{1/2}^{(2)}$$

In this case the equilibrium stablish b/w the no. of parent & daughter nuclei is called Sequlan eqm.
Since the half life of parent nuclei is very long so during the time t much more lesser than half life of Parent.

$$t << t_{1/2}^{(1)}$$

$$\lambda_1 t << 1$$

$$e^{-\lambda_1 t} \longrightarrow 1 \quad , \text{ Min value of } \lambda_1 \text{ is } 0.$$

t is much more lesser than $t_{1/2}$ of (1) (Parent)
" " " greater " $t_{1/2}$ of (2) (Daughter)

i.e. $$t_{1/2}^{(2)} << t << t_{1/2}^{(1)}$$

So

$$N_2(t) = \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2} \left[e^{-\lambda_2 t} - 1\right]$$

$$= \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10}\left(1 - e^{-\lambda_2 t}\right)$$

But $t >> t_{1/2}^{(2)}$

$$\Rightarrow \lambda_2 t >> 1 \Rightarrow e^{-\lambda_2 t} \longrightarrow 0$$

~~for a good approx.~~

So $$\boxed{N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10}}$$ N_2 is independent on time.

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& $N_1(t) = N_{10} e^{-\lambda_1 t}$

$N_1(t) = N_{10}$

To a good approximation the no. of parent nuclei are constant.

No. of parent nuclei → Constant
daughter " → constant
& their ratio

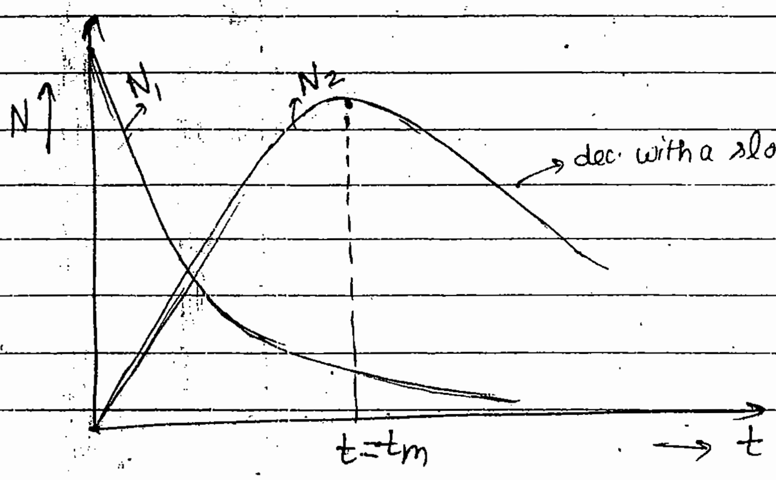
$$\boxed{\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1}} \quad \text{or} \quad \boxed{\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2}} \quad \lambda_2 >> \lambda_1$$

Such kind of eq^m is called Secular Eq^m.

Note → Only differce b/w Transient & Secular eq^m is that
→ In Transient eq^n → N_1 & N_2 varies with time but ratio → const.
→ In Seculer eq^m → N_1 & N_2 both constant & ratio → const.

Prob :- ^{231}Pa is in Seculer equilibrium with rare isotope ^{235}U (0.7 % of Uranium). The measurement of the ratio of the atoms of natural uranium to ^{231}Pa gives a value 2.89×10^6 years. Calculate the half life of uranium-235 assuming that the half of ^{231}Pa to be 3.3×10^4 years.

$\lambda_1 > \lambda_2$
$t_{1/2}^{1} < t_{1/2}^{2}$
daughter will be longer live.



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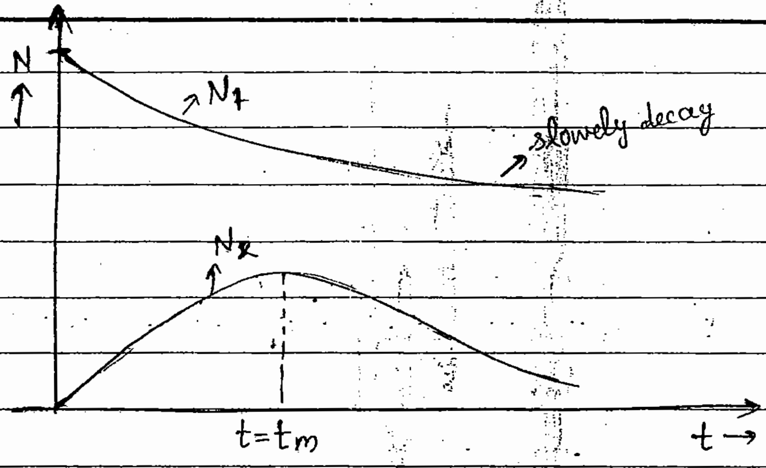
$\lambda_1 < \lambda_2$
$t_{1/2}^{(1)} > t_{1/2}^{(2)}$
Parent will be longer live.



sol^n:- for Secular eq^m, $\frac{N_1}{N_2} = \frac{\lambda_2}{\lambda_1} = \frac{(t_{1/2})^{(1)}}{t_{1/2}^{(2)}}$

$\rightarrow t_{1/2}^{(2)} = 3.3 \times 10^4$ y.

$\rightarrow \frac{N_1}{N_2} = 2.89 \times 10^6 \times \frac{0.7}{100}$ $\quad N_1 \rightarrow U$, $N_2 \rightarrow Pa$

{ ^{235}U is 0.7 % of natural U }

$\frac{N_1}{N_2} = 2.023 \times 10^4$

Now, $\frac{N_1}{N_2} = \frac{t_{1/2}^{(1)}}{t_{1/2}^{(2)}}$

$(t_{1/2})^{(1)} = \frac{N_1}{N_2} \times t_{1/2}^{(2)} = 2.023 \times 10^4 \times 3.3 \times 10^4$

$t_{1/2}^{(1)} = 6.6759 \times 10^8$ years Ans

Prob:- A radioactive element has a half life of 25 hours.
(i) After what time will 1/8 th of initial no. of its atoms disintegrate.
(ii) After what time will 1/32 th of the initial no. remain unchanged.

(ii) $t_{1/2} = 25$ h. , $N = N_0 \left(\frac{1}{32}\right)$

~~$\lambda = \frac{0.693}{25}$~~ $\quad N = N_0 \left(\frac{1}{2}\right)^n$

$\frac{1}{32} \cancel{N_0} = \cancel{N_0} \left(\frac{1}{2}\right)^n$ or $\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^5$

$n = 5$

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$n\, t_{1/2} = T$

$5 \times 25\ hr = T$

$\boxed{T = 125\ Hrs.}$

(i) $\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{25}$ (4 Hr, 49 min.)

$\lambda = 0.02772$

$N = N_0 e^{-\lambda t}$

$\frac{7}{8} \cancel{N_0} = 1 \cancel{N_0}\, e^{-0.02772 t}$

$\log \frac{7}{8} = -0.02772 t$

$t = \frac{\log 8/7}{0.02772} = \frac{\log 8 - \log 7}{0.02772}$

$t = \frac{2.0794 - 1.9459}{0.02772}$

$= \frac{0.1335}{0.02772} = 4.81$ hours.

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Radioactive Branching :- There are certain radioactive isotopes in which both α & β transformations are observed with defenate branching ratios. If a radioactive substance exhibit both α & β activities then there are definate probabilities of decay in the two branches. Let λ_α & λ_β be the decay constants of the two branches.

Then the probability that a nucleus will undergo α decay in time $dt = \lambda_\alpha dt$

The prob. " " " " " "
β decay in time $= \lambda_\beta dt$

Total prob. of decay $\lambda dt = \lambda_\alpha dt + \lambda_\beta dt$

$$\boxed{\lambda = \lambda_\alpha + \lambda_\beta}$$

In terms of half life,

$$\frac{0.693}{t_{1/2}} = \frac{0.693}{t_{1/2}^{\alpha}} + \frac{0.693}{t_{1/2}^{\beta}}$$

$$\Rightarrow \boxed{\frac{1}{t_{1/2}} = \frac{1}{t_{1/2}^{\alpha}} + \frac{1}{t_{1/2}^{\beta}}}$$

α - Decay :- α - decay is due to short range of nuclear forces.

The value of nuclear forces is limited to the smaller region (certain limit)

P - P repell to each other by columnbian force

Because of heavy size of α - particle

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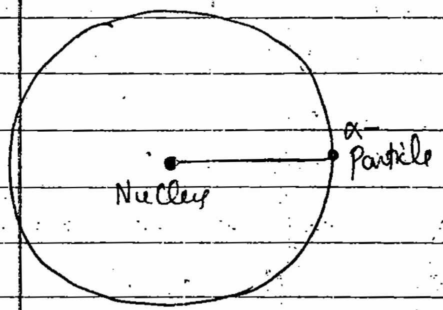
α - particle emits with energy → 4 − 9 MeV



Nucley & α - particle are bound with strong attractive forces.

$U = 30$ MeV
40 MeV

This energy is required to emit the α-particle out of nucleus.

It have only 4 to 9 MeV energy. To tunnel the barrier & come out is difficult,

Acc. to classical mech., α - particle can not cross the barrier.

Acc. to Q.M., there is small prob. but definate prob. to cross the barrier.

Prob:- Nuclei which contain 210 or more nucleon are so large that the short range nuclear force that hold them together are unable to counter balance a repulsion b/w the protons.

α - decay occurs in such nuclei as a means of increasing their stability by reducing their size.

$${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}X' + {}^{4}_{2}He$$

Q value :- $Q = M_n(X) - M_n(X') - M_n({}^{4}_{2}He)$

These are the nuclear masses.

Atomic Mass = Nuclear Mass + Mass of e^-

⇒ Nuclear Mass = Atomic Mass − Mass of e^-

$$M_n = M(A) - M_e$$

So $Q = M(X) - M_e - M(X') + M_e - M({}^{4}_{2}He) - M_e$

$= M(X) - M(Z) - M(X') + M(Z-2) - M({}^{4}_{2}He) - 2M$

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$$Q = M(X) - M(X') - M({}_2^4He) \quad \text{———— (1)}$$

In terms of K.E., from the conservation of K.E.,

$$Q = K_{X'} + K_\alpha \quad \text{———— (2)}$$

From the conservation of momentum,

$$0 = \vec{p}_{X'} + \vec{p}_\alpha$$

$$\vec{p}_{X'} = -\vec{p}_\alpha \quad \text{or} \quad |\vec{p}_{X'}| = |\vec{p}_\alpha|$$

$$p_{X'} = p_\alpha$$

from eq^n (2) $\Rightarrow$

$$Q = K_{X'} + K_\alpha$$

$$= \frac{p_{X'}^2}{2M_{X'}} + K_\alpha$$

$$= \frac{p_\alpha^2}{2M_{X'}} + K_\alpha \qquad (p_{X'} = p_\alpha)$$

$$= \frac{p_\alpha^2}{2M_{X'}} \cdot \frac{m_\alpha}{m_\alpha} + K_\alpha$$

$$= \frac{m_\alpha}{M_{X'}} K_\alpha + K_\alpha \qquad \left\{ \frac{p_\alpha^2}{2m_\alpha} = K_\alpha \right\}$$

$$Q = K_\alpha \cdot \frac{M_{X'} + M_\alpha}{M_{X'}}$$

Masses are almost equal to the mass nos.
So,

$$Q = K_\alpha \left(\frac{A - 4 + 4}{A - 4} \right)$$

$$\boxed{Q = \frac{A}{A-4} K_\alpha}$$

The K.E. of α-particle will be,

$$\boxed{K_\alpha = \frac{(A-4)}{A} Q}$$

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Aavilable Q energy is shared by the K.E of α-particle & K.E of daughter nuclei.
K.E will never exceed the Q-value.

Prob - Find the K.E. of the α-particle emitted in the α-decay of ${}^{226}_{88}Ra_{138}$.

$${}^{226}_{88}Ra_{138} \longrightarrow {}^{222}_{86}Rn_{136} + \alpha$$

Given that $m(Ra) = 226.025403\ u$
$m(Rn) = 222.017571$
$m(\alpha) = 4.002603\ u$

$K_\alpha = ?$

$Q = m(Ra) - m(Rn) - m(\alpha)$
$= 226.025403 - 222.017571 - 4.002603\ u$
$= 0.005229\ \underset{\times 931.5\ MeV}{u} \simeq 4.871\ MeV$

$\cancel{Q = K(\alpha) - K(Ra) - K(Rn)}$
$\cancel{0.005229 = K_\alpha -}$

$$K_\alpha = \left(\frac{A-4}{A}\right) Q$$

$= \frac{226-4}{226} \times 0.005229$

$= \frac{222}{226} \times 0.005229$

$= 0.9823 \times 0.005229$

$= 0.00513645 \times 931.5$

$= \cancel{0.00770468}$

$\simeq 4.78460010$

$\simeq 4.785\ MeV$ = Ans

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Qu 1- $^{210}_{84}Po$ isotope is unstable & emits a 5.30 MeV α-particle. The atomic mass of Po is 209.9829 u & $m(^4_2He) = 4.0026$ u
Identify the daughter nuclei & its mass.

$M(X') = ?$

$$^{210}_{84}Po \longrightarrow \, ^{210-4}_{84-2}X' + \alpha \, ^4_2He$$

$$= \, ^{206}_{82}X' = \, ^{206}_{82}Pb \quad (\text{lead})$$

Daughter nuclei = Lead

$$Q = K_\alpha \left(\frac{A}{A-4}\right)$$

$$= 5.30 \left(\frac{210}{206}\right) = 5.40 \text{ MeV}$$

$$Q = M(Po) - m(Pb) - m(^4_2He)$$

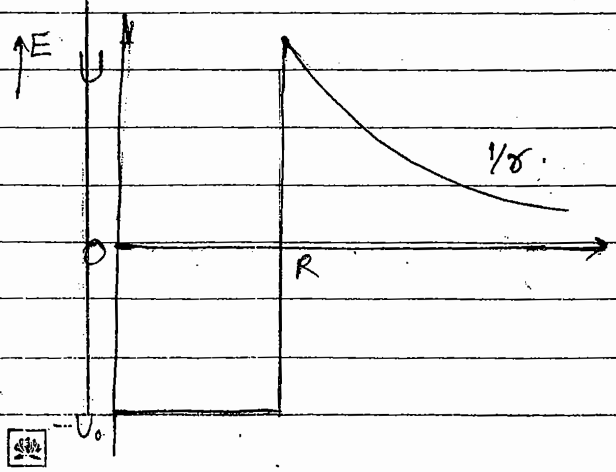
$$5.40 = \cancel{M}(209.9829) \times 931.5 - m(Pb) - 4.0026 \times 931.5$$

$$m(Pb) = 205.0004 \times 931.5 - 5.40$$

$$= 190957.8726 - 5.40$$

$$= 190952.4726 \text{ MeV} = \frac{190952.4726}{931.5 \text{ MeV}}$$

$$m(Pb) = 205.9744 \, \cancel{\text{MeV}} \, u$$



Behaviour of α-particle inside & outside the nucleus :-

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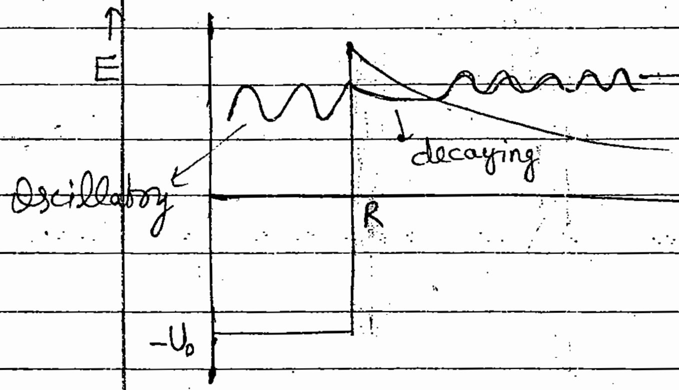
Particle is localised & wave which is not localised.



The wave after crossing barrier again oscillatory wave but its amp. is smaller than orignal. So it indicates that there is small but finite prob. to tunnel the barriers.

$$\frac{d^2\psi}{dx^2} + \underbrace{\frac{2m}{\hbar^2}(E-U)}\,\psi = 0$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

where $k = \sqrt{\frac{2m}{\hbar^2}(E-U)} = \sqrt{(-)\frac{2m}{\hbar^2}(U-E)}$ $\quad (E>U)$

This will give the oscillatory sol^n.

If we solve the Sch^r wave eq^n for a particle tunneling through the barrier of height U_0 (amount of energy) then the probability of tunneling is equal to

$$\text{Probability} = e^{-2k_2 L}$$

$L \rightarrow$ thickness of barrier.

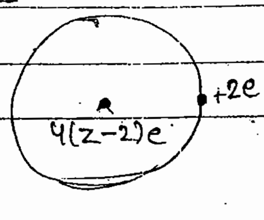
$$k_2 = \sqrt{\frac{2m}{\hbar^2}(U_0 - E)}$$

For α-particle, Energy = 4 MeV – 9 MeV

But for the heavy Nuclei,

$$\text{Barrier Potential} = \frac{1}{4\pi\epsilon_0} \times \frac{2(Z-2)e^2}{R}$$

$$\simeq 25 - 40 \text{ MeV}$$



If min ; we take 25 MeV

& α = 4 MeV

i.e. α particle has 4 MeV & to cross barrier need 25 MeV. So 21 MeV energy must be produced by some other means.

$Q = 25 - 40$ MeV

This variation for different heavy nuclei.

α-particle strikes for 10^{21} time on barrier. In 10^{21} times, only one chance to come out of nuclei.

- Probability per unit time λ for α-particle to decay (or to come out of nucleus) is equal to the prob. of penetrating the barrier $\times$ no. of strikes per sec.

~~$\lambda =$~~ If v is the velocity of α-particle & R is the radius of the nucleus then time taken in one strike,

Time taken in one strike $= \dfrac{2R}{v}$

No. of strikes in one sec., freq $= \dfrac{v}{2R}$

So prob. per unit time λ for α-decay

$$\boxed{P = \frac{v}{2R} \cdot e^{-2k_2 t}}$$

The is known as Gamow theory of α-decay. (1928)

Prob: A nucleus having mass no. 240 decays by the emission of to the ground state of its daughter nucleus. The Q value of the process is 5.26 MeV. The energy in MeV for α-particle will be

(i) 5.26 (ii) 5.17 (iii) 5.13 (iv) 5.09

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$A = 240$, $Q = 5.26$ MeV

$$K_\alpha = \left(\frac{A-4}{A}\right) Q = \frac{240-4}{240} \times 5.26$$

$$K_\alpha = \frac{236}{240} \times 5.26$$

$$K_\alpha = 5.1723 \text{ MeV}$$ Ans

Que:- The activity R of a sample of unknown radioactive nuclei is measured at hourly interval. The result in Mega Bquarel (MBq) are

80.5   36.2   16.3   7.3   3.3

Find the half life of the radioactive substance.

$$A = A_0 e^{-\lambda t}$$

Here activity $\rightarrow R$

so $R = R_0 e^{-\lambda t}$

$$\ln R = \ln R_0 - \underbrace{\lambda}_{\text{slop}} t \qquad y = mx + c$$

$$\Rightarrow \lambda = \frac{\ln R_0 - \ln R}{t}$$

$$= \frac{1}{t} \times \ln\left(\frac{R_0}{R}\right)$$

$$= \frac{1}{1\text{ Hr}} \times \ln\left(\frac{80.5}{36.2}\right)$$ {Take any 2 values}

$$= \frac{1}{60} \times 0.79919807 = 0.01331997$$

$$t_{1/2} = \frac{0.693}{\lambda} = 52 \text{ min.}$$

Q.

α-D

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Que:- The activity of a sample of unknown radioactive nuclei is measured in ~~ddi~~ daily intervals. The results in MBq are,

Activity (MBq)	32.1	27.2	23.0	19.5	16.5
(days) t	0	1	2	3	4

find half life $t_{1/2}$?:

$$A = A_0 e^{-\lambda t}$$

$$\ln A = \ln A_0 - \lambda t$$

$$\lambda = \frac{\ln A_0 - \ln A}{t}$$

$$= \frac{1}{t} \ln\left(\frac{A_0}{A}\right)$$

$$= \cancel{\frac{1}{1} \ln\left(\frac{32.1}{27.1}\right)} \quad \cancel{\ln} \quad \frac{1}{1} \ln\left(\frac{32.1}{27.2}\right)$$

$$= \ln(1.18015)$$

$$= 0.16564$$

$$t_{1/2} = \frac{0.693}{0.16564} = \underline{4.18377 \text{ days}}. \text{ Ans}$$

α-Decay

$${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}X' + {}^{4}_{2}He \ \{\alpha\text{-particle}\}$$

This is purely a Nuclear phenomenon

K.E. of α-particle, $K_\alpha = Q\left\{\frac{A-4}{A}\right\}$

Mostely K.E. is less than Q-value.

If α-particle having K.E. K_α & is moving then X' must also move.

Initially assuming X^* to be at rest, then

$$Q = K_{X'} + K_\alpha \quad \text{——— (1)}$$

$$= K_{X'} + \frac{p_\alpha^2}{2m_\alpha}$$

from the conservation of momentum,

$$\vec{P}_{X'} + \vec{P}_\alpha = 0 \qquad \text{(initial mom. = 0)}$$

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$|\vec{p}_\alpha| = |\vec{p}_{x'}|$

$$\therefore Q = K_{x'} + \frac{p_{x'}^2}{2m_\alpha} \times \frac{m(x')}{m(x')}$$

where $\frac{p_{x'}^2}{2m_{x'}} = K_{x'}$

So $$\boxed{Q = K_{x'} + \frac{K_{x'}\, m(x')}{m_\alpha}}$$

$$Q = K_{x'}\left[\frac{m_\alpha + m(x')}{m_\alpha}\right]$$

$$Q = K_{x'}\left[\frac{4 + A - 4}{4}\right] = K_{x'}\left[\frac{A}{4}\right]$$

$$\boxed{K_{x'} = \frac{4}{A}Q} \quad \text{——— (2)}$$

This is the K.E. of Recoil nucleus.

So $$\frac{K_\alpha}{K_{x'}} = \frac{Q\left\{\frac{A-4}{A}\right\}}{\frac{4}{A}Q}$$

$$\boxed{\frac{K_\alpha}{K_{x'}} = \frac{A-4}{4}}$$

β-Decay :- In β-decay a neutron in the nucleus changes into a proton or proton into a neutron. Z and N changes by one unit. But A is invarient.

There are 3 types of β-decay observed :-

i) β^- decay
ii) β^+ "
iii) Electron Capture

(i) **β^- Decay :-** In β^--decay, Z increases by 1 unit & Hence N decreases by 1 unit in order to make A constant.

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$$_Z^AX \longrightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e \qquad ({}_{-1}^{0}e \rightarrow \beta^-)$$

Here $n \longrightarrow p + e^-$

Q-value corresponding to β^--decay will be

$$Q_{\beta^-} = [M_{nuc}(X) - M_{nuc}(Y) - m_e]\,c^2$$

Convert this nuclear mass in terms of atomic mass

$$Q_{\beta^-} = [M(X) - \cancel{Z m_e} - M(Y) + (\cancel{Z+1}) m_e - \cancel{m_e}]\,c^2$$

Now these are the atomic masses.

{Mass of Nucleus = Mass of atom − Mass of e^- }

$$Q_{\beta^-} = [M(X) - M(Y)]\,c^2$$

$$\boxed{Q_{\beta^-} = [M(Z,A) - M(Z+1,A)]\,c^2}$$

Q-value is +ve only if Mass of parent atom > Mass of daughter

i.e. $Q > 0$ for $M(Z,A) > M(Z+1,A)$

(ii) β^+ decay :- Z ↓ by 1 unit & N ↑ by 1 unit so that to make A constant.

$$_Z^AX \longrightarrow {}_{Z-1}^{A}Y + {}_{+1}^{0}e \qquad ({}_{+1}^{0}e \rightarrow \beta^+)$$

positron

proton is the lighest baryon.
p can be decay only into a lighter particle than p but such a particle does not exist. So this rec^n does not possible in free space.

Q-value,

$$Q_{\beta^+} = [M_{nuc}(X) - M_{nuc}(Y) - m_e]\,c^2$$

$$= [M(Z,A) - Z m_e - M(Z-1,A) + (Z-1) m_e - m_e]\,c^2$$

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$$Q_{\beta^+} = [M(Z,A) - M(Z-1,A) - 2m_e]\,c^2$$

In case of nuclear phenomenon It hase to be modified.

Q-value will be +ve if & only if
The difference in the mass of parent & daughter is greater that by an amount of at least equal to 2 times of mass of e^-.

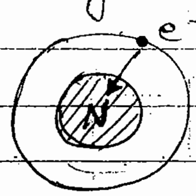
i.e. β^+ disintegration takes place only when
Mass of parent atom > Mass of the daughter atom
at least 2 times of m_e.

$m_e = 0.511$ MeV

$2m_e = 1.022$ MeV ie (Mass of parent > 1.022)

(iii) **Electron Capture** :- The nucleus capture the e^- moving in the orbit & after capturing the e^- nucleus X is converted into Y

$$^{A}_{Z}X + e^- \longrightarrow {}^{A}_{Z-1}Y$$



Q-value in this case,

$$Q_{e^-\text{Capt.}} = [M_{nuc.}(X) + m_e - M_{nuc.}(Y)]$$

$$= [M(Z,A) - Zm_e + m_e - M(Z-1,A) + (Z-1)m_e]\,c^2$$

$$\boxed{Q_{e^-\text{capt.}} = [M(Z,A) - M(Z-1,A)]\,c^2}$$

This is analouges to the Ist case.
Q will be +ve, if
Mass of parent > Mass of daughter. (simply)

Electron-Capture process is possible only when the mass of the parent atom is greater than the daughter atom.

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Ques. 1 :- The only known nuclei with $A = 7$ are ${}^{4}Li_{3}$ whose atomic mass $M(7,3) = 7.016\,u$ and ${}^{3}Be_{4}$, $M(7,4) = 7.0169\,u$. Which of these nuclei is stable to β-decay. What process (e⁻-capture or positron emission) is employed in β-decay of the unstable nucleus to the stable nucleus.

$M(7,3) = 7.016\ u \longrightarrow Li$
$M(7,4) = 7.0169\ u \longrightarrow Be$

Assume $M(7,3) \rightarrow$ unstable & $M(7,4) \rightarrow$ stable then
$M(A,Z) \longrightarrow M(A, Z+1)$
Mass is ↑ by 1 unit so it must by β^- decay but for β^- decay, Mass of parent should be greater than mass of daughter. But here mass of daughter is greater than parent. so This recⁿ is not possible. $\beta^- \rightarrow$ Not possible.
i.e. Mass of ${}^{4}Li_{3} <$ ${}^{3}Be_{4}$ so β^- is not permitted.

Therefore Li is stable to β-decay
So Be can be decay into lithum.

$${}^{3}Be_{4} \longrightarrow {}^{4}Li_{3}$$

Here Z is ↓ by 1 unit So 2 possibilities.
β^+ & e^--capture.

Mass difference $= 7.016\,u - 7.0169$
$= 0.0009\,u$
$= 0.0009 \times 931.5$ MeV
$= 0.83835$ MeV

{ diff should be > 1.022 MeV } ⇒ (for β^+ decay)

So this decay will undergo e^--capture.
Ans

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Q.2 :- Masses are given, $_{28}Ni^{64} \equiv 63.9280\ u$
$_{29}Cu^{64} \equiv 63.9298\ u$
Which of them is unstable & what kind of decay will take place ~~form~~ the unstable one.

Assume Ni is unstable then

$$_{28}Ni^{64} \longrightarrow {}_{29}^{64}Cu$$

Here $Z \uparrow$ by 1 unit so β^- decay but

Mass of $_{28}Ni^{64} < {}_{29}Cu^{64}$

$\beta^- \rightarrow$ Not possible.

Now suppose Cu is unstable

$$_{29}Cu^{64} \longrightarrow {}_{28}Ni^{64} \Rightarrow 2 \text{ possibilities}$$
β^+-decay & e^--capture.

Mass diff $= 63.9298 - 63.9280$
$= 0.0018\ u = 0.0018 \times 931.5 = 1.67$ MeV

~~0.0018 < 1.002~~

Mass diff > 1.002 MeV

~~1.67 MeV~~ > 1.002 MeV

β^+ decay will dominate.

- Before, the concept of particle physics, n is decayed into p as

$$\underset{\frac{1}{2}}{n} \longrightarrow \underbrace{\underset{\frac{1}{2}}{p} + \underset{\frac{1}{2}}{e^-}}_{0,1}$$

It violets the ~~con~~ angular momentum conservation so we have to add another particle in L.H.S whose spin is 1/2.

Q-value of this

$Q = 0.782$ MeV

* Positron & Neutrino were discovered ~~by~~ after ~~the~~ a long time of their prediction.

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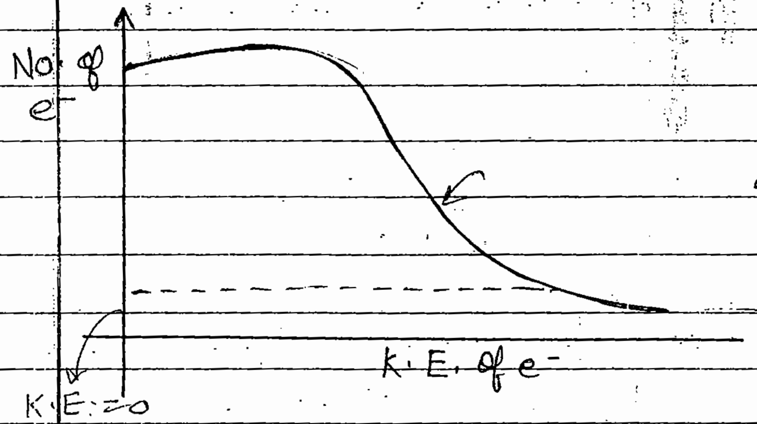
Most of the Q-value i.e. K.E. is given to the e^- & Y is bulky

$$X \longrightarrow Y + e^-$$

So e^- which are coming out of nucleus are monoenergetic & their energy will be 0.782 MeV & all the e^- have equal amount of energy.

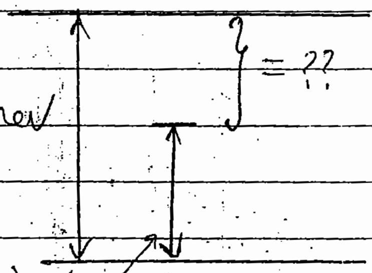


Most of e^- have K.E. = 0

← but ↑ forward very few e^- have maxi. energy = 0.782 MeV

available energy for Q-value = 0.782 MeV

T.E. = 0.782 MeV



But β-particle will emit by this much energy.

What is The Rest energy = ?

This rest energy is given to another particle called Neutrino — This was Pauli Hypothesis.

$$n \longrightarrow p + e^- + \bar{\nu}_e \text{ (antineutrino)}$$

$$\tfrac{1}{2} \qquad \tfrac{1}{2} \quad \tfrac{1}{2} \quad \tfrac{1}{2}$$

<u>Neutrino</u> :- charge = 0

Its name is Italian.

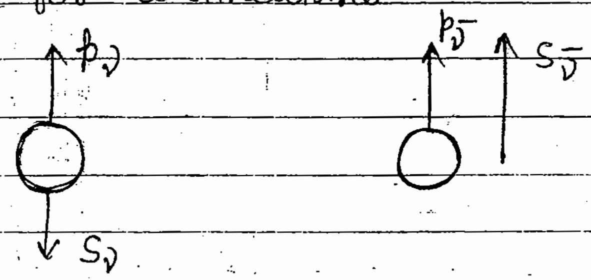
Neutrino ≡ little neutral one

Acc. to Dirac, for every particle, there ~~shd~~ should be antiparticle also.

How can we distinguish them?

Neutrino	Antineutrino
Rest Mass $= 0$	Rest mass $= 0$
charge $= 0$	charge $= 0$

The angular momentum vector p_ν and spin S_ν are antiparallel in the case of neutrino while they are parallel for antineutrino.



Therefore we calle neutrino - Left handed particle & antineutrino - Right handed particle. We define the handedness of neutrino & antineutrino by introducing a parameter called "Helicity."

It is defined as,

$$H = \frac{\vec{p} \cdot \vec{s}}{|\vec{p}|\,|\vec{s}|}.$$

$H = -1$ for Neutrino
$H = +1$ " Antinutrino

Que:- ^{23}Ne is decays into ^{23}Na by -ve β emission (β^-). What is the maximum K.E. of emitted e^- if M of

$^{23}_{10}Ne \equiv 22.994465$ u
$^{23}_{11}Na \equiv 22.989768$ u

$$^{23}_{10}Ne \longrightarrow {}^{23}_{11}Na$$

$Z \longrightarrow Z+1$ & Mass of Parent > Mass of daughter

So β^- decay is possible

$$^{23}_{10}Ne \longrightarrow {}^{23}_{11}Na + {}^{0}_{-1}e$$

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Q-value, $Q_{\beta^-} = [M(Z,A) - M(Z+1,A)]\,c^2$

$= [22.994465 - 22.989768]\,c^2$

$= 0.0047 \times 931.5$

$= 4.37805$

Maxi K.E. of $e^- = 4.37805$

Ques:- ^{40}K is an unusual isotope in that it decays by -ve β^--decay, β^+-decay & e^--capture. Find the Q-values for these decays. Given that - mass of

$^{40}_{19}K = 39.96399$

$^{40}_{20}Ca = 39.962591$

$^{40}_{18}Ar = 39.962384$

For β^--decay :- $^{40}_{19}K \longrightarrow {}^{40}_{20}Ca + {}^{\,0}_{-1}\beta + \bar{\nu}_e$

$Q_{\beta^-} = [M(Z,A) - M(Z+1,A)]\,c^2$

$= [39.96399 - 39.962591]\,c^2$

$= 0.00149 \times 931.5$

$\boxed{Q_{\beta^-} = 1.38793}$

for β^+-decay :- $^{40}_{19}K \longrightarrow {}^{40}_{18}Ar + {}^{\,0}_{+1}\beta + \nu_e$

$Q_{\beta^+} = [M(Z,A) - M(Z-1,A) - 2m_e]\,c^2$

$= [(39.96399 - 39.962384)_u - 1.022]\,c^2$

$= (0.00169 \times 931.5) - 1.022$

$= 1.57424 - 1.022$

$\simeq \cancel{0.55224}$ $\boxed{0.473989 = Q_{\beta^+}}$

Here diff. of masses of parent & daughter > 1.022 so β^+-decay is possible.

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for e^- - capture :- ${}^{40}_{19}K + {}^{\ \ }_{-1}e^{-} \longrightarrow {}^{40}_{18}Ar + \nu_e$ ${}_{-1}\beta^{0}$

$Q_{value} = [M({}^{40}K) - m({}^{40}Ar)]c^2$

$= (39.96399 - 39.962384) \times 931.5$

$Q = 1.495989$ MeV

$\boxed{Q_e \simeq 1.5 \text{ MeV}}$

Ques :- ${}^{12}N$ beta decays to an excited state of ${}^{12}C$ which subsequently decays to the ground state with the emission of γ-photon of 4.43 MeV energy. What is the maximum K.E. of the emitted β particle. $M({}^{12}N) = 12.0186134$

Mass of Carbon atom ${}^{12}_{6}C = 12$

${}^{12}N$ decay to excited state of ${}^{12}C$,

$${}^{12}_{7}N \longrightarrow {}^{12}_{6}C^{*} + {}^{0}_{+1}\beta + \nu_e$$

Neutrino shared no energy so Maxi. K.E. of β-particle will be equal to the Q-value.

C^* decay to its ground state,

$${}^{12}_{6}C^{*} \longrightarrow {}^{12}_{6}C + \gamma$$

Mass of ${}^{12}_{6}C^{*}$ = Mass of ${}^{12}_{6}C$ + Mass of γ

$= 12u + 4.43$ MeV

$= (12 \times 931.5 + 4.45)$ MeV

$= 11182.43$ MeV

$Q_{value} = [m({}^{12}N) - m({}^{12}C^{*}) - 2m_e]$

$= [12.018613 \times 931.5 - (11182.43) - 1.022]$

$= 11195.33801 - 11182.43 - 1.022$

$= 11.886$

$\boxed{Q_{value} \simeq 11.89}$ Ans

Que :- The masses of ^{11}C & ^{11}B are respectively 11.0114 u & 11.0093u. Find the maxi. energy a positron can have in β^+-decay of ^{11}C to ^{11}B.

$$^{11}_{6}C \longrightarrow {}^{11}_{5}B + {}^{0}_{1}\beta + \nu_e$$

$$Q\text{-value} = [M({}^{11}_{6}C) - M({}^{11}_{5}B) - 2m_e]$$

$$= [(11.0119 - 11.0093)u - 1.022]$$

$$= \cancel{\ldots}\ 1.95615 - 1.022 \text{ MeV}$$

$$= 0.93415 \text{ MeV} \quad \underline{\text{Ans}}$$

Que : Calculate the Q-values in the following decays:

(a) $^{19}O \longrightarrow {}^{19}F + {}^{0}_{-1}e + \bar{\nu}_e$

(b) $^{25}Al \longrightarrow {}^{25}Mg + e^+ + \nu_e$

The atomic masses needed are as follows:

^{19}O	^{19}F	^{25}Al	^{25}Mg	
19.003576	18.998403	24.990432	24.985839	(u)

(a) $^{19}O \longrightarrow {}^{19}F + {}^{0}_{-1}e + \bar{\nu}_e$

$$Q_{\beta^-} = [M({}^{19}O) - M({}^{19}F)]c^2$$

$$= [19.003576 - 18.998403] \times 931.5$$

$$= 0.005173 \times 931.5$$

$$\boxed{Q_{\beta^-} = 4.8186 \text{ MeV}}$$

(b) $^{25}Al \longrightarrow {}^{25}Mg + e^+ + \nu_e$

$$Q_{\beta^+} = [[M({}^{25}Al) - M({}^{25}Mg)]u - 1.022]$$

$$= [(24.990432 - 24.985839)\times 931.5 - 1.022] \text{ MeV}$$

$$= 0.004593 \times 931.5 - 1.022$$

$$= 4.2783795 - 1.022$$

$$\boxed{Q_{\beta^+} = 3.2563 \text{ MeV}}$$

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Que:- Find the maximum K.E. that a β-particle can have in the following reeⁿ :

$$^{176}Lu \longrightarrow {}^{176}Hf + e^- + \bar{\nu}_e$$

$$\underset{175.9426944}{\downarrow} \qquad \underset{175.9414204}{\downarrow}$$

$$Q\text{-value} = [m(^{176}Lu) - m(^{176}Hf)]c^2$$

$$= [175.9426944 - 175.9414204] \times 931.5$$

$$\boxed{\text{Maxi K.E of } \beta = 1.186731 \text{ meV}} \text{ Ae}$$

Que: Consider the β-decay,

$$^{198}Au \longrightarrow {}^{198}Hg^* + \beta^- + \bar{\nu}_e$$

where Hg^* repⁿ a mercury nucleus in excited state at energy 1.088 MeV above the ground state. What can be the maxi. K.E. of e^- emitted if the atomic mass of ^{198}Au = 197.968233 u
^{198}Hg = 197.966760 u

$$Hg^* \longrightarrow Hg + \gamma$$

$$M(Hg^*) = M(Hg) + M(\gamma)$$

$$= (197.966760 \times 931.5 + 1.088) \text{ meV}$$

$$= 184407.1249 \text{ meV}$$

$$Q = [M(^{198}Au) - M(^{198}Hg^*)]c^2$$

$$= (197.968233) \times 931.5 - 184407.1249$$

$$= 184407.409 - 184407.1249$$

$$= 0.2841395 \text{ meV}$$

$$\boxed{Q \simeq 0.28 \text{ meV}} \text{ Ae}$$

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Gamma Decay :-

α - decay & β - decay are always followed by γ decay bcoz after the emission of α & β then nucleus is in excited state. And to go to the ground state nucleus emit γ - photon.

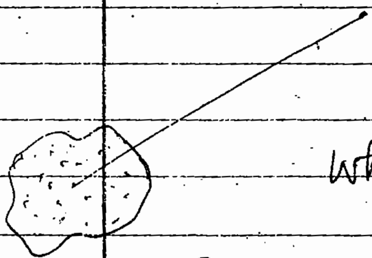
Inside the nucleus, electric & magnetic both type of dipole exist.

Following α or β - decay the final nucleus may be left in excited state & it will reach to ground state after emitting one or more γ - photons.

Assuming that the nucleus to be at continous distribution of charge the potential at a certain distance from the nucleus centre can be expended in terms of various ~~pot~~ powers of r distance.

$$V = \frac{1}{4\pi\varepsilon_0}\left[\sum_i \frac{q_i}{R_0} + \frac{1}{R_0^2}\sum_i q_i r_i \cos\theta + \frac{1}{R_0^3}\sum_i q_i r_i^2 \left(\frac{3}{2}\cos^2\theta - 1\right) + \cdots\right.$$

$$\left. \cdots + \frac{1}{R_0^{L+1}}\sum_i q_i r_i^L P_L(\cos\theta) + \cdots\right]$$



where $P_L \longrightarrow$ Legendre's Polynomials.

For $L=0$, I - term $\Rightarrow$ Pot^n due to monopole. (south pole, North pole, $+q$, $-q$)

~~$X \neq X$~~ $\qquad V = \frac{1}{4\pi\varepsilon_0}\frac{\Sigma_i q_i}{R_0}$

$L = 1$, corresponds pot^n due to dipole

$$V = \cancel{\frac{1}{R_0^2}}\frac{1}{4\pi\varepsilon_0}\frac{\Sigma_i q_i r_i \cos\theta}{R_0^2}$$

$L = 2$, corresponds pot^n due to Quadrupole.

If charge distribution has time varing moments, then it will radiate energy. In general V_L

Yes → means Parity change
e.g. $1^+ \to 2^-$
No → means No parity change

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corresponds to the radiation field of an electric 2^L pole & magnetic 2^{L-1} pole transition.
L → is called the multipolarity of the radiation & it is a measure of the no. of units of angular momentum transferred in radiative transitions.
The parity selection rules states that in electric multipole transition, the parity is even when L is even & odd when L is odd
i.e. $(-1)^L$

Magnetic Multipole transition, the parity is given by
$(-1)^{L+1}$

I^P	Transition	Parity Change	Forward multipoles
(i)	$0 \longleftrightarrow 0$	Particle in G.S. can't radiate itself	forbidden
(ii)	$1 \longleftrightarrow 0$	Yes	E1
		No	M1

$\left\{ \text{if } I_i = 1,\ I_f = 0 \text{ then } L = I_i - I_f \text{ to } I_i + I_f;\ 1-0 \text{ to } 1+0 \Rightarrow L = 1 \right\}$, $L = 1$

	Transition	Parity Change	Forward multipoles
(iii)	$1 \longleftrightarrow 2$	Yes	E1 M2 E3
		No	M1 E2 M3

$\left\{ I_i = 1,\ I_f = 2 \right\}$
$\left\{ L \equiv |1-2| \text{ to } |1+2| \Rightarrow 1,2,3 \Rightarrow L \equiv 1,2,3 \right\}$

	Transition	Parity Change	Forward multipoles
(iv)	$1 \longleftrightarrow 3$	Yes	M2 E3 M4
		No	E2 M3 E4

Allowed L values $\equiv |1-3|$ to $|1+3| = 2,3,4$.

	Transition	Parity Change	Forward multipoles
(v)	$2 \longleftrightarrow 3$	Yes	E1 M2 E3 M4 E5
		No	M1 E2 M3 E4 M5

L values $\equiv |2-3|$ to $|2+3| = 1,2,3,4,5$

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NEE :- No, Even, Electric
YEM :- Yes, Even, magnetic

Ques - Find the allowed multipole transitions in the formula :-

(i) $2^+ \longrightarrow 0^+$

No parity change.

$L \equiv |2-0|$ to $|2+0| = 2 \Rightarrow E2$

(ii) $4^+ \longrightarrow 2^+$ Parity No $L \equiv 2, 3, 4, 5, 6 \Rightarrow E2\ M3\ E4\ M5\ E6$

E2 dominent one

It is found that the probability of multipole emission decreases rapidly with increasing L. For same L, electric radiation is more probable than magnetic radiation.

(iii) $2^+ \rightarrow 1^-$ Yes $L \equiv 1, 2, 3$ E1 M2 E3

E1 Highly favoured

(iv) $1^- \longrightarrow 0^+$ Yes $L \equiv 1$ E1

(v) $2^+ \longrightarrow 1^+$ No $L \equiv 1, 2, 3$ M1 E2 M3

(vi) $1^+ \longrightarrow 0^+$ No $L \equiv 1$ M1

(vii) $\frac{1}{2}^- \longrightarrow \frac{1}{2}^+$ Yes $L \equiv 0, 1$ ~~M0~~ E1

L=0 corresponds to monopole

(viii) $\frac{3}{2}^+ \longrightarrow \frac{1}{2}^+$ No $L \equiv 1, 2$ M1 E2

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Que :- The spin, parity of the ground state nucleus is 0^-. It absorbs a γ-ray & goes to the excited state. If the transition is magnetic dipole then the spin parity of the excited state will be

(a) 0^- (b) 1^- (c) 1^+ (d) 0^+

0 is forbidden so (a) & (d) $\rightarrow$ x

Transition is mag. diople, it corresponds to $L=1$ i.e. M1 so from 0^- to which state so that M_1 emit so NEE.

$$0^- \longrightarrow 1^- \Rightarrow M1$$

(b) ✓

Que :- ^{10}Be in its I-excited state has spin parity 2^+. It gets de-excited to the ground state which has spin parity 0 by γ-emission. The multipole carried by γ-radiation, are

(a) E2 (b) M2 (c) E2 M2 (d) E4

$$2^+ \longrightarrow 0^+ \Rightarrow E2$$

Que - Which one of the following corresponds to the electric dipole γ-transition

(a) $\frac{3}{2}^+ \longrightarrow \frac{1}{2}^-$ (c) $1^+ \longrightarrow 1^+$

(b) $\frac{3}{2}^+ \longrightarrow \frac{1}{2}^+$ (d) $3^+ \longrightarrow 0^-$

(a) $L = 1, 2$ Yes E1 M2

(c) $L = 0, 1, 2$ No M1 E2

(b) $L = 1, 2$ No M1 E2

(d) $L = 3$ Yes E3

~~E1 M2 is for elec dipole γ-trans.~~

(a) ✓

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Q: What are the possible electric & magnetic multipolarities involve in the transition

$$h_{11/2} \longrightarrow d_{3/2}$$

Parity $\equiv (-1)^l$

for $h_{11/2} \longrightarrow l = 5$ so Parity $= (-1)^5 = -1$ odd

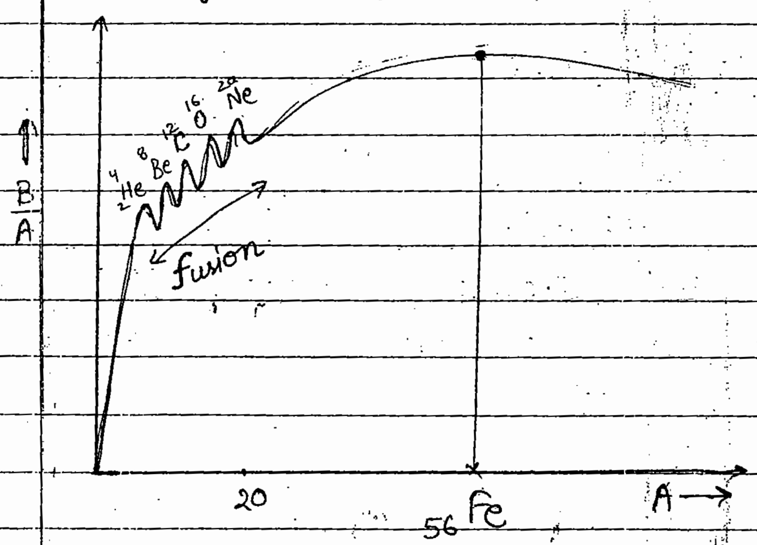
$d_{3/2} \longrightarrow l = 2$ so Parity $= (-1)^2 = 1$ even

so $\frac{11}{2}^- \longrightarrow \frac{3}{2}^+$ Parity change $\rightarrow$ Yes

$L = \left|\frac{11}{2} - \frac{3}{2}\right|$ to $\left|\frac{11}{2} + \frac{3}{2}\right|$

$= 4$ to 7 $= 4, 5, 6, 7$ $\Rightarrow$ M4 E5 M6 E7

Binding Energy Curve :-



Features of Curve : (1) Curve is almost continous except at peaks ^4He, ^8Be, ^{12}C, ^{16}O & ^{20}Ne

(2) In light nuclei for which $A < 30$, the maximum Binding energy is for ^4_2He, ^8_4Be, $^{12}_6C$, $^{16}_8O$, $^{20}_{10}Ne$ which posses α - clusters α, 2α, 3α, 4α, 5α.

(3) The maximum B.E. is 8.79 MeV for $_{56}Fe$.

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(4) When 2 light nuclei combine to form a heavy nuclei, B.E. per nucleon i.e. B/A increases. But in the same way, when a heavy nuclei is broken into 2 light nucli B.E./A increases. The process are called Fusion & fission respecitvely.

(5) Nuclear forces are short range forces & B.E. must be supply to make its constituents free. Just like in the case of liquid, Wander Wall forces are short range forces. (The energy supplied to change liquid to gasseous state is → latent heat of vapourisation.) & Latent heat of Vapourisation must be supplied to make their constituents (molecules of water) free, thus a nucleus is analogous to a drop of liquid & Hence Liquid Drop Model was proposed.

(6) Even N & Even Z nuclei are most stable bcoz 2 protons & 2 neutrons of opposite spin form a pair.

(7) For Z = 2, 8, 20, 28, 50, 82, 126, they posses more B.E. ~~Hence~~ & Hence Shell model was proposed as these Magic no's could not be explained by L.D.M.

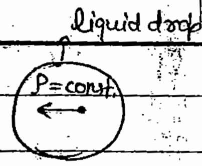
Liquid Drop Model

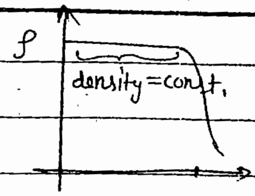
It is based on two properties which are common to almost all the nuclei :-

1- Interior mass densities are app. same (const.)

2- B.E. are app. proportional to mass no.

$\frac{B}{A}$ – constant





$\boxed{B \propto A}$

Comparision to an incompressible liquid drop :-

1- Interior density is constant.

2- Their heats of vapourisation are proportional to their masses. ($Q = mL$)

These are the only two things bcoz of which we consider a liquid drop as nucleus.

B.E. & mass relationship.

$$\boxed{M(Z,A) = ZM_H + (A-Z)M_n - B(Z,A)}$$

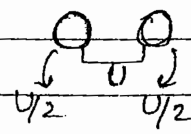
Therefore, B.E. can be written as

$$\boxed{B(Z,A) = ZM_H + (A-Z)M_n - M(Z,A)}$$

This was given by Weizsacker, called Semi-imperical mass formula.

Binding Energy Terms :-

(i) Volume Energy :- Interaction energy b/w 2 nuclei is U & this energy is shared by both nuclei so energy shared by each nuclei is U/2.



To cover one sphere, we need 12 more sphere so energy shared by 1 nuclei $= \frac{U}{2} \times 12$
$= 6U$

If there are A no. of nucleons then volume energy $B_v = \underbrace{6A U}_{\text{constant}}$

$$\boxed{B_v = a_v A}$$

$\overline{B} =$ const. $\Rightarrow \frac{B}{A} =$ const. $\Rightarrow B \propto A$

$$\boxed{B_v = a_v A}$$

same thing.

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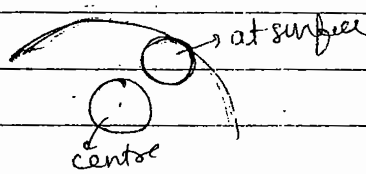
2) Surface Energy :-

Centre sphere touches all the 12 spheres but the sphere at boundary touches ~~only~~ slightly less no of spheres. So B.E ↓.

Surface energy term will ↓ the B.E.

& larger the sphere at surface i.e. larger the surface area, larger will be the decrease in B.E.



$B_S \propto$ Surface Area

$B_S \propto 4\pi R^2$

$\propto 4\pi R_0^2 A^{2/3}$

$B = -\text{const.}\ A^{2/3}$

$$\boxed{B_S = -a_S A^{2/3}}$$

-ve sign shows ↓ in B.E.

3) Coulombian Energy :-

proton is inside the nucleus have +ve charge so there will be coulomb repulsion b/w p & p.

The coulomb attraction will ↑ the B.E & ||ly

" repulsion " ↓ " "

So this Coulomb energy will ↓ the B.E.

$$B_c = -\frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

$$= -\frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R_0 A^{1/3}}$$

$$\boxed{B_c = -a_c A^{-1/3} Z(Z-1)} \Rightarrow \boxed{B_c = -a_c Z(Z-1) A^{-1/3}}$$

or $B_c = -a_c Z^2 A^{-1/3}$

where $a_c = \frac{3}{5}\frac{\cancel{Z(Z-1)}\, e^2}{4\pi\varepsilon_0 R_0}$

These 3 terms are classical terms.

4) Assymmetric Energy :- In light nuclei, there is a tendency for neutrons & protons to be equal. $\underbrace{N = Z}_{\text{This is symmetry.}}$, to form most stable configuration.

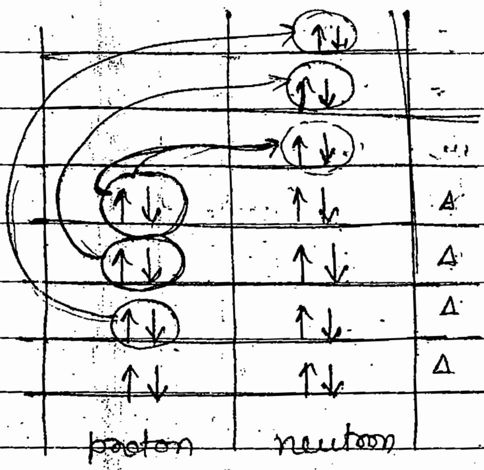
As we go to the higher nuclei, increase in the no. of protons decreases the B.E. bcoz of Colombain repulsion so some extra neutrons must be present to provide aditional n-n bonds to compensate it.

However this disturbs the condition of $N \equiv Z$

$N \equiv Z \rightarrow$ Most stable

$N \not\equiv Z \rightarrow$ Asymmetry will ↓ the B.E.

We introduce asymmetry -



$N - Z = 10 - 6 = 4$ $\quad \Delta E = 2\Delta$

$N - Z = 12 - 4 = 8$ $\quad \Delta E = 6\Delta + 2\Delta$, $\Delta E = 8\Delta$

$N - Z = 14 - 2 = 12$ $\quad \Delta E = 10\Delta + 8\Delta$, $\Delta E = 18\Delta$

general Expression

$$\Delta E = \frac{(N-Z)^2}{8} \Delta$$

Thus we see that, the asymmetry energy

$$B_{as} \propto \frac{(N-Z)^2 \Delta}{8}$$

The speration b/w the energy level decreases with increasing ~~a~~ A. Thus

$$\Delta \propto \frac{1}{A}$$

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Thus $B_{as} \propto \frac{(N-Z)^2}{8A}$

$$B_{as} = -a_{as}\frac{(N-Z)^2}{A}$$

$$\boxed{B_{as} = -a_{as}\frac{(A-2Z)^2}{A}}$$

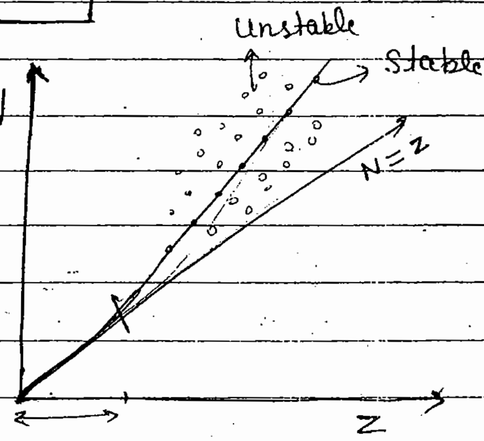
• Nuclei which are β^- emitters lie -

(a) below the line of β stability

(b) on the " " "

✓(c) above " " "

(d) below N = Z line



The neuclei having greater no. of ~~e~~ neutrons will emit β^--particle to get stability. & nuclei above the stable line ~~are~~ have more no. of n.

Paring energy :- It has been observed experimentaly that even-even nuclei (i.e. no. of n & p both are even) are more stable as compared to that of even-odd or odd-odd nuclei.

n : even, p : even	most stable	odd, odd — least stable	Even odd OR odd even — Odd A (intermediate stable)

Paring, Binding Energy is

$$B_p = \begin{cases} +\delta & \text{for even-even} \\ 0 & \text{for odd A (even-odd or odd-even)} \\ -\delta & \text{for odd-odd} \end{cases}$$

Fermi Proposed that, the paring energy δ is directely proportional to $A^{-3/4}$

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$\delta \propto A^{-3/4}$

Therefore,

$$B_p = \begin{cases} a_p A^{-3/4} & \text{even - even} \\ 0 & \text{Odd } A \\ -a_p A^{-3/4} & \text{odd - odd} \end{cases}$$

Quantum terms → Asymmetric & Parity energy :

Shell Effect :- comes to conserve magic no.

Deformed nuclei effect :- deviation from spherical nuclei.

These 2 effects are contributing negligible.

Total B.E.

$$B(Z,A) = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{as}\frac{(A-2Z)^2}{A} \pm \delta$$

$$\delta = \pm a_p A^{-3/4}$$

→ Now, Semi Empirical Mass formula,

$$M(Z,A) = ZM_H + (A-Z)M_n - B(Z,A)$$

$$\boxed{M(Z,A) = ZM_H + (A-Z)M_n - a_v A + a_s A^{2/3} + a_c Z(Z-1)A^{-1/3} + a_{as}\frac{(A-2Z)^2}{A} \mp a_p A^{-3/4}}$$

Mass Parabola :-

$$M(Z,A) = ZM_H + (A-Z)M_n - a_v A + a_s A^{2/3} + a_c Z(Z-1)A^{-1/3} + a_{as}\frac{(A-2Z)^2}{A} \mp \delta$$

$$M(Z,A) = \alpha A + \beta Z + \gamma Z^2 \mp \delta$$

If A is fixed, then this eq^n is quadratic in Z

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i.e. eqn is linear in 1 variable (A) & quadratic in Z.
then conic cross-section will be parabola.
In Semi empirical mass formula,

$$\alpha = M_n - a_v + a_s A^{-1/3} + a_{as}$$

$$\beta = M_H - M_n - a_c A^{-1/3} - 4a_{as}$$

$$\gamma = a_c A^{-1/3} + 4a_{as} A^{-1}$$

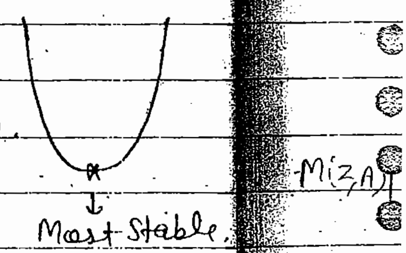
Plot of M v/s Z $\longrightarrow$ Parabola.

Therefore for a given value of A eqn starts represents a parabola b/w Mass M v/s Z for a fixed value of A.

Nuclei having same A are called Isobars.

$$M(A,Z) = \cdots\cdots - B(A,Z)$$

* More ~~lesser~~ the B.E., lesser the mass.
* Nuclei have less mass → More stable.



$$\left.\frac{\partial M}{\partial Z}\right|_{Z=Z_0} = \beta + 2\gamma Z_0 = 0$$

$$\Rightarrow \beta = -2\gamma Z_0$$

$$Z_0 = -\frac{\beta}{2\gamma}$$

$(Z = Z_0 = Z_{min})$

Putting the value of β ⇒

$$\boxed{Z_0 = \frac{a_c A^{-1/3} + 4a_{as} - (M_H - M_n)}{2a_c A^{-1/3} + 8a_{as} A^{-1}}}$$

On solving, Approximately

$$\boxed{Z_0 \simeq \frac{A}{2}}$$

The nuclei will be most stable whose Z value will be half of A value.

$\frac{11+12+13}{3}$ $\frac{36}{3}$ (12)

$A>72 \to \beta^-$ emitter
$A<72 \to \beta^+$ emitter

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in Z.

Que 1- For $A = 25$, Available Isobars are
$^{25}_{11}Na$, $^{25}_{12}Mg$, $^{25}_{13}Al$

$A = 25$

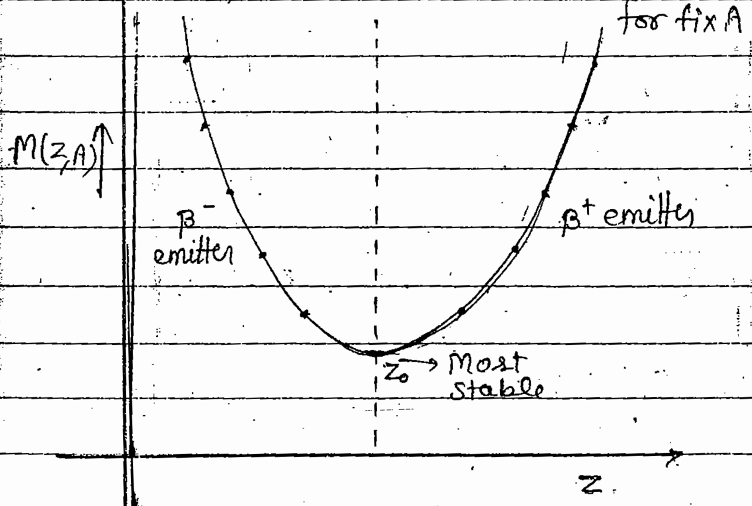
$Z_0 \simeq 11.7$

β^- emitter   Most Stable β^+ emitter

~~$M(Z,A) = \alpha A + \beta Z + \gamma Z^2 \mp \delta$~~

for Odd A, $M(Z,A) = \alpha A + \beta Z + \gamma Z^2$
for Even A, $M(Z,A) = \alpha A + \beta Z + \gamma Z^2 \mp \delta$

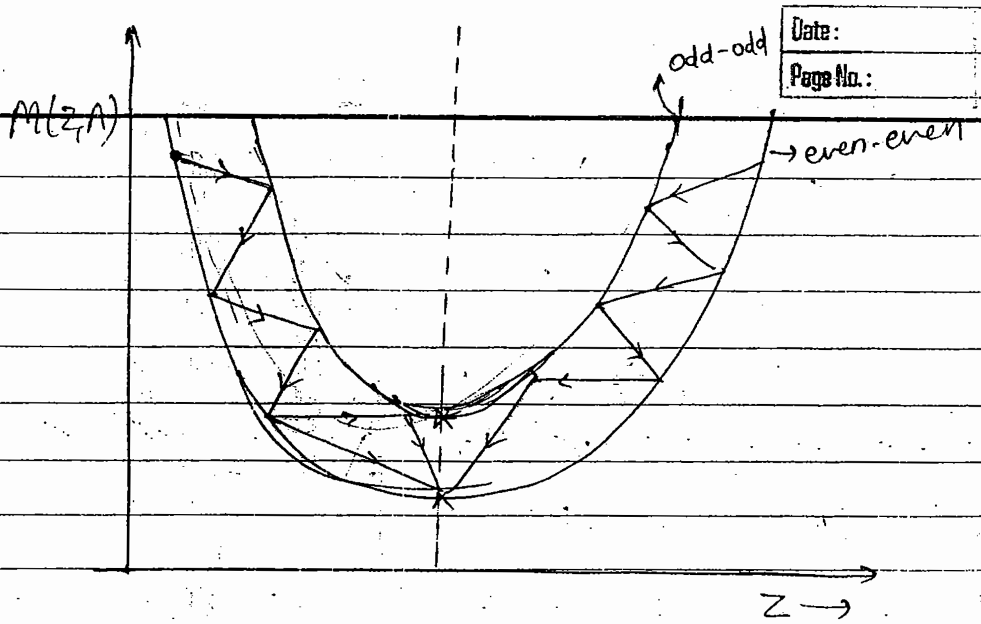
When A → odd, there will be a single parabola & for A → Even " " " 2 parabolas, one corresponding to $+\delta$ & one to $-\delta$.



(β^- decay)
for left side nuclei
$M(Z,A) \to M(Z+1,A) + {}_{-1}^{0}\beta + \bar{\nu}_e$
for right side nuclei
$M(Z,A) \to M(Z-1,A) + {}_{+1}^{0}\beta + \nu_e$
(β^+ decay)

Even A, $M(Z,A) = \alpha A + \beta Z + \gamma Z^2 - \delta$ even-even
$= \alpha A + \beta Z + \gamma Z^2 + \delta$ odd-odd

$M \downarrow$, B.E. $\uparrow$ $\Rightarrow$ $+\delta$ even-even
$M \uparrow$, B.E. $\downarrow$ $\Rightarrow$ $-\delta$ odd-odd



In Case of Cu, It can emit β^- & β^+ decay → both probability → Dual beta decay.

In case of Te, Simultaneously tow β^- particle emit → Double beta decay.

Mirror Nuclei :-

$${}^{A}_{Z}X_{A-Z} \quad \Big| \quad {}^{A}_{A-Z}Y_{Z}$$

Mass diff of 2 mirror can be calculated by SIMF.

Two ~~isotopes~~ isobars of a nuclei are said to be mirror nuclei if no. of protons of in one is equal to the no. of nutrons of in other & vice versa.

e.g. ${}^{11}_{6}C_{5}$, ${}^{11}_{5}B_{6}$

${}^{17}_{9}F_{8}$, ${}^{17}_{8}O_{9}$

for the mirror nuclei, $Z \longrightarrow A-Z$

$$M'(Z,A) = (A-Z)M_H + ZM_n - a_v A + a_s A^{2/3} + a_c [A-Z](A-Z-1)A^{-1/3}$$
$$+ a_{as}\frac{(A-2A+2Z)^2}{A} \mp a_p A^{-3/4}$$

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Difference of masses, $M(z,A) - M'(z,A)$

$$\Rightarrow M(z,A) - M(A-z,A) = \{\cancel{(2z-A)M_H + (2A-2z)M_n}\}$$
$$= (2z-A)M_H - (2z-A)M_n$$
$$\cancel{+ a_c A^{-1/3}\{z(z-1) - (A-z)(A-z-1)\}}$$

$$\Delta = (2z-A)(M_H - M_n) + a_c A^{-1/3}[z^2 - z - (A-z)^2 + A - z]$$
$$= [\cancel{z^2} - z - A^2 - \cancel{z^2} + 2Az + A - z]$$
$$= [-A^2 + 2Az - 2z + A]$$
$$= [A(2z-A) - 1(2z-A)]$$
$$= (2z-A)(A-1)$$
$$= (2z-A)(M_H - M_n) + a_c A^{-1/3}(2z-A)(A-1)$$

$$\boxed{\Delta = (2z-A)\left[a_c A^{-1/3}(A-1) + (M_H - M_n)\right]}$$

$(2z-A) = 1$

(50)

Que 1: The mass difference b/w the pair of mirror nuclei ${}_6^{11}C$ & ${}_5^{11}B$ is given to be $\Delta MeV/c^2$. Acc. to the semi impirical mass formula, the mass difference b/w the pair of mirror nuclei ${}_9F^{17}$ and ${}_8O^{17}$ will approximately be (rest mass of proton $m_p = 938.27\ MeV/c^2$ and rest mass of nutron $m_n = 939.57\ MeV/c^2$

(a) $1.39\ \Delta\ MeV/c^2$ (b) $(1.39\Delta + 0.5)\ MeV/c^2$
(c) $0.86\ \Delta\ MeV/c^2$ (d) $(1.6\Delta + 0.78)\ MeV/c^2$

$$m_n = 939.57\ MeV/c^2$$
$$m_H \simeq m_p = 938.27\ MeV/c^2$$

$$\Delta = (2z-A)\left[a_c A^{-1/3}(A-1) + (M_H - M_n)\right]$$

${}_6^{11}C$ & ${}_5^{11}B$

$$\Delta = (2\times 6 - 11)\left[a_c \frac{(11-1)}{\sqrt[3]{11}} + \underbrace{M_H}_{m_p} - M_n\right]$$

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$$\Delta = 1 \cdot \left[a_c \frac{10}{\sqrt[3]{11}} + 938.57 - 939.57 \right]$$

$$= a_c \frac{(10)}{\sqrt[3]{11}} - 1.3$$

$$\Delta + 1.3 = \frac{a_c (10)}{\sqrt[3]{11}} \quad \text{———} (1)$$

Let the mass diff b/w ${}_9F^{17}$ & ${}_8O^{17}$ is Δ'.

$$\Delta' = (1) \left[a_c \frac{16}{\sqrt[3]{17}} - 1.3 \right]$$

$$\Delta' + 1.3 = a_c \frac{16}{\sqrt[3]{17}} \quad \text{———} (2)$$

$$(1) \Rightarrow \quad a_c = \frac{\sqrt[3]{11}}{10} (\Delta + 1.3)$$

$$(2) \Rightarrow \quad \Delta' + 1.3 = \frac{16}{\sqrt[3]{17}} \frac{\sqrt[3]{11}}{10} (\Delta + 1.3)$$

$$\Delta' = \sqrt[3]{\frac{11}{17}} \frac{16}{10} (\Delta + 1.3) - 1.3$$

$$= \frac{2.223 \times 16}{2.57 \times 10} (\Delta + 1.3) - 1.3$$

$$= \frac{35.568}{25.7} (\Delta + 1.3) - 1.3$$

$$= 1.38397 (\Delta + 1.3) - 1.3$$

$$= 1.38 \Delta + 1.79916 - 1.3$$

$$= 1.38 \Delta + 0.499$$

$$\boxed{\Delta' = (1.38 \Delta + 0.5) \text{ MeV}/c^2}$$

✓ (b)

Qu - Which of the following nucleus is more stable

(i) ${}_3^7Li$ ${}_3^8Li$

(3, 4) (3, 5)

✓ odd even odd odd

${}_3^7Li$ is more stable.

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(ii) $^{13}_{6}C$ $\quad$ $^{15}_{6}C$

(6, 7) $\quad$ (6, 9)

even odd $\quad$ ~~odd even~~ odd even

Nuclei having more no. of n will be more stable.

So $^{15}_{6}C \longrightarrow$ More stable.

Que :- The difference in the coulomb energy b/w the mirror nuclei $^{49}_{24}Cr$ & $^{49}_{25}Mn$ is 6.0 MeV. Assuming that the nuclei have spherically symmetric charge distribution & the mass difference b/w the mirror nuclei depends only on columbian term. Estimate the radius of ~~25~~ $^{49}_{25}Mn$. given that $\frac{e^2}{4\pi\varepsilon_0} \equiv 1.0$ MeV-fm

(a) 4.9×10^{-13} m $\quad$ (b) 4.9×10^{-15} m

(c) 5.1×10^{-13} m $\quad$ (d) 5.1×10^{-15} m

$$E_c = B_c = -\frac{3}{5}\frac{Z(Z-1)\,e^2}{4\pi\varepsilon_0 R}$$

$$E_{c_1} = \frac{3}{5}\cdot\frac{1}{4\pi\varepsilon_0}e^2\,\frac{24(23)}{R_1}$$

$$E_{c_2} = \frac{3}{5}\cdot\frac{1}{4\pi\varepsilon_0}e^2\,\frac{25(24)}{R_2}$$

$$R_1 = R_2 = R$$

$$E_{c_2} - E_{c_1} = \frac{3}{5}\frac{e^2(24)}{4\pi\varepsilon_0}\left[\frac{25-23}{R}\right] = 6.0 \text{ MeV}$$

$$\Rightarrow \frac{3}{5}(1)(24)\left[\frac{2}{R}\right] = 6.0$$

$$\Rightarrow \frac{72 \times \cancel{2}}{5R} = \cancel{6}\,3 \Rightarrow 72 = 15R$$

$$R = \frac{72}{15} = 4.8 \text{ fm}$$

Not exact

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or $$E_c = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0}\frac{Z^2}{R}$$

$$E_{c_1} = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0}\frac{(24)^2}{R}, \quad E_{c_2} = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0}\frac{(25)^2}{R}$$

$$E_{c_2} - E_{c_1} = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0}\left[\frac{(25)^2-(24)^2}{R}\right]$$

$$6.0 = \frac{3}{5}(1)\left[\frac{625-576}{R}\right]$$

$$6 = \frac{3}{5}\frac{49}{R}$$

$$R = \frac{3\times 4.9}{5\times 6} = \underline{4.9\ fm}\ \text{Ans}$$

(b)

Que: Given that, the mass difference b/w $^{27}_{14}Si$ & $^{27}_{13}Si$ is 6 MeV. Estimate its radius? Assuming that b/w a pair of mirror nuclei is entirely due to the coulomb energy.

$$E_c = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0}\frac{Z^2}{R}$$

$$E_{c_1} = \frac{3}{5}(1.6\times10^{-19})^2\times9\times10^9\ \frac{(14)^2}{R}$$

$$E_{c_2} = \frac{3}{5}(1.6\times10^{-19})^2\times9\times10^9\ \frac{(13)^2}{R}$$

$$E_{c_1} - E_{c_2} = \frac{3}{5}(1.6\times10^{-19})^2\times9\times10^9\left[\frac{(14)^2-(13)^2}{R}\right]$$

$$6\times10^6\times1.6\times10^{-19} = \frac{3}{5}\ \cancel{14\times4\times}\ 23.04\times10^{-29}\left[\frac{196-169}{R}\right]$$

$$10^{-13}\times6\times1.6 = \frac{3}{5}\times23.04\times10^{-29}\left[\frac{27}{R}\right]$$

$$R = \frac{3\times23.04\times27\times10^{-29}}{6\times5\times1.6\times10^{-13}} = \frac{1866.24\times10^{-29}}{1.6\times30\times10^{-13}}$$

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No. of Nucleons $\Rightarrow 2n^2 = 2(2)^2 = 8$ (in terms of n)
$\Rightarrow 2(2\times0+1) + 2(2\times1+1) = 8$ (" " l)

$n = 4$,
$l = 0, 1, 2, 3$
$m_l = 0, \pm 1, 0, -1, +2, +1, 0, -1, -2, +3, +2, +1, 0, -1, -2, -3$

~~$S = \pm\frac{1}{2}$ for each m_l.~~

$2n^2 = 2(4)^2 = 2\times16 = 32$

$2(2l+1) = 2(2\times0+1) + 2(2\times1+1) + 2(2\times2+1) + 2(2\times3+1)$
$= 2 + 6 + 10 + 14 = 32$

SHELL MODEL

To explain Magic no.s, shell model was discovered. The assumption behind Liquid drop model was that the nucleon interacts with its nearest neighbour only. This model was unable to provide several phenomenons such as, Magic No.s

$\boxed{2, 8, 20, 28, 50, 82, 126}$

Then another hypothesis was proposed acc. to which each neucleon inside the nucleus interacts with the general force field produced by all other nucleons.

Sch^r eq^n, $\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$

We use hit & trail method, i.e. different pot^{n}s to solve the sch^r eq^n bcoz we don't know the exact form of pot^n.

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$$= \frac{1866.24}{1.6 \times 30 \times 10} \times 10^{-15} = \frac{38.88}{10} \times 10^{-15}$$

$$= 3.8 \times 10^{-15} m \; Ans = 3.8 \, fm$$

OR By $E_C = \frac{3}{5} \frac{e^2}{4\pi\varepsilon_0} \frac{Z(Z-1)}{R}$

$E_{C_2} = E_{C_1} \Rightarrow 6.0 \times 10^6 \times 1.6 \times 10^{-19} = \frac{3}{5} \times 9 \times 10^9 \frac{(1.6 \times 10^{-19})^2}{R} [14 \times 13 - 13 \times 12]$

$\Rightarrow R = 3.74 \, fm$ Ans

Hydrogen Atom Problem :-

^1_1H : 1p & 0n

$n = 1, 2, 3, 4 \ldots\ldots$

$l = 0$ to $n-1$

$m_l = -l$ to $+l$

$s = \pm 1/2$

$l = 0 \longrightarrow$ S- state Sharp - Series
$= 1 \longrightarrow$ P - state Principal Series
$= 2 \longrightarrow$ d - " Diffusion Series
$= 3 \longrightarrow$ f - " fundamental Series
$= 4 \longrightarrow$ g - "
$= 5 \longrightarrow$ h - "
$= 6 \longrightarrow$ i - "
⋮

In term of principal Q. No. $\Rightarrow$ no. of nucleons $= 2n^2$

In terms of $l \Rightarrow$ Max. no. of nucleons $= 2(2l+1)$

In $n = 2$ state

$l = 0, 1$, $m_l = 0, +1, 0, -1$

$S = +\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}$

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$V \equiv$ Linear harmonic potⁿ
$\equiv$ Yukawa potⁿ
$\equiv$ Wood saxon potential
$\equiv$ Square well potⁿ with rounded corners.

→ To fix Maria Geopert & H. J. D. Jenson use the technique of spin-orbit Interaction, to explain the Magic nos ($\vec{l}$ interacts with $\vec{s}$).

→ The spin-Orbit Interaction Potential can be written as

$$V_{ls} = -\phi(r)\,\vec{l}\cdot\vec{s}$$

$$\vec{j} = \vec{l} + \vec{s}$$

$$\vec{j}\cdot\vec{j} = (\vec{l}+\vec{s})\cdot(\vec{l}+\vec{s})$$

$$\vec{j}\cdot\vec{j} = \vec{l}\cdot\vec{l} + 2\vec{l}\cdot\vec{s} + \vec{s}\cdot\vec{s}$$

$$\vec{l}\cdot\vec{s} = \frac{\vec{j}\cdot\vec{j} - \vec{l}\cdot\vec{l} - \vec{s}\cdot\vec{s}}{2}$$

$$= \frac{j(j+1) - l(l+1) - s(s+1)}{2}$$

There are 2 orientation permitted,

Say $\quad \varepsilon_{ls} = -\langle\phi(r)\rangle\,\vec{l}\cdot\vec{s}$

for $j = l + \frac{1}{2}$

$$\vec{l}\cdot\vec{s} = \frac{(l+\frac{1}{2})(l+\frac{3}{2}) - l(l+1) - \frac{1}{2}(\frac{1}{2}+1)}{2}$$

$$= \frac{\cancel{l^2} + 2l + \cancel{\tfrac{3}{4}} - \cancel{l^2} - l - \cancel{\tfrac{3}{4}}}{2} = \frac{l}{2}$$

& for $j = l - \frac{1}{2}$

$$\vec{l}\cdot\vec{s} = \frac{(l-\frac{1}{2})(l+\frac{1}{2}) - l(l+1) - \frac{1}{2}(\frac{3}{2})}{2}$$

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$= l$

$\vec{l}\cdot\vec{s} = -\frac{(l+1)}{2}$

i.e. $\vec{l}\cdot\vec{s} = \frac{l}{2}$ for $j = l+\frac{1}{2}$

$\vec{l}\cdot\vec{s} = -\frac{(l+1)}{2}$ $j = l-\frac{1}{2}$

$\epsilon_{ls} = -\langle\phi(r)\rangle\frac{l}{2}$ $j = l+\frac{1}{2}$

$= +\langle\phi(r)\rangle\frac{(l+1)}{2}$, $j = l-\frac{1}{2}$

$\Delta\epsilon = \epsilon_{ls}(j=l-\frac{1}{2}) - \epsilon_{ls}(j=l+\frac{1}{2})$

$$\boxed{\Delta\epsilon_{ls} = \frac{(2l+1)}{2}\langle\phi(r)\rangle}$$

Observed Empirical formula,

$$\boxed{\Delta\epsilon_{ls} = 10\,(2l+1)\,A^{-2/3}\ \text{MeV}}$$

Magic Numbers :

2, 8, 20, 28, 50, 82, 126 → neutrons as magic No.

2, 8, 20, 28, 50, 82 → protons " "

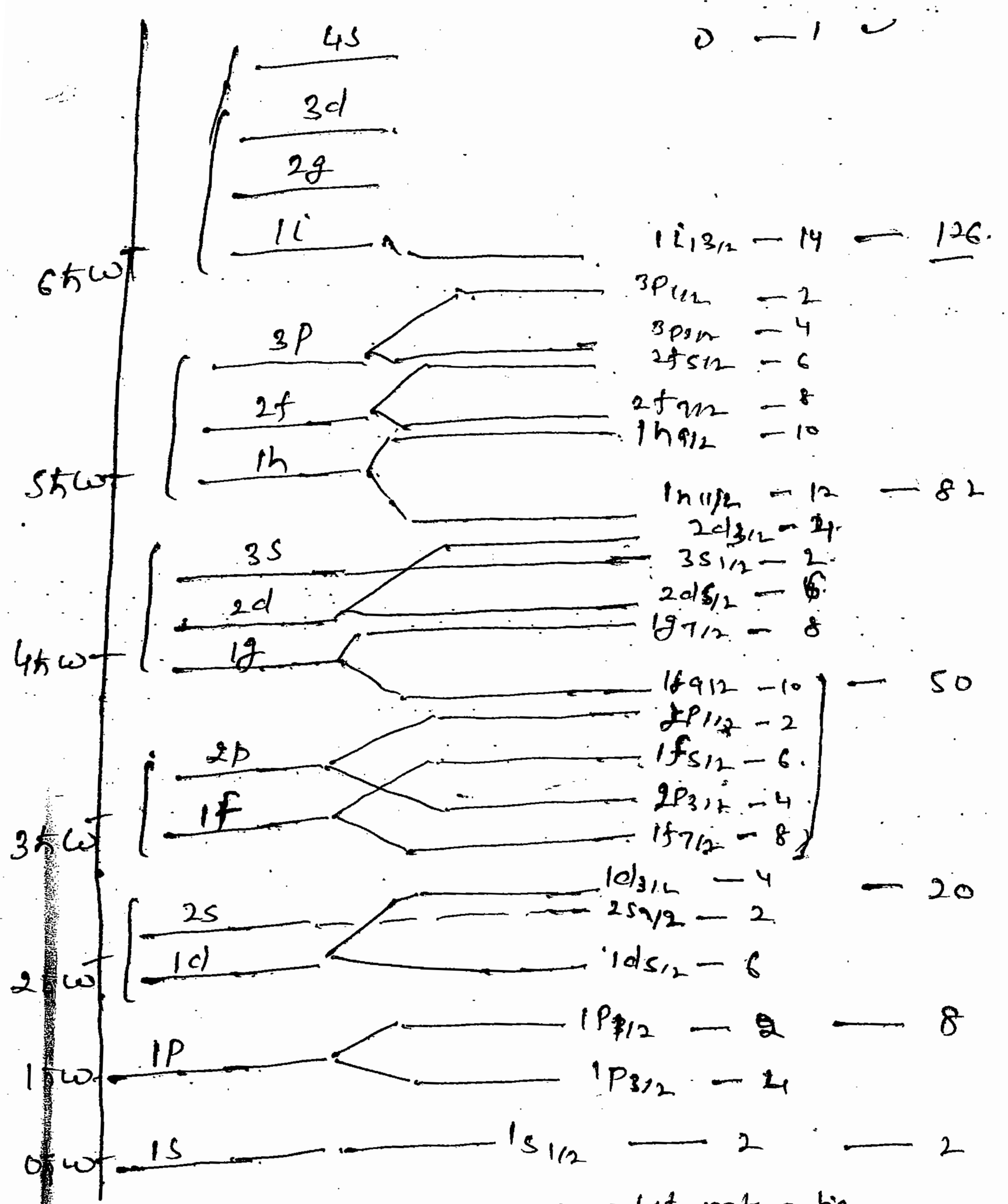
Splitting of Energy levels without spin-orbit Interaction :-

This is extreme single particle shell model. Maximum value of ~~Isospin~~ I is 9/2.

0, 1 → 1 2,

s p d f g h i
0 1 2 3 4 5 6

Energy level splitting
5 — 3
4 — 3
3 — 2
2 — 2
1 — 1
0 — 1



Energy level   splitting   spin-orbit interaction

(Without sp

(1) 2

1 S lev

$l = 0$

Capac

(2) 8

$1s^{*}$,

⇓

$l = 0$

2 (2X

(3) 20

level splitting

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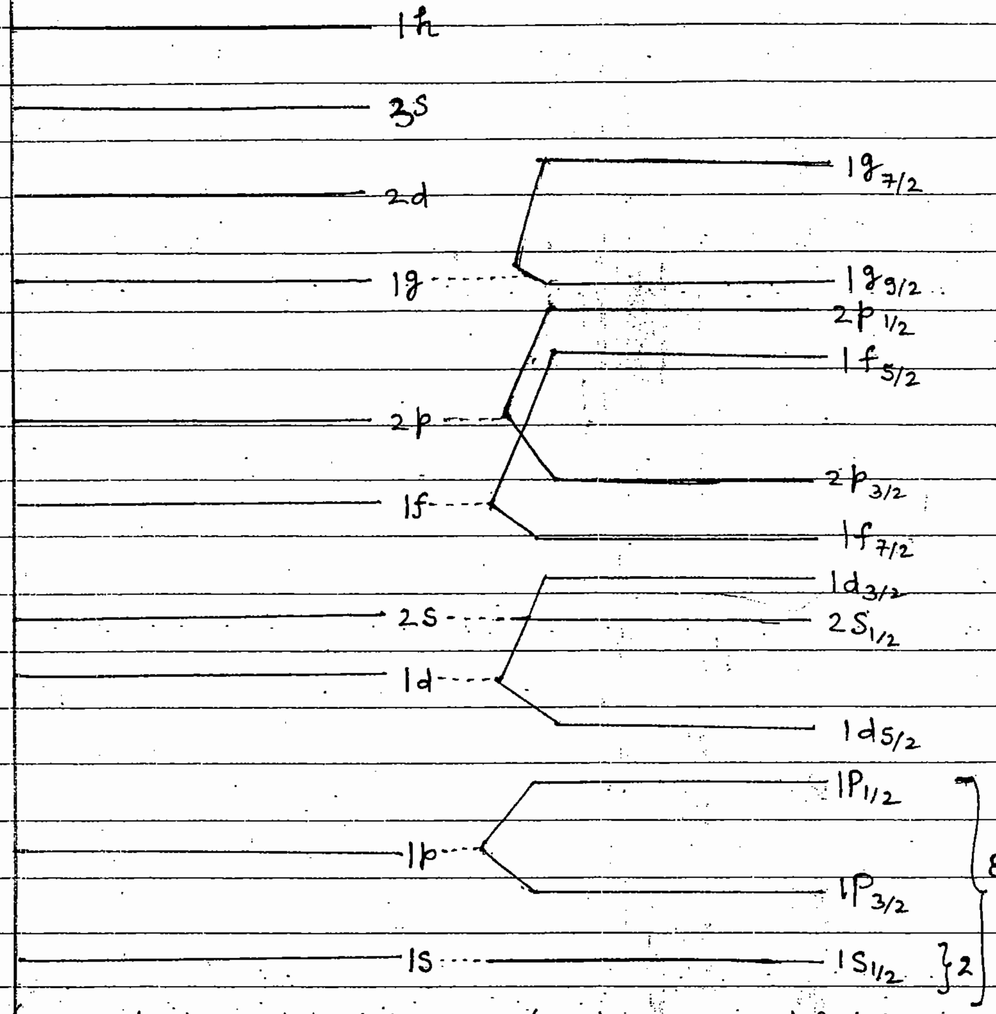

(Without spin-orbit Int.) (With spin-orbit Int.)

126
82
50
20
8
2

(1) 2

1 s level

$l = 0$

Capacity $2(2l+1) = (2\times 0+1)\,2 = 2$

(2) 8

1s, 1p

$\Downarrow$ $\Downarrow$

$l=0$ $l=1$

$2(2\times 0+1) + 2(2\times 1+1) = 2+6 = 8$

(3) 20

1s 1p 1d 2s

$\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$

$2 + 6 + 10 + 2 = 20$

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28

1S 1p 1d 2S 1f

$2+6+10+2+14 = 34$ Not a magic No.

Only 3 Magic No's can be explain by this method.

~~To~~

All the higher magic No's disagree with those that are observed & there is no way to remove the discrepancy by ordering the nuclear levels.

In the presence of Spin-Orbit interaction nuclear levels are splitted & the level is defined by j value.

$$j = l \pm s$$

$j = l + 1/2$

$= l - 1/2$

Capacity $= 2j+1$

for S $\rightarrow l = 0$ so $j = \frac{1}{2}$, $j \neq -\frac{1}{2}$ cant be -ve

so S-level is unsplitted.

for p $\rightarrow l = 1$, $j = 1 + 1/2 \rightarrow 3/2$

$= 1 - 1/2 \rightarrow 1/2$

for d $\rightarrow l = 2$, $j = 2 + 1/2 \rightarrow 5/2$

$= 2 - 1/2 \rightarrow 3/2$

With Spin-Orbit Interaction :-

2

$1S_{1/2}$ capacity $= 2j+1 = 2 \times \frac{1}{2} + 1 = 2$

$1S_{1/2}$, $1p_{3/2}$, $1p_{1/2}$

$2 + 4 + 2 = 8$

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(3) 20

$1s_{1/2}$ $1p_{3/2}$ $1p_{1/2}$ $1d_{5/2}$ $2s_{1/2}$ $1d_{3/2}$

$2+4+2+6+2+4 = 20$

(4) 28

$1s_{1/2}$ $1p_{3/2}$ $1p_{1/2}$ $1d_{5/2}$ $2s_{1/2}$ $1d_{3/2}$ $1f_{7/2}$

$2+4+2+6+2+4+8 = 28$

(5) 50

$1s_{1/2}$ $1p_{3/2}$ $1p_{1/2}$ $1d_{5/2}$ $2s_{1/2}$ $1d_{3/2}$ $1f_{7/2}$ $2p_{3/2}$ $1f_{5/2}$ $2p_{1/2}$ $1g_{9/2}$

$2+4+2+6+2+4+8+4+6+2+10 = 50$

Pridictions :-

Spin :- is decided by odd no. of nucleon (p or n)

(1) If both n & p are even then

spin = 0

$0^+ \longrightarrow$ even - Even nuclei

• $^{15}_{7}N$

$N = 15 - 7 = 8$ (Even) ×

$Z = 7$ (odd) ✓ $\equiv 1s_{1/2}^2 \; 1p_{3/2}^4 \; 1p_{1/2}^1$ $(l=1)$

~~spin $I = \frac{1}{2}$~~

Parity $= (-1)^l = (-1)^1 = -1$ (odd)

Spin $= \frac{1}{2}^-$

• $^{17}_{8}O$, $Z = 8$

$N = 17 - 8 = 9$ ✓ $\equiv 1s_{1/2}^2 \; 1p_{3/2}^4 \; 1p_{1/2}^2 \; 1d_{5/2}^1$

for d, $l = 2$

Parity $= (-1)^2 = 1$ Even

Spin $= \frac{5}{2}^+$

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- ${}^{39}_{19}K$, $Z \equiv 19$ $\quad 1S_{1/2}^2 \; 1p_{3/2}^4 \; 1p_{1/2}^2 \; 1d_{5/2}^6 \; 2S_{1/2}^2 \; 1d_{3/2}^3$

  spin $= 3/2$

  Parity $= (-1)^l = (-1)^2 = 1$ even

  spin $\equiv \frac{3}{2}^+$ //

- ${}^{16}_{8}O \longrightarrow Z = 8, N = 8$

  Parity → Even

  spin $= 0^+$

- ${}^{18}_{8}O \longrightarrow Z = 8 ; N = 10$, Parity → even

  spin $= 0^+$

- ${}^{14}_{7}N \longrightarrow Z = 7 \equiv 1S_{1/2}^2 \; 1p_{3/2}^4 \; 1p_{1/2}^1$

  $N = 7 \equiv 1S_{1/2}^2 \; 1p_{3/2}^4 \; 1p_{1/2}^1$

  $I_1 = \frac{1}{2} \qquad I_2 = \frac{1}{2}$

  $I = I_1 + I_2$ or $I_1 - I_2$

  $I = 1, 0 \longrightarrow$

<u>Nordheim Rule</u> :- (for both n & p → odd)

→ If $j_1 + j_2 + l_1 + l_2 \equiv$ Even

spin $\equiv |j_1 - j_2|$

→ If $j_1 + j_2 + l_1 + l_2 \equiv$ odd

spin $\equiv |j + j_2|$

Parity $(\pi) = (-1)^{l_1} * (-1)^{l_2}$

It is multiplicative Q. No.

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→ for ${}^{14}_{7}N$ → $j_1 + j_2 + l_1 + l_2 = \frac{1}{2} + \frac{1}{2} + 1 + 1 = 3$ (odd)

Spin $= \left|\frac{1}{2} + \frac{1}{2}\right| = 1$

Parity → $(-1)^1 \times (-1)^1 = +1$ even

i.e. $\boxed{\text{Spin} = 1^+}$

• ${}^{64}_{29}Cu$ → $Z = 29 \equiv \ldots \quad 2p^1_{3/2}$

$N = 35 \equiv \quad 1f^3_{5/2}$

$I_1 = \frac{3}{2}$, $I_2 = \frac{5}{2}$; $l_1 = 1$, $l_2 = 3$

$j_1 + j_2 + l_1 + l_2 = \frac{3}{2} + \frac{5}{2} + 1 + 3 = \frac{8}{2} + 4 = 8$ (even)

spin $= |j_1 - j_2| = \left|\frac{3}{2} - \frac{5}{2}\right| = 1$

Parity $= (-1)^1 \times (-1)^3 = +1$ even

$\boxed{\text{Spin} = 1^+}$

• ${}^{7}_{3}Li$ → $N = 4$

$Z = 3 \equiv 1s^2_{1/2}, 1p^2_{3/2}$

$I = 3/2$

$P = (-1)^l = (-1)^1 = -1$

$I = \frac{3}{2}^-$

• ${}^{13}_{6}C$ → $N = 7 \equiv 1s^2_{1/2}, 1p^4_{3/2}, 1p^1_{1/2}$

$Z = 6$

$P = (-1)^1 = -1$

$I = \left(\frac{1}{2}\right)^-$

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- $^{31}_{16}P$ $Z = 16$
  $N = 15 \equiv 1S_{1/2}^{2}\ 1P_{3/2}^{4}\ 1P_{1/2}^{2}\ 1d_{5/2}^{6}\ 1S_{1/2}^{2}$
  $P = (-1)^0 = 1$
  $I = \frac{1}{2}^+$

Que :- Total B.E. of ^{15}O ^{16}O ^{17}O are 111.96 MeV, 127.62 MeV, 131.76 MeV i.e.

^{15}O → 111.96, ^{16}O → 127.62, ^{17}O → 131.76

~~Que~~ :- Energy Gap b/w $1P_{1/2}$ & $1d_{5/2}$ neutron shells for the nuclei whose mass no. is close to 16.

~~111.~~ (a) 4.1 (b) 11.5
(c) 15.7 ✓(d) 19.8

$^{15}_{8}O$, $^{16}_{8}O$, $^{17}_{8}O$ → Isotopes

$^{15}O \rightarrow 1S_{1/2}^{2}\ 1P_{3/2}^{4}\ 1P_{1/2}^{1}$
$^{16}O \rightarrow 1S_{1/2}^{2}\ 1P_{3/2}^{4}\ 1P_{1/2}^{2}$
$^{17}O \rightarrow 1S_{1/2}^{2}\ 1P_{3/2}^{4}\ 1P_{1/2}^{2}\ 1d_{5/2}^{1}$

Energy gap b/w $1P_{1/2}$ & $1d_{5/2}$ = ?
^{15}O & ^{17}O are differing by no. of neutrons. The energy gap b/w them is simply the B.E. difference b/w P & d states.

$= 131.76 - 111.96$ (MeV)
$= 19.8$ MeV Ans

✓(d)

Exp

Exceptional case for shell model:

F	Tl	Na	As	Cu	Ni	An	Mn	Ta	Sc
$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	$\frac{5}{2}^-$	$\frac{5}{2}$	$\frac{7}{2}^-$	$\frac{7}{2}^+$

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- $^{14}_{6}C \rightarrow Z=6$
  $N=8$ $\quad I = 0^+$

s 0
p 1
d 2
f 3

- $^{9}_{4}Be \rightarrow Z=4$
  $N=5 \equiv 1S_{1/2}^2 \; 1P_{3/2}^3$
  $P = (-1)^1 = -1$
  $I = \frac{3}{2}^-$

7.6 MeV,

- $^{41}_{20}Ca \longrightarrow Z=20$
  $N=21 \equiv 1S_{1/2}^2 \; 1P_{3/2}^4 \; 1P_{1/2}^2 \; 1d_{5/2}^6 \; 1S_{1/2}^2 \; 1d_{3/2}^4 \; 1f_{7/2}^1$
  $P = (-1)^3 = -1$
  $J = \frac{7}{2}^-$

shells 6

- $^{15}_{8}O \longrightarrow Z=8$
  $N=7 = 1S_{1/2}^2 \; 1P_{3/2}^4 \; 1P_{1/2}^1$
  $P = (-1)^1 = -1$
  $I = \frac{1}{2}^-$

- $^{23}_{11}Na \longrightarrow Z=11 \equiv 1S_{1/2}^2 \; 1P_{3/2}^4 \; 1P_{1/2}^2 \; 1d_{5/2}^1$
  $N=12$
  $P = (-1)^2 = 1$
  $I = \frac{5}{2}^+ \longrightarrow$ X wrong

Exceptional cases — This is exceptional case.

energy b/w

* • $^{23}_{11}Na = 1S_{1/2}^2 \; 1P_{3/2}^4 \; 1P_{1/2}^2 \; 1d_{5/2}^3 \; 2S_{1/2}^{\cancel{2}}$
$\Downarrow$
$1d_{5/2}^2 \; 1d_{3/2}^1$

deciding factor is $1d_{3/2}$

$I = \frac{3}{2}^+$

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- ${}^{75}_{33}As \Rightarrow Z = 33,\ N = 42$

$33 \equiv 1s_{1/2}^2\ 1p_{3/2}^4\ 1p_{1/2}^2\ 1d_{5/2}^6\ 1s_{1/2}^2\ 1d_{3/2}^4\ 1f_{7/2}^8\ 2p_{3/2}^4\ 1f_{5/2}^1$

Predicted $\rightarrow \frac{5}{2}^-$

observed $\rightarrow \frac{3}{2}^-$

$2p_{3/2}^4\ 1f_{5/2}^1 \longrightarrow 2p_{3/2}^3\ 1f_{5/2}^2$

$\Downarrow$

$\frac{3}{2}^-$

- ${}^{61}_{28}Ni \Rightarrow Z = 28,\ N = 33$

33 same as above $I = \frac{3}{2}^-$

- ${}^{55}_{25}Mn \Rightarrow Z = 25,\ N = 30$

$25 \equiv 1s_{1/2}^2\ 1p_{3/2}^4\ 1p_{1/2}^2\ 1d_{5/2}^6\ 1s_{1/2}^2\ 1d_{3/2}^4\ 1f_{7/2}^5$

Last nucleon enters in $1f_{7/2}^5$ state but experimentally it has been observed, the spin parity to be $\frac{5}{2}^-$ i.e. $1f_{7/2}^5$

$\Downarrow$

$1f_{7/2}^2,\ 1f_{5/2}^3$

$\Downarrow$

$\frac{5}{2}^-$

Que 1- Calculate the energy difference b/w $1d_{3/2}$, $1f_{7/2}$, neutrons cells corresponding to a neutron no. 20 given that the masses of

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$^{40}_{20}Ca$ 39.962589 u

$^{41}_{20}Ca$ 40.962275 u

$^{39}_{20}Ca$ 38.970691 u

$m_n \equiv 1.008665$ u

$^{41}_{20}Ca \equiv 1s_{1/2}\ 1p_{3/2}\ 1p_{1/2}\ 1d_{5/2}\ 1s_{1/2}\ 1d_{3/2}\ 1f_{7/2}$

$^{39}_{20}Ca \equiv 1s_{1/2}\ 1p_{3/2}\ 1p_{1/2}\ 1d_{5/2}\ 1s_{1/2}\ 1d_{3/2}$

energy gap $=$ B.E. diff b/w $1d_{3/2}$ & $1f_{7/2}$

$= 40.962275 - 38.970691$

$= 1.$

Classification of Elementry Particles

- Boson
  - Photon
  - Graviton
  - Measons
    - Pions (π^-, π^0, π^+)
    - kaons $(K^+, K^0, K^-, \bar{K}^0)$
    - η^0
- Fermions
  - Leptons
    - Electron (e^-, e^+)
    - muon (μ^-, μ^+)
    - Tau (τ^-, τ^+)
    - $(\nu, \bar{\nu})$
  - Bryons
    - Nucleons $(p^+, p^-, n^0, \bar{n}^0)$
    - Hyprons
      - Lamda Λ^0
      - ksi Ξ $[\Xi^-, \Xi^0]$
      - Sigma Σ $[\Sigma^-, \Sigma^0, \Sigma^+]$
      - Omega (Ω^-)

* Hadrons ⇒ All hyprons and measons together involving in the strong interaction are formed a group called Hadrons and composition of further elementry particles called quarks.

Measons –
- Pions (π): π^-, π^0, π^+
- Kions (K): K^+, K^0, $\bar{K}^0$, $\bar{K}$
- η^0: $(\eta^0$

leptons

		Q	L_e	Antiparticles	Q	L_e
1. Electron	e^-	-1	1	e^+	+1	-1
	ν_e	0	1	$\bar{\nu}_e$	0	-1
2. muon	μ^-	-1	1	μ^+	+1	-1
	ν_μ	0	1	$\bar{\nu}_\mu$	0	-1
3. Tauon	τ^-	-1	1	τ^+	+1	-1
	ν_τ	0	1	$\bar{\nu}_\tau$	0	-1

Bryons
- Nucleons → p^+, p^-, n^0, $\bar{n}^0$
- Hyprons

Hyprons
- Lamda (Λ^0) → Λ^0
- ksi (Ξ) → Ξ^0, Ξ^-
- Sigma (Σ) → $\Sigma^-, \Sigma^0, \Sigma^+$
- Omega (Ω) → Ω^-