

- CAREER ENDEAVOUR -

THERMODYNAMICS

By

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∴ Thermodynamics :-

← Thermodynamic property →

Free Energy = $\begin{cases} \rightarrow \text{Gibbs free Energy [G]} \\ \rightarrow \text{Helmholtz free Energy [A]} \end{cases}$

Internal Energy or intrinsic Energy
[E or U]

Enthalpy (H)

Entropy (S)

work

Heat

State function

Euler's theorem

Joule's Thompson Effect

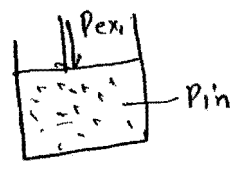
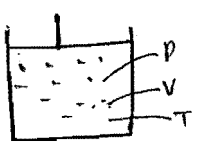
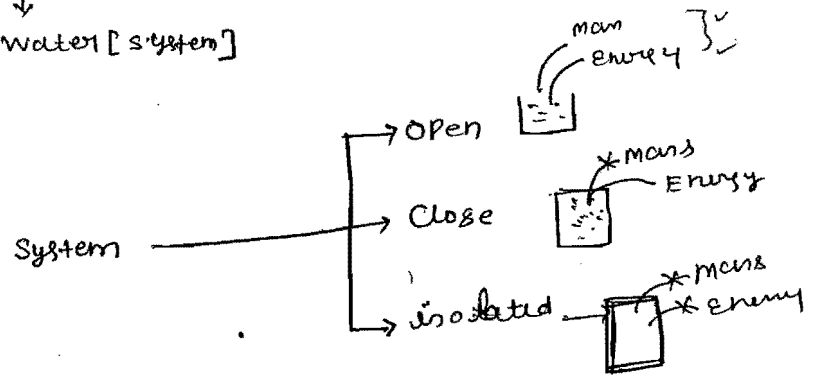
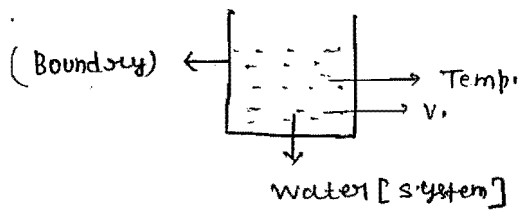
Carnot Engine

Zeroth, Ist, IInd & IIIrd law of thermo.

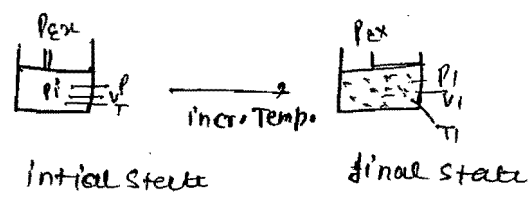
fugacity

chemical potential

thermochemistry



$P_{ex} > P_{in} \Rightarrow$ compression
 $P_{in} < P_{ex} \Rightarrow$ expansion
 $P_{in} = P_{ex}$ [system as it]
 [P, V, T - fixed]
 (Existence of syst.)



$$\Delta V = V_f - V_i$$

$$\Delta T = T_f - T_i$$

Thermodynamics:-

②

"It is branched of science which study the quantitative relationship b/w heat and another form of energy.

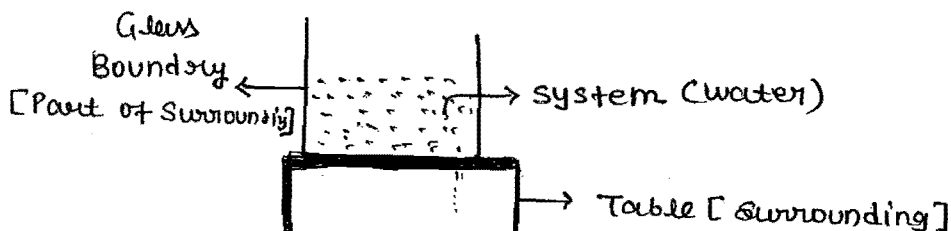
For e.g. \Rightarrow we study the Energy term like

- \Rightarrow Gibbs free Energy
- \Rightarrow Helmholtz free Energy
- \Rightarrow U
- \Rightarrow ΔS
- \Rightarrow H

There are the two basic concept in the thermodynamics one is System and another is Surrounding.

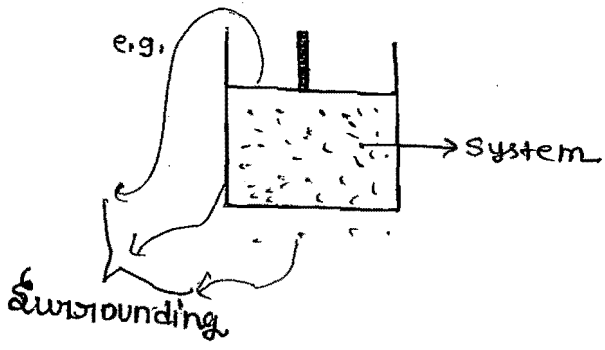
System: \Rightarrow It is a part of universe which under investigation experimentally or ~~theoretically~~ theoretically.

e.g. if we take a glass of water and if u study the properties [P, V, & T] of water then water is considered as a system.



Surrounding: \Rightarrow

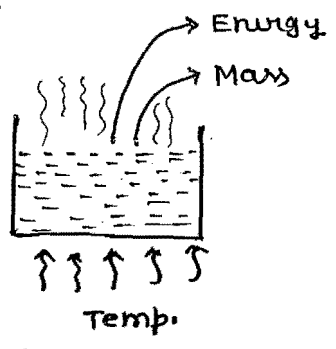
It is a part of universe other than system is called surrounding. For practical purpose, Surrounding is the region which is around the system or which is in vicinity of the system.



there are three types of System

- (i) open System
- (ii) Close System
- (iii) Isolated System

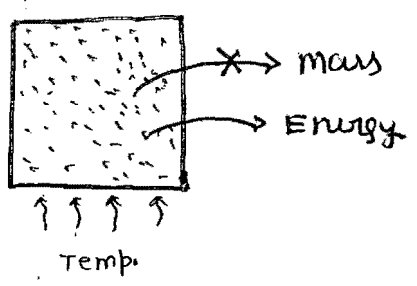
Open System: →



The System which Exchange mass as well as Energy to the Surrounding is called open system.

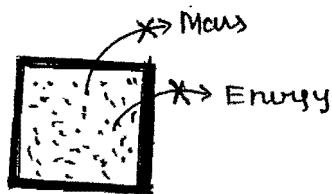
Close System:-

the System which Exchange Energy not mass to the Surrounding is called close system.



the System which neither Exchange Energy nor mass with the Surrounding that is called Isolated System.

Note \Rightarrow Practically completely isolated system is not possible

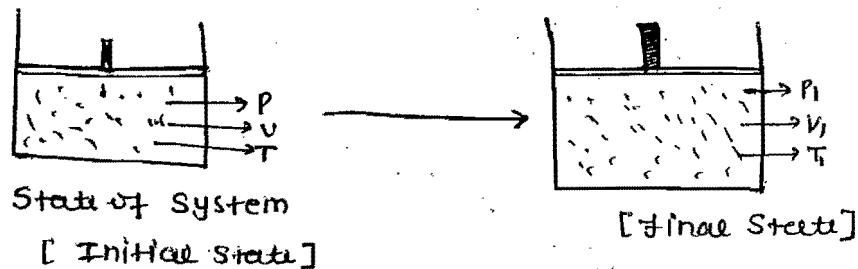


State of the System \Rightarrow

It is the condition of existence of a particular System when its macroscopic property have a definite value

i.e. Suppose a Gaseous System which having a piston, these pressure, volume, temp. etc. are fixed. ^{fixed Properties} this shows that system has a particular state of existence.

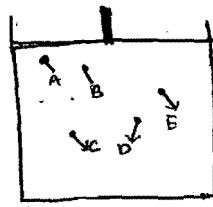
e.g. \Rightarrow



Macroscopic properties \Rightarrow

The properties of a System which contains [atom/molecules/ions], these properties is arise due to ^[P, V, & T] the collective behaviour of all the molecules, ions, atom, from which the System is made.

e.g. \Rightarrow Volume is a macroscopic property which arise



$$V = V_A + V_B + V_C + V_D + V_E$$

e.g. $\Rightarrow P, V, T, E, Q, S$

*** EXTENSIVE AND INSTENSIVE PROPERTY ***

EXTENSIVE PROPERTY [Depends upon Amount]

\Rightarrow The property of a system which depend upon the quantity of matter contained. or which depends upon Amount.

e.g. \Rightarrow Energy [H, G, A, E, S]

- No. of moles
- Volume
- Heat capacity [molar heat capacity is Int. prop.]
- Molarity [उत्तर Data वदुत वरिसे] $C = \frac{d}{m \cdot \Delta t}$

INTENSIVE PROPERTY:-

\Rightarrow The property of a system which is independent upon the quantity of matter, don't depend upon the amount.

- e.g. \Rightarrow
- \Rightarrow Temp. \Rightarrow molarity
- \Rightarrow Molar Volume \Rightarrow molarity
- \Rightarrow Molar Volume \Rightarrow Dielectric Const.
- \Rightarrow Activity pressure \Rightarrow mole fraction
- \Rightarrow Density \Rightarrow conc. [उत्तर Data वदुत वरिसे]
- \Rightarrow pH [Uniform distribution]
- \Rightarrow Viscosity \Rightarrow free Energy per mole
- \Rightarrow Refractive index

⇒ The ratio of two Extensive become Intensive.

(6)

$$\text{e.g.} \rightarrow \text{Density} \quad = \frac{\text{Mass [Ext.]} }{\text{Volume [Ext.]}} \\ \text{[Intensive]}$$

←: TYPES OF PROCESS :→

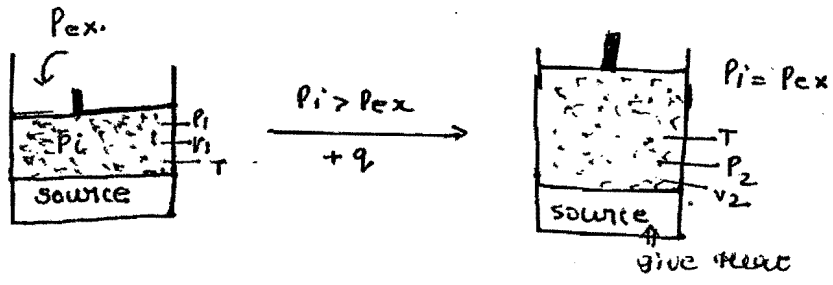
- Isobaric process
- Isochoric process
- Isothermal process
- Adiabatic process
- Polytropic process
- Reversible process
- Irreversible process [Spontaneous process]
 ↓
 [Natural process]
- Cyclic process
- Entropic process

Isenthalmal Process \Rightarrow

When a System change from one state to another state, during this the temp remain same then the process is called

there is two type of possibility of isenthalmal process one is isenthalmal expansion and another is isenthalmal compression.

\Rightarrow Isenthalmal Expansion

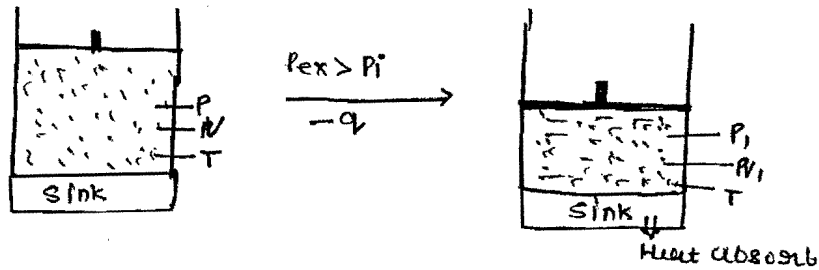


in isenthalmal Expan. work is done by the System during expansion the System absorb heat from the Surrounding.

- in isenthalmal Ex. temp. only remain same when a System absorb heat from outside but ^{if} the System expand among itself then temp. definitely lower at final state.
- we use the source for Heat absorbance.

Isothermal Compression: →

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In isothermal compression work is done on the system by the surrounding. In this case heat is rejected to the surrounding.

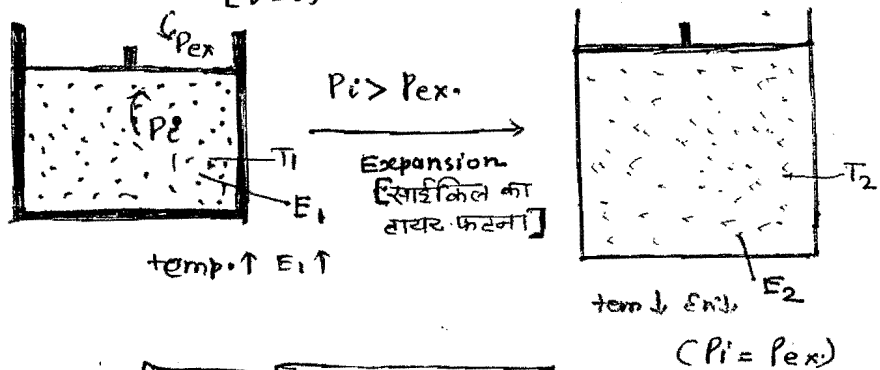
- In isothermal compression work is done on the system is converted into the heat and ultimately in order to maintain the temp. const. heat is rejected to the surrounding.
- In this case work is done by the surrounding on the system.

Adiabatic process: →

[No complete Adiabatic process possible]

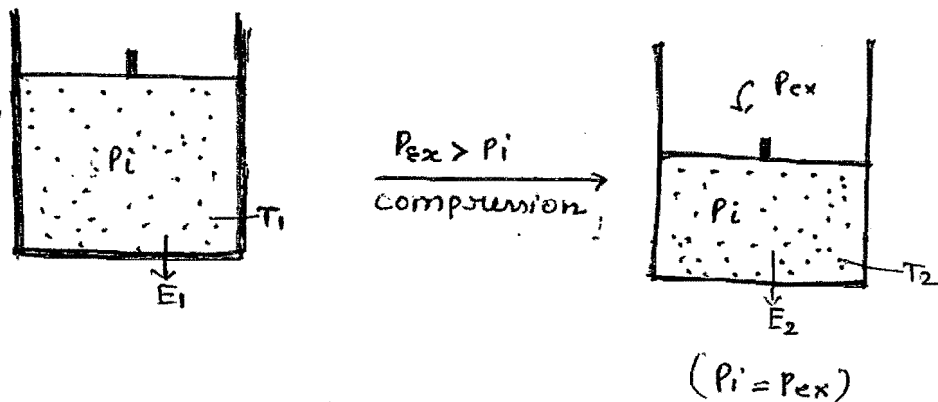
[there is no exchange of heat b/w system & surrounding]

$[q=0]$



$\Delta E_{int} \Rightarrow E_2 - E_1 = -W_e$

$T_1 > T_2$



	$T_2 > T_1$
$\Delta E \Rightarrow$	$E_2 - E_1 \Rightarrow \text{+ive}$

" The System which Neither Exchange Heat with the Surrounding then System is called Adiabatic."

- " In adiabatic process there is two possibility-
- (1) Adiabatic compression
 - (2) " Expansion

" In Adiabatic Expansion System Expend on the Expense of Internal Energy becoz there is no other possibility of Energy"

- ΔE decreases
- Final temp. lower down.

" In adiabatic compression System work is done by the system surrounding.

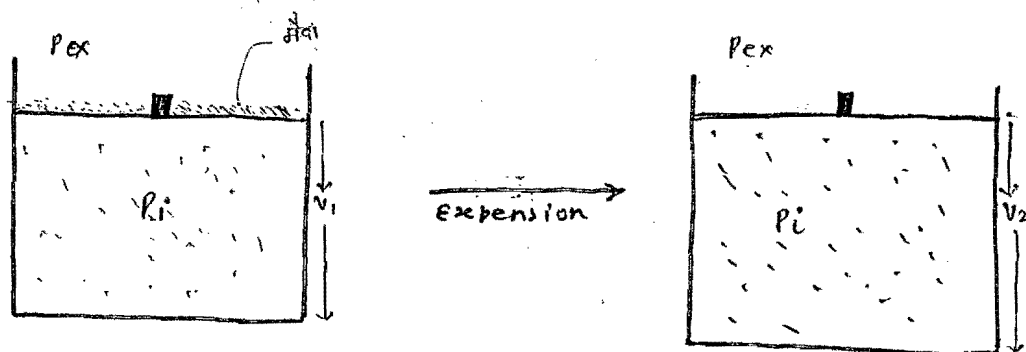
Heat up the molecule of System, there is no possibility of heat rejection to the given temp.

... system increases

one life time possible - T.E.D.E.

(10)

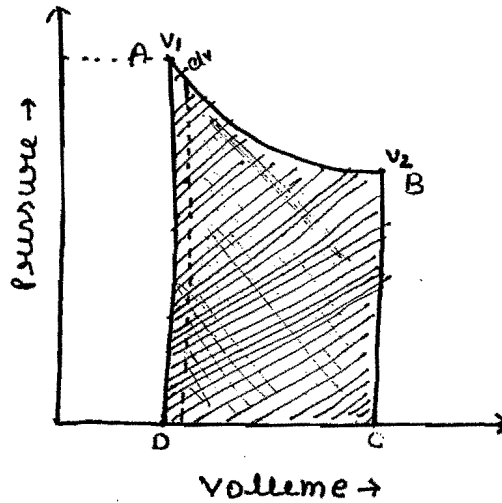
Reversible Process: \Rightarrow (Ideal, Imagination work)



$$P_i' = P_{ex} + dp$$

$$P_{ex} = P_{in}$$

- \Rightarrow It is hypothetical and imaginary process, practically it is not possible.
- \Rightarrow Opposing force and driving force are comparable, so work done is maximum.
- \Rightarrow It takes infinite time to complete & infinite steps to complete.
- \Rightarrow It takes place ~~to~~ infinite slowly in such a manner the system remains at equilibrium at each & every step.
- \Rightarrow There is a lot of steps of work done take place when you combined all the steps then we get total work done.
- \Rightarrow It is reverse. at each & every step



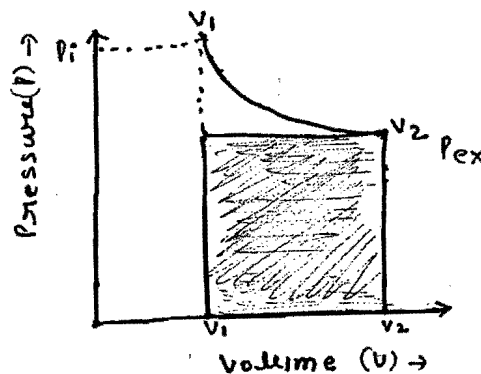
Area of ABCD = Total work done in expansion from v_1 to v_2

[Graph for work done in Revers. process]

Irreversible Process :-

(सिद्धांत की वजह से) - due to the expansion [internal pressure is very high]

- ⇒ Gas. process are very rapid process is very rapid.
- ⇒ there is very large diff. b/w oppo. & driving force
- ⇒ Gas. process are also called Natural process and spontaneous process (जो स्वतः ही होता है)



⇒ Total work done (w) = $-P_{ex} [v_2 - v_1]$
 It is not reversible, it only reverse when

e.g. \Rightarrow combustion of Paper, Gas

(12)

\Rightarrow flowing of heat from high temp. to low temp.

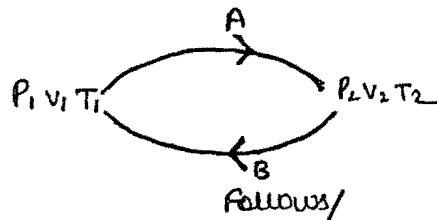
\Rightarrow flow of water from the slope.

\Rightarrow All the natural reaction

\Leftarrow POLYTROPIC PROCESS \Rightarrow

\Rightarrow "The process in which, ^{when} there is no change in heat capacity take place, then this process is called Polytropic process."

\Leftarrow Cyclic Process \Rightarrow (जहाँ से गये वही वापस)



When a system travels many steps/state and finally return to the original state then the process is called cyclic process.

* Isobaric Process:-

→ "When there is no change in pressure, then this process is called Isobaric process."

$$(P_1, V_1, T_1) \longrightarrow (P_1, V_2, T_2)$$

$$\Delta P = 0$$

When a system change from one state to another state during this process pressure is not change then this process is called Isobaric process.

* Isochoric Process:-

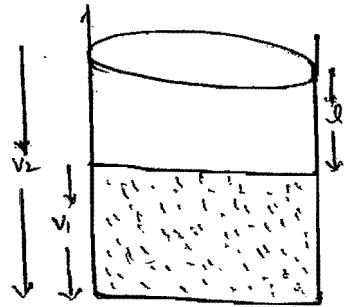
When the system change from one state to another state during this process volume is not change then this process is called Isochoric process.

$$(P_1, V_1, T_1) \longrightarrow (P_2, V_1, T_2)$$

$$\Delta V = 0$$

* Isoentropic process:-

When the system change from one state to another state, when there is no change in Entropy take place ($\Delta S = 0$) is called isentropic process.



$P = F/a$

$\Delta V = V_2 - V_1$

$w = F \times \text{distance moved}$

- work is a organised form of Energy
- Its standard unit is Joule
- It is path dependent function becoz the value of work is different for different path so it is not state function also
- In general work is done on the system internal energy increases.
- if work is done by the system, internal energy decreases

Since we know that work (w) = force x displacement

$w = F \times d$

$w = P \times a \times l$

$w = P \cdot \Delta V$

↓
change in V.

($\Delta V \Rightarrow \text{in diff.}$)

$$W = -P(V_2 - V_1)$$

(15)

Sign conversion for work -

if work is done by the system then we always consider -ive sign, ~~but~~ if work is done on the system then we always consider +ive sign.

Notes: →

we always consider work in the form of $-PdV$.

HEAT : → [absorb (+ive) release (-ive)]

- Heat is a Unorganized or Random form of Energy.
- if Heat ^{is} absorb by the system then internal Energy of system increases.
- if Heat is releases by the system then the internal Energy of a system is decrease.
- the value of heat depend upon the path so that that is path dependence function - - - - -

Sign conversion for heat

- if Heat is given out of the system then the sign of heat is considered negative and if Heat ~~is~~ absorbed by the system then the sign of heat is considered +ive.

The only two parameter in thermodynamics is a path dependence
 one is Heat another is work (16)

⇨ EULER'S THEOREM: ⇒

if Z is a function which depend upon x and

y -

then,

$$Z = f(x, y)$$

$$\frac{\partial Z}{\partial x} = \left(\frac{\partial Z}{\partial x} \right)_y \partial x + \left(\frac{\partial Z}{\partial y} \right)_x \partial y$$

Total diff. ↓ (M) ↓ (N) Partial diff. at const. x
↓ Partial differential ⇒ Z changes with changes of y .
↓ at const. y which means that the Z
changes with changes x

which means that the value of Z changes
 with simultaneously change of x and y .

$$\partial Z = M dx + N dy$$

$$\left(\frac{\partial M}{\partial y} \right)_x = \left(\frac{\partial N}{\partial x} \right)_y$$

$$\frac{\partial}{\partial y} \left(\frac{\partial Z}{\partial x} \right) = \frac{\partial}{\partial x} \left(\frac{\partial Z}{\partial y} \right)$$

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{\partial^2 Z}{\partial y \partial x}$$

⇒ This conditions shows that Z is a State function.

⇒ This is the conditions of Euler's theorem.

$$\underline{Q} \quad dz = \underbrace{x}_{M} dy + \underbrace{y}_{N} dx$$

(17)

it must be follow Euler's theorem then

the function is state function

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$

[By Euler's theorem]

$$\underbrace{x}_{M} dy \quad \underbrace{y}_{N} dx$$

$$\left(\frac{\partial z}{\partial y}\right)_x \quad \left(\frac{\partial z}{\partial x}\right)_y$$

$$\frac{\partial}{\partial x} \left[\frac{\partial z}{\partial y} \right] = \frac{\partial}{\partial y} \left[\frac{\partial z}{\partial x} \right]$$

$$\frac{\partial^2 z}{\partial x \partial x} = \frac{\partial}{\partial x} [x] = \frac{\partial}{\partial y} [y] = \frac{\partial^2 z}{\partial y \partial y}$$

$$1 = 1$$

follow Euler's theorem

so that this is state function

$$\underline{Q} \quad dz = \underbrace{(51x^2y + 47y^4)}_M dx + \underbrace{(17x^3 + 188y^3)}_N dy$$

$$\frac{\partial}{\partial y} [M] = \frac{\partial}{\partial x} [N]$$

$$\frac{\partial}{\partial y} [51x^2y + 47y^4] = \frac{\partial}{\partial x} [17x^3 + 188y^3]$$

$$[51x^2 + 188y^3] = [51x^2 + 188y^3]$$

Q. $\partial z = x^2 dy + y^2 dx$

(18)

$$\frac{\partial}{\partial x} [x^2] = \frac{\partial}{\partial y} [y^2]$$

$$2x = 2y$$

$$x = y$$

this is not ~~not~~ equal

So that doesn't follow Euler's theorem

∴ Not a state function.

Q4. $\partial z = x dx + y dy$

$$\frac{\partial}{\partial y} [x] + \frac{\partial}{\partial x} [y]$$

$$0 = 0$$

∴ Z follow Euler's theorem

So that is state function.

~~Q. $\partial z = 5t$~~

Net-2014

Qw. the exact diff. of

$f(x, y)$ among the following is a state function.

(a) $x dy$ (b) $\frac{x}{y} dy - \frac{y}{x} dx$ (c) $y dx - x dy$ (d) $\frac{1}{y} dx - \frac{x}{y^2} dy$
 ~~$\frac{1}{y} dx - \frac{x}{y^2} dy$~~
 ~~$= \frac{1}{y^2} dx - \frac{x}{y^2} dy$~~

$\partial z = x dy + y dx$

$\frac{\partial}{\partial y} [x] = 0$

$\frac{\partial}{\partial x} [y] = 1$

$\frac{\partial}{\partial x} [\frac{x}{y}] = \frac{1}{y}$

$\frac{\partial}{\partial y} [y] = 1$

$\frac{\partial}{\partial x} [x] = 1$

$\frac{\partial}{\partial x} [\frac{1}{y}] = -\frac{1}{y^2}$

$\frac{\partial}{\partial x} [-\frac{x}{y^2}] = -\frac{1}{y^2}$

Q.11

सब एक दूसरे के विभिन्न हैं जो But one is a function of the other two

← CYCLIC RULE ⇒ (3 independent variables)

The relation b/w three variables can be represented in well define relation Suppose if z is a function of x and y then -

$$z = f(x, y)$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

Suppose z is a constant function

$$\text{So } dz = 0$$

$$\therefore dz = 0$$

$$0 = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

divide both side by dy at a const. z,

$$\cancel{\left(\frac{\partial z}{\partial y}\right)_x} = \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z + \cancel{\left(\frac{\partial z}{\partial y}\right)_x} \left(\frac{\partial y}{\partial y}\right)_z$$

$$0 = \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial z}{\partial y}\right)_x$$

$$-\left(\frac{\partial z}{\partial y}\right)_x = \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z$$

$$\boxed{\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x = -1}$$

this relation can be represented in terms of
 P, V & T .

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$$\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$$

$$\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial P}\right)_T \left(\frac{\partial T}{\partial V}\right)_P = -1$$

$$PV = RT$$

$$\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial V}{\partial P}\right)_T$$

Q4. Prove that cyclic Rule verify for Ideal Gas -

$$PV = RT$$

$$\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial V}{\partial P}\right)_T = -1$$

$$\left(\frac{\partial P}{\partial T}\right)_V$$

$$PV = RT$$

$$P = \frac{RT}{V}$$

$$\Rightarrow \left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V} \frac{\partial T}{\partial T} = \frac{R}{V}$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V}$$

$$\left(\frac{\partial T}{\partial V}\right)_P = \frac{R}{P}$$

$$PV = RT$$

$$\left(\frac{\partial V}{\partial P}\right)_T = \frac{RT}{P^2} = -RT \cdot \frac{1}{P^2}$$

$$T = \frac{PV}{R}$$

$$\left(\frac{\partial T}{\partial V}\right)_P = \frac{P}{R} \left(\frac{\partial V}{\partial V}\right)$$

$$= -\frac{RT}{P^2}$$

Clausius - Clapeyron Equation

$$\ln \frac{P_2}{P_1} = \frac{\Delta H}{R} \left[\frac{T_2 - T_1}{T_1 \cdot T_2} \right]$$

⇒ Scattering ~~का~~ पर ⇒ (Ice)

press. ↑ करने से Temp ↓
[m.pt. ↓] ⇒ (variation of temp. with pressure)

⇒ water जिल्ल बन जाती है।

for an Ideal Gas \Rightarrow

Ques:- $\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial V}\right)_T = ?$

(2)

(i) $-\frac{R^2}{P^2}$ (ii) -1 (c) $\frac{V}{T} \cdot \frac{R}{V^2} = -\frac{R^2}{V^2}$

$$PV = RT$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V}$$

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P}$$

$$\left(\frac{\partial P}{\partial V}\right)_T$$

$$PV = RT$$

$$\partial P = + \frac{RT}{V}$$

$$\left(\frac{\partial P}{\partial V}\right)_T = RT \cdot \frac{\partial V}{\partial V} \cdot \frac{1}{V}$$

$$\left(\frac{\partial P}{\partial V}\right)_T = RT \cdot -\frac{1}{V^2}$$

$$= \frac{R}{V} \cdot \frac{R}{P} \cdot -\frac{RT}{V^2}$$

$$= \frac{-R^2 \cdot RT}{PV \cdot V^2}$$

$$= \frac{-R^2 \cdot RT}{RT \cdot V^2}$$

$$= -\frac{R^2}{V^2}$$

$$P(V-b) = RT$$

$$PV - Pb = RT$$

$$PV = RT + Pb$$

$$V = \frac{RT + Pb}{P}$$

$$dV = \frac{RT}{P} + b$$

$$\left(\frac{dV}{dT}\right)_P = \frac{R}{P} \frac{\partial T}{\partial T} + 0$$

$$\left(\frac{dV}{dT}\right)_P = \frac{R}{P} \quad \text{--- (1)}$$

$$P = \frac{RT}{(V-b)}$$

$$\left(\frac{dP}{dT}\right)_V = \frac{R}{(V-b)} \frac{\partial T}{\partial T}$$

$$\left(\frac{dP}{dT}\right)_V = \frac{R}{(V-b)} \quad \text{--- (2)}$$

$$T = \frac{P(V-b)}{R}$$

$$\left(\frac{dT}{dP}\right) = \frac{(V-b)}{R} \frac{dP}{dP}$$

$$\left(\frac{dT}{dP}\right)_V = \frac{(V-b)}{R}$$

$$PV - Pb = RT$$

$$P(V-b) = RT$$

$$P = \frac{RT}{V-b}$$

$$\left(\frac{dP}{dV}\right)_T = RT \frac{dV}{dV} \left(\frac{1}{(V-b)}\right)$$

$$= RT \left(\frac{-1}{(V-b)^2}\right)$$

$$\left(\frac{dP}{dV}\right)_T = -\frac{RT}{(V-b)^2}$$

$$\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V \left(\frac{\partial P}{\partial V}\right)_T$$

$$\frac{R}{P} \times \frac{-RT}{(V-b)^2} \times \frac{(V-b)}{R}$$

$$-RT$$

$$-RT$$

$$-RT$$

hence proved

[16.10.15]

Net-2010 [Sub.]

Integrating factor

the factor which making the function become exact by simply multiplying with an exact function then that function factor is called Integrating factor.

Let us consider -

$$dz = y dx + (-x) dy$$

applying Euler's theorem -

$$\frac{\partial^2 z}{\partial x \partial y} \neq \frac{\partial^2 z}{\partial y \partial x}$$

$$1 \neq -1$$

it doesn't follow Euler's theorem.

Multiplying in Equation by one by x^2 [$\frac{1}{x^2}$] then the function become -

$$\frac{dz}{x^2} = \frac{y}{x^2} dx + \frac{x}{-x^2} dy$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = \frac{1}{x^2}$$

Now the function follow Euler's theorem -

then that this is state function.

Now the function become exact

So $\frac{1}{x^2}$ is Integration factor.

STATE - FUNCTION :→

- * The value of state function depend upon initial state and final state.
- * the value of state function path independent
- * It must be Exact differential.
- * It must follow Euler's theorem.
- * their cyclic integral is zero.

$$Z = T, P, V, G, H, A, E \text{ Etc.}$$

$$Z \neq q, w \text{ [Path dependent function]}$$



$$\oint dE = 0$$

$$\oint dV = 0$$

but

$$\oint dq \neq 0$$

$$\oint dw \neq 0$$

∴ INTERNAL ENERGY :-

or "Intrinsic Energy"

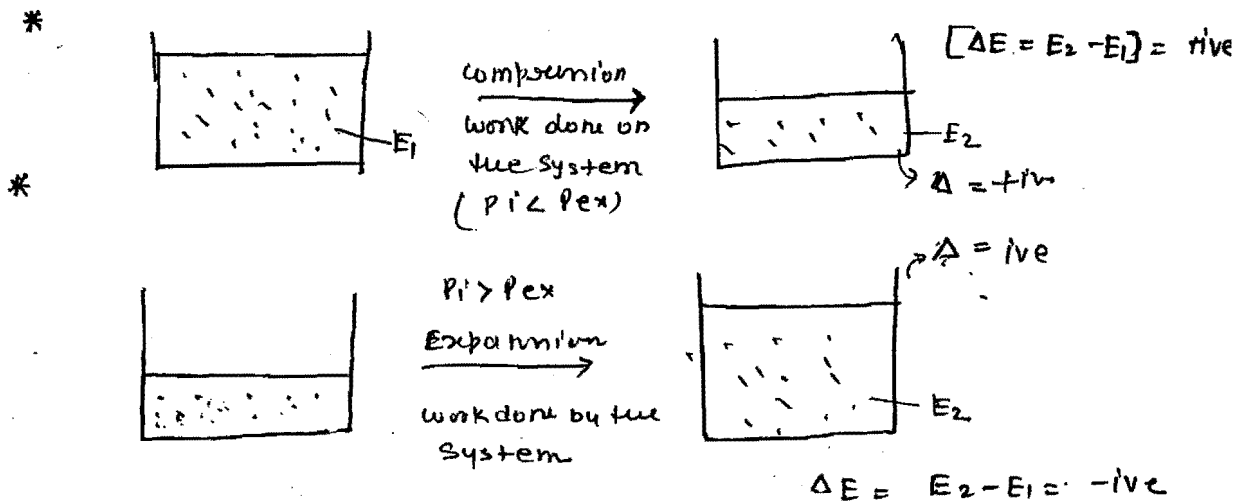
[E or U]

(Per mole)

* Internal Energy is State function

* Int. Enrg. is an Extensive property.

* The value of Int. Enrg. following system depend up chemical nature of the substance of a System.

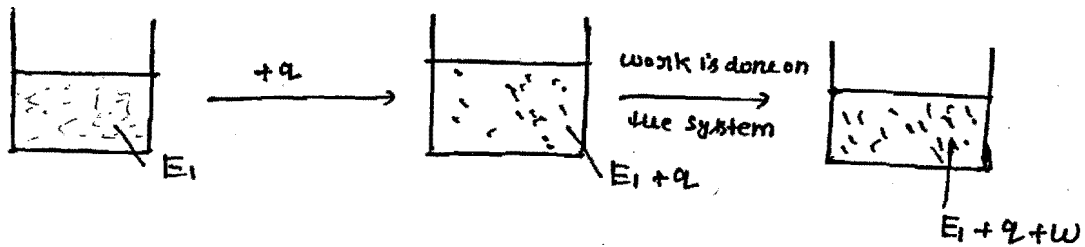


In Gen.

If work is done on the system internal energy increases.

If work is done by the system internal energy decreases.

⚡ FIRST-LAW OF THERMODYNAMICS ⚡



$$E_2 = E_1 + q + w$$

$$E_2 - E_1 = q + w$$

$$\Delta E = q + w$$

Energy can neither be created nor be destroyed
It can only be converted into one form to another
form.

It is not possible to create energy.

Energy always be conserved

SIGNIFICANCE OF INTERNAL ENERGY \Rightarrow or Heat chng at constt volume

Heat change at constant volume is called
Change in Internal Energy.

$$\Delta E = q + w$$

$$\Delta E = \int -P \cdot dV$$

$$\Delta E = 0$$

at constant volume-

$$\Delta E \cong q - 0$$

$$\Delta E = q$$

$$\Delta E = q_v$$

workdone नही हुआ तो
जितनी Heat change उतनी Internal Energy

or

$$dE = dq_v \text{ - small change}$$

or,

that then there is not $(P \cdot dV)$ work.

$(dv=0)$, then the Heat change is called
change in internal Energy.

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P$$

↓
(Isobaric Expansion coefficient)

$$\beta = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$$

Isothermal Compressibility coefficient

$$\gamma = \frac{1}{P} \left(\frac{\partial P}{\partial T} \right)_V$$

Isochoric thermal-pressure coefficient

Isobaric Expansion Coefficient $[\alpha]$ $\left[\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P \right]$

α - Shows that Increase in temp. take place with increase in volume.

V & T having direct relation

i.e we can say that α is Isobaric Expansion Coefficient.

- It having a dimension $\frac{L}{T}$
(Temp.)
- The value of α for an ideal gas = $\frac{1}{T}$

Proof. since we know that

$$PV = RT$$

$$V = \frac{RT}{P}$$

$$\left(\frac{dV}{dT} \right)_P = \left[\frac{d}{dT} \left(\frac{R}{P} \right) T \right]_P$$

$$\left(\frac{dV}{dT} \right)_P = \frac{R}{P} \cdot 1$$

$$\left(\frac{dV}{dT} \right)_P = \frac{R}{P}$$

$$\therefore \alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P$$

$$= \frac{1}{V} \cdot \frac{R}{P} = \frac{R}{PV} = \frac{R}{RT} = \frac{1}{T}$$

Q4. Calculate the value of α for a gas which having
Equation $P(V-b) = RT$

$$P(V-b) = RT$$

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P$$

$$PV - Pb = RT$$

$$PV = RT + Pb$$

$$V = \frac{RT}{P} + b$$

$$\left(\frac{\partial V}{\partial T} \right)_P = \left(\frac{d}{dT} \left[\frac{R}{P} T + b \right] \right)_P$$

$$\left(\frac{\partial V}{\partial T} \right)_P = \frac{R}{P} \cdot 1 + 0$$

$$\left(\frac{\partial V}{\partial T} \right)_P = \frac{R}{P}$$

$$\alpha = \frac{1}{V} \cdot \frac{R}{P} = \frac{R}{PV}$$

Q4. Find out the value of α for vander-well Equation:

$$(P + \frac{a}{V^2})(V-b) = RT \quad \text{--- (1)} \quad \therefore \alpha = \frac{1}{V} \left(\frac{dV}{dT} \right)_P$$

We can find out the value of $\frac{dV}{dT}$ at constant Pressure - by (1)

$$\frac{d}{dt} \left[(P + \frac{a}{V^2})(V-b) \right]_P = \frac{d}{dt} [RT]_P$$

$$(P + \frac{a}{V^2}) \frac{d}{dt} (V-b) + (V-b) \frac{d}{dt} (P + \frac{a}{V^2}) = R$$

$$(P + \frac{a}{V^2}) \left(\frac{dV}{dt} \right)_P + (V-b) \left[0 + -\frac{2a}{V^3} \right] \left(\frac{dV}{dt} \right)_P = R$$

$$(P + \frac{a}{V^2}) \left(\frac{dV}{dt} \right)_P + (V-b) \left(-\frac{2a}{V^3} \right) \left(\frac{dV}{dt} \right)_P = R$$

$$\left(\frac{dV}{dT} \right)_P \left[(P + \frac{a}{V^2}) - \frac{2a(V-b)}{V^3} \right] = R$$

$$\left(\frac{dV}{dT} \right)_P = \left[\frac{R}{(P + \frac{a}{V^2}) - \frac{2a(V-b)}{V^3}} \right]$$

$$\alpha = \frac{1}{V} \left[\frac{R}{(P + \frac{a}{V^2}) - \frac{2a(V-b)}{V^3}} \right]$$

$$\alpha = \frac{1}{V} \left[\frac{R}{\frac{RT}{(V-b)} - \frac{2a(V-b)}{V^3}} \right]$$

$$\alpha = \frac{1}{V} \left[\frac{R}{\frac{RTV^3 - 2a(V-b)^2}{V^3}} \right]$$

$$\alpha = \frac{RV^2}{RTV^3 - 2a(V-b)^2}$$

Isothermal Compressibility Coefficient

$$\beta = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$$

- β - Shows that pressure increases with decrease in volume that's why β has -ive sign
- Dimension P^{-1}
↳ pressure.
- the value of β for an ideal gas [$\beta = \frac{1}{P}$]

Proof:-

$$PV = RT$$

$$V = \frac{RT}{P}$$

$$\left(\frac{\partial V}{\partial P} \right)_T = \left(\frac{\partial}{\partial P} \frac{RT}{P} \right)_T$$

$$\left(\frac{\partial V}{\partial P} \right)_T = RT \cdot \frac{-1}{P^2}$$

$$\beta = -\frac{1}{V} \cdot RT \cdot \frac{-1}{P^2}$$

$$= \frac{RT}{PV \cdot P}$$

$$= \frac{RT}{RT \cdot P}$$

$$\beta = \frac{1}{P}$$

Isochoric thermal-pressure coefficient

$$\gamma = \frac{1}{P} \left(\frac{\partial P}{\partial T} \right)_V$$

- γ shows that pressure & temp. having direct relation.
- It having dimension T^{-1}
- the value of γ for ideal gas. $\left[\frac{1}{T} \right]$

Proof.

$$PV = RT$$

$$P = \frac{RT}{V}$$

$$\left(\frac{dP}{dT} \right)_V = \left[\frac{d}{dT} \left(\frac{RT}{V} \right) \right]_V$$

$$\left(\frac{dP}{dT} \right)_V = \left(\frac{R}{V} \right)$$

$$\gamma = \frac{1}{P} \cdot \frac{R}{V}$$

$$\gamma = \frac{R}{RT} = \frac{1}{T}$$

$$\boxed{\gamma = \frac{1}{T}}$$

Enthalpy :→

- Enthalpy is a State function, it is path independent
- Enthalpy is a Heat content of a system at a constant pressure and temp. when system is open.

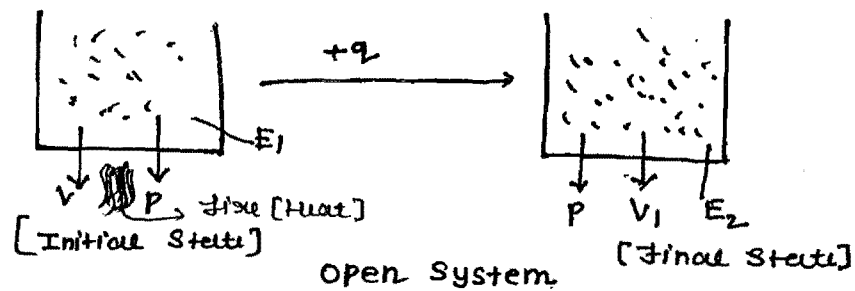
Mathematically, Enthalpy is sum of Internal Energy and PV .

$$H = U + PV$$

we can't measure the Enthalpy, we only measure the change in Enthalpy.

change in Enthalpy at const. temp., pressure is equal to Heat change so,

e.g. in liquid. Having Internal Energy E



∴ First-law of thermodynamics

$$\Delta E = q - P \Delta V$$

$$E_2 - E_1 = q - P(V_2 - V_1)$$

$$E_2 - E_1 = q - PV_2 + PV_1$$

$$E_2 + PV_2 = q + E_1 + PV_1$$

$(E_2 + PV_2)$ or $(E_1 + PV_1)$ is represented by a term called Enthalpy (H)

$(E+PV)$ is also called Heat content of the system at constant and pressure.

Now,

$$H_2 = q + H_1$$

$$H_2 - H_1 = q_p$$

$$\boxed{\Delta H = q_p}$$

Heat change at a constant pressure is called Enthalpy change.

October, 22 [2015]

Imp.

Max-well-Equation: \Rightarrow

Max well Equation in term of change in
Internal Energy.

this Equation Shows that Internal Energy
changes with simultaneously change of
S and V

$$dE = Tds + (-P)dv \quad \text{--- ①}$$

We can also calculate -

[at a constant S]

$$\left(\frac{dE}{dv} \right)_S = -P$$

or

$$-\left(\frac{dE}{dv} \right)_S = P$$

$$\left(\frac{dE}{dv} \right)_S = Tds + (-P) \frac{dv}{dv}$$

$$\left(\frac{dE}{dv} \right)_S = -P$$

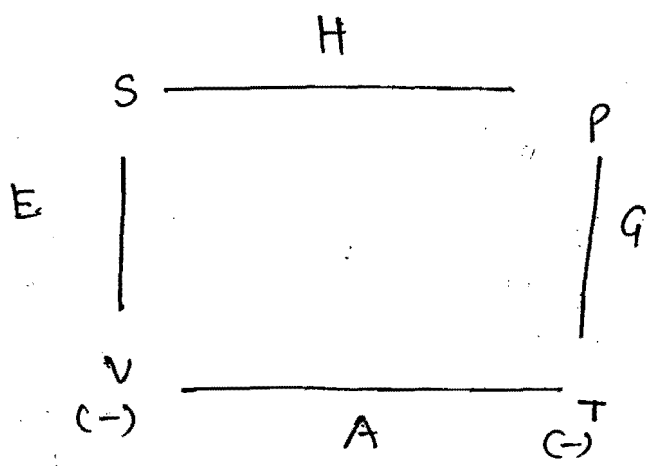
we can also calculate

$$\left(\frac{dE}{ds} \right)_V = T$$

$$dE = Tds + (-P)dv \quad \text{const} \rightarrow V$$

$$\left(\frac{dE}{ds} \right)_V = T \left(\frac{ds}{ds} \right)$$

we can also calculate max-well relation \Rightarrow



Note \Rightarrow diagonal multiply में sign

Include नहीं करना

$$dH = TS + VP$$

$$\therefore dE = Tds + (-p)dv \quad \text{---(i)}$$

$$E = f(s, v)$$

above Equation shows that E is function of s and v

$$\therefore \frac{dE}{ds} = \left(\frac{dE}{ds} \right)_v ds + \left(\frac{dE}{dv} \right)_s dv \quad \text{(ii)}$$

Compare (i) and (ii)

$$dE = Tds + (-p)dv$$

\therefore E is state function, so it must follow Euler's theorem, Applying Euler's theorem \rightarrow

$$\boxed{\left(\frac{dT}{dv} \right)_s = - \left(\frac{dp}{ds} \right)_v}$$

Max-well Eqn in terms of change in Enthalpy \rightarrow

$$\therefore dH = Vdp + Tds$$

$$\left(\frac{dv}{ds} \right)_p = \left(\frac{dT}{dp} \right)_s$$

Above relation shows that Enthalpy change with simultaneously change in s and p

$$\begin{aligned} dH &= Vdp + Tds \\ \left(\frac{dH}{dp} \right)_s &= V \end{aligned}$$

$$\left(\frac{dH}{ds} \right)_p = T$$

$$\begin{aligned} dH &= Vdp + Tds \\ \left(\frac{dH}{dp} \right)_s &= V \\ \left(\frac{dH}{ds} \right)_p &= T \end{aligned}$$

Max-well relation - (Prove by Euler theorem)

$$\left(\frac{dv}{ds}\right)_P = \left(\frac{dT}{dP}\right)_S$$

Prove \rightarrow It is

$$\left(\frac{dv}{ds}\right)_P$$

Max well Equation in terms of G . [Gibbs free Energy]

$$dG = VdP - SdT$$

$$\therefore \left(\frac{dG}{dP}\right)_T = V$$

$$\begin{aligned} \left(\frac{dG}{dT}\right)_P &= V \frac{dP}{dT} - S \overset{0}{dT} \\ &= V \end{aligned}$$

$$\therefore \left(\frac{dG}{dT}\right)_P = -S$$

$$\left(\frac{dG}{dP}\right)_P = V \overset{0}{dP} - S \left(\frac{dT}{dT}\right) = -S$$

max well - Relation

$$\left(\frac{dV}{dT}\right)_P = -\left(\frac{dS}{dP}\right)_T$$

Prove \rightarrow

Max well Equation in terms of Helmholtz-free Energy \Rightarrow

$$dA = -Pdv - SdT$$

$$\left(\frac{dA}{dv}\right)_T = -P$$

$$\left(\frac{dA}{dT}\right)_v = -S$$

Max well relation \Rightarrow

$$\boxed{\left(\frac{dP}{dT}\right)_v = \left(\frac{dS}{dv}\right)_T}$$

Proof \Rightarrow

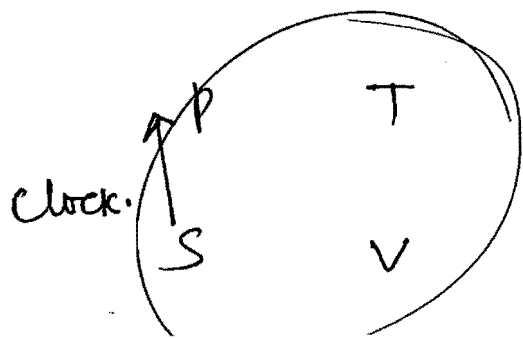
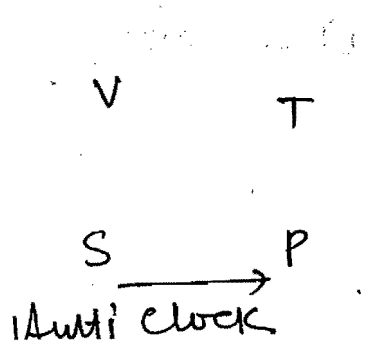
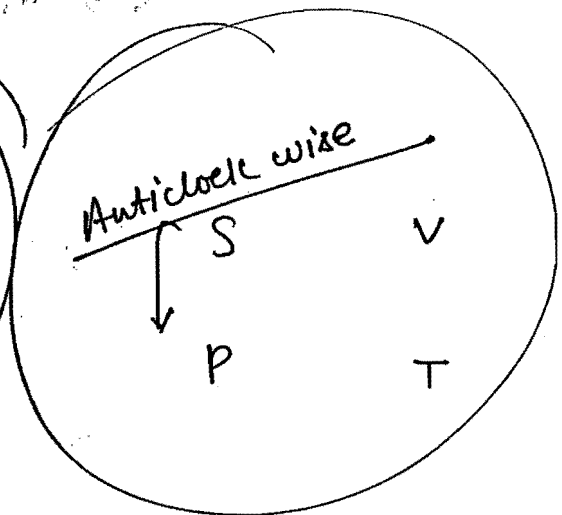
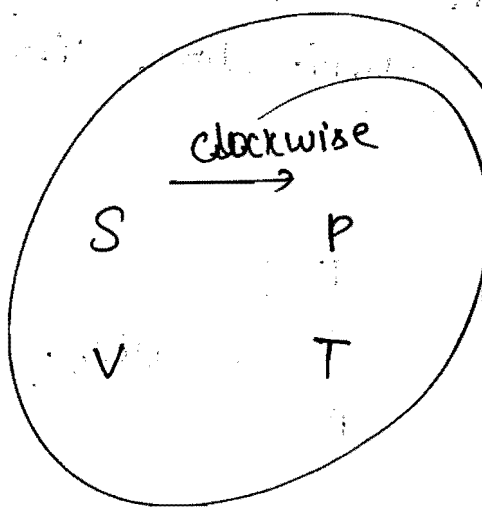
Maxwell Relation

$$-\left(\frac{\partial v}{\partial s}\right)_p = \left(\frac{\partial T}{\partial p}\right)_s$$

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$$

$$\left(\frac{\partial p}{\partial s}\right)_v = \left(\frac{\partial T}{\partial v}\right)_p$$

$$\ominus \left(\frac{\partial s}{\partial p}\right)_T = \left(\frac{\partial v}{\partial T}\right)_p$$



Ques: \Rightarrow For a System of constant composition

[No. of moles remain same]

the pressure P is given by \Rightarrow

(a) $-\left(\frac{du}{ds}\right)_v$

~~(b)~~ $-\left(\frac{du}{dv}\right)_s$

(c) $\left(\frac{\partial v}{\partial s}\right)_T$

(d) $\left(\frac{du}{dv}\right)_T$

$dU = T \boxed{dS} - P dV$ \downarrow at const S

$-\left(\frac{du}{dv}\right)_s = P$

\Rightarrow क्यों कि P ठीक है ही
और अगर const. v हो तो P का
 P का zero हो जाएगा

2012 June (Net)

Q. Indicate which one of the following relation is NOT correct. (सही विकल्पों में सही उत्तर चुनें)

(a) $-\left(\frac{dT}{dV}\right)_S = \left(\frac{\partial P}{\partial S}\right)_V$ (Correct)

~~(b)~~ $-\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$ [Not correct]

(c) $T\left(\frac{\partial S}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V$ (Correct)

(d) $-\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{\partial V}{\partial T}\right)_P$ (Correct)

(2012) Net (June)

Q. The correct thermodynamics relation among the following is

~~(a)~~ $\left(\frac{dE}{dV}\right)_S = -P$

(b) $\left(\frac{dH}{dV}\right)_S = -P$

(c) $\left(\frac{\partial H}{\partial V}\right)_S = -P$

(d) $\left(\frac{dA}{dV}\right)_S = -S$

$dH = T ds + V dp$
 $\left(\frac{dH}{dS}\right)_P = T$

2013

Q. The maxwell relationship derived from the Equation

$dg = vdp - (s)dT$

(i) $\left(\frac{dv}{dT}\right)_P = \left(\frac{\partial s}{\partial P}\right)_T$

$\left(\frac{dv}{dT}\right)_P = -\left(\frac{\partial s}{\partial P}\right)_T$

~~(ii)~~ $\left(\frac{dv}{dT}\right)_P = -\left(\frac{\partial s}{\partial P}\right)_T$

(iii) $\left(\frac{\partial v}{\partial P}\right)_T = -\left(\frac{\partial s}{\partial T}\right)_P$ (iv) $\left(\frac{\partial v}{\partial P}\right)_T = -\left(\frac{\partial s}{\partial T}\right)_P$

(2014) For a process in a closed system Temp. is Equal to -

(a) $\left(\frac{\partial H}{\partial P}\right)_S$

(b) $\left(\frac{dG}{dP}\right)_T$

(c) $-\left(\frac{dA}{dV}\right)_T$

~~(d)~~ $\left(\frac{dH}{dS}\right)_P$

$$dH = T ds + v dp$$

$$\left(\frac{dH}{dS}\right)_P = T$$

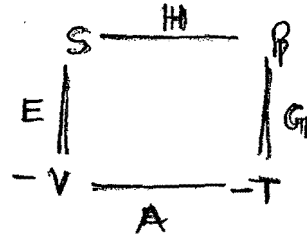
$$\boxed{\left(\frac{dH}{dS}\right)_P = T}$$

First thermodynamic Equation of a State

We know that from the max well eqⁿ-

$$dE = T ds - PdV$$

Diff. Both side with respect to V , at constant temp.



$$\left(\frac{dE}{dV}\right)_T = T \left(\frac{ds}{dV}\right)_T - P \left(\frac{dV}{dV}\right)_T$$

$$dE = T ds - PdV$$

$$\left(\frac{dE}{dV}\right)_T = T \left(\frac{ds}{dV}\right)_T - P$$

$$\boxed{\left(\frac{dE}{dV}\right)_T = T \left(\frac{dP}{dT}\right)_V - P}^*$$

$$\therefore \left(\frac{ds}{dV}\right)_T = \left(\frac{dP}{dT}\right)_V$$

$\left(\frac{dE}{dV}\right)_T \Rightarrow$ Internal pressure or π [unit]

$\left(\frac{dE}{dV}\right)_T$ at a const. T is called

change in Internal Energy with change in volume at a const. temp. is called internal pressure

Calculation of internal pressure for an ideal gas

$$PV = RT$$

$$P = \frac{RT}{V}$$

$$\left(\frac{dE}{dV}\right)_T = T \times \left(\frac{dP}{dT}\right)_V - P$$

\downarrow
 $\frac{RT}{V}$

$$\left(\frac{dP}{dT}\right)_V = \frac{R}{V} \times \frac{dT}{dT}$$

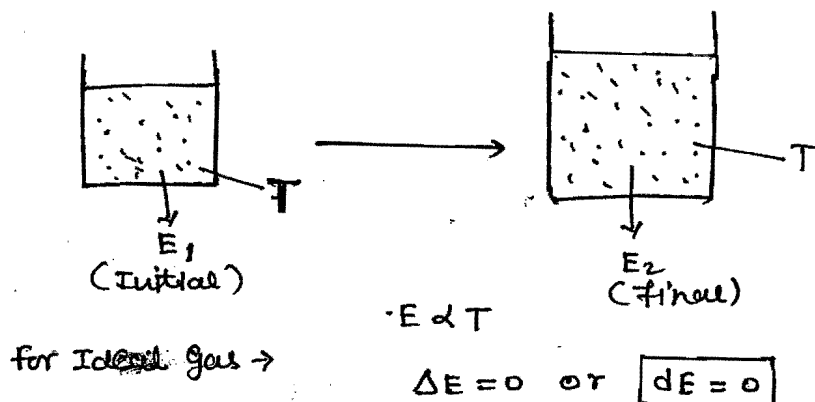
$$\left(\frac{dP}{dT}\right)_V = \frac{R}{V}$$

$$\left(\frac{dE}{dV}\right)_T \Rightarrow \frac{RT}{V} - \frac{RT}{V}$$

$$\left(\frac{dE}{dV}\right)_T = \cancel{TR} - \cancel{RT} = 0$$

← PHYSICAL SIGNIFICANCE OF INTERNAL PRESSURE FOR

IN IDEAL GAS: ⇒



Since we know that in ideal gas there is no interaction b/w molecule, so no energy is wasted in^{order to} overcome the interaction. In isothermal expansion for an ideal gas there is no change in internal energy b/w initial & final state.

So,

$$\Delta E = 0$$

or

$$\partial E = 0$$

∴ i.e. the change in internal energy with change in volume at const. temp. is

Equal to zero.

$$\left(\frac{\partial E}{\partial V} \right)_T = 0$$

Ideal Gas

∴ Internal pressure for an ideal gas zero (π)

← Calculation of Internal pressure for an Real Gas: ⇒

$$\left(\frac{\partial E}{\partial V}\right)_T = T \left(\frac{\partial P}{\partial T}\right)_V - P \quad \text{--- (1)}$$

$$(P + \frac{a}{V^2})(V-b) = RT$$

To find $\left(\frac{\partial P}{\partial T}\right)_V$

$$\left[\frac{d}{dT} (P + \frac{a}{V^2})(V-b)\right]_V = R \frac{dT}{dT}$$

$$(P + \frac{a}{V^2}) \frac{dP(V-b)}{dT} + \left(\frac{dP}{dT}\right)_V (V-b) \times (1) = R$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{R}{V-b}\right)$$

Put the value in (1) then,

$$\left(\frac{\partial E}{\partial V}\right)_T = \frac{T \times R}{(V-b)} - P$$

$$= \frac{(P + \frac{a}{V^2})(V-b)}{(V-b)} - P$$

$$= P + \frac{a}{V^2} - P$$

$$\left(\frac{\partial E}{\partial V}\right)_T = \frac{a}{V^2}$$

⇒ bcoz the interaction b/w molecules ^{in Real Gas} ~~is not~~ internal pressure of real gas is ...

Internal pressure for ~~an~~ Real Gas is $\frac{a}{V^2}$ which signifies that ~~work~~ ^{work} ~~is~~ ^{is} present in the Real Gas, so when you supply the energy, some of the energy is wasted in order to overcome the interaction b/w the Gas molecule.

So we conclude that due to interaction b/w the ^{real} Gas molecule internal pressure can never be zero but it is equal to a/V^2

Qn: calculate the internal pressure for a Gas Equation

$$P(V-b) = RT$$

$$PV - Pb = RT$$

$$P = \left(\frac{RT}{V-b} \right)$$

$$\left(\frac{dP}{dT} \right)_V = \left(\frac{R}{(V-b)} \right) \times \frac{dT}{dT}$$

$$\left(\frac{dP}{dT} \right)_V = \frac{R}{(V-b)}$$

$$\left(\frac{dE}{dV} \right)_T = \frac{T \times R}{(V-b)} - P$$

$$= \frac{RT}{(V-b)} - P$$

$$= \frac{P(V-b)}{(V-b)} - P$$

$$= P - P$$

$$\left(\frac{dE}{dV} \right)_T = 0$$

← Second thermodynamic Equation of State: →

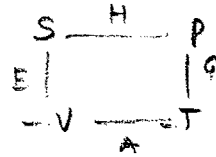
We know that, from Maxwell Equation ⇒

$$dH = Tds + Vdp$$

Diff. w.r.t. to P at const. temp.

$$\left(\frac{dH}{dP}\right)_T = T\left(\frac{ds}{dP}\right)_T + V\left(\frac{\partial P}{\partial P}\right)_T$$

$$\left(\frac{\partial H}{\partial P}\right)_T = T \cdot \left(\frac{\partial s}{\partial P}\right)_T + V$$



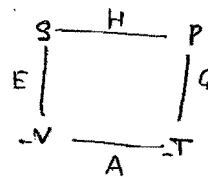
$$dH = Tds + Vdp$$

$$\left[\because \left(\frac{\partial s}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P \right]$$

$$\therefore \left(\frac{\partial H}{\partial P}\right)_T = -T \left(\frac{\partial V}{\partial T}\right)_T + V$$

Criteria for Spontaneity : →

$$\boxed{(\Delta S)_{U,V} > 0}$$



By seeing the sign of change in thermodynamic parameter we can predict the process spontaneous, reversible, or equilibrium.

Entropy change: →

$$(\Delta S)_{U,V} > 0 \quad [\text{Spontaneous}]$$

$$(\Delta S)_{U,V} = 0 \quad [\text{Reversible or Equilibrium}]$$

Change in Enthalpy

$$(\Delta H)_{S,P} < 0 \quad [\text{Spontaneous}]$$

$$(\Delta H)_{S,P} = 0 \quad [\text{Reversible or Equilibrium}]$$

Change in Gibbs free Energy

$$(\Delta G)_{P,T} < 0 \quad [\text{Spontaneous}]$$

$$(\Delta G)_{P,T} = 0 \quad [\text{Reversible or Equi.}]$$

Change in Helmholtz free Energy

$$(\Delta A)_{V,T} < 0 \quad [\text{Spon.}]$$

$$(\Delta A)_{V,T} = 0 \quad [\text{Rev. + Equi.}]$$

Change in Internal Energy

$$(\Delta E)_{V,S} < 0 \quad [\text{Spon.}]$$

$$(\Delta E)_{V,S} = 0 \quad [\text{Rev. + Equi.}]$$

October, 23 2015

Relation b/w C_p and C_v

$$C_p - C_v = T \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial T} \right)_P$$

$$C_p - C_v = R \quad [\text{for an ideal gas}]$$

$$C_p - C_v = R \left[1 + \frac{\alpha^2 T V}{\beta} \right] \rightarrow \text{for an } \text{Real} \text{ Gas}$$

$$C_p - C_v = \frac{\alpha^2 \cdot T V}{\beta}$$

$$C_p - C_v = R \rightarrow \text{for an Gas whose Eqn is } \boxed{P(V-b) = RT}$$

$$C_p - C_v = -T \left(\frac{\partial P}{\partial T} \right)_V^2 \left(\frac{\partial V}{\partial P} \right)_T$$

$$\textcircled{*} \quad C_p - C_v = T \left(\frac{\partial P}{\partial T} \right)_V \cdot \left(\frac{\partial V}{\partial T} \right)_P$$

Proof $\Rightarrow S = f(T, V)$

Total differential

$$ds = \left(\frac{ds}{dT} \right)_V dT + \left(\frac{ds}{dV} \right)_T dV$$

Divide both side by ∂T at constant pressure

$$\left(\frac{ds}{dT} \right)_P = \left(\frac{ds}{dT} \right)_V + \left(\frac{ds}{dV} \right)_T \left(\frac{\partial V}{\partial T} \right)_P$$

Multiply both side by T

$$T \left(\frac{ds}{dT} \right)_P = T \cdot \left(\frac{ds}{dT} \right)_V + T \cdot \left(\frac{ds}{dV} \right)_T \cdot \left(\frac{\partial V}{\partial T} \right)_P$$

$\therefore T ds = dq$ (we know that from Entropy change)

$$\left(\frac{dq_p}{dT} \right)_P = \left(\frac{dq_v}{dT} \right)_V + \left(\frac{dq}{dV} \right)_T \left(\frac{\partial V}{\partial T} \right)_P$$

[$\because dq$ at const. $P = \partial H$, $\therefore dq$ at const. $V = \partial E$]

$$\left(\frac{\partial H}{\partial T} \right)_P = \left(\frac{\partial E}{\partial T} \right)_V + T \cdot \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial T} \right)_P$$

[\because From maxwell relation -

$$\left[\left(\frac{ds}{dV} \right)_T = \left(\frac{\partial P}{\partial T} \right)_V \right]$$

$$C_p = C_v + T \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial T} \right)_P$$

$$\boxed{C_p - C_v = T \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial T} \right)_P}$$

$$* C_p - C_v = R \text{ [for an Ideal-gas]}$$

Proof:→

$$C_p - C_v = T \cdot \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial T} \right)_P$$

$$PV = RT \text{ [Ideal Gas Equation]}$$

$$\therefore \left(\frac{dP}{dT} \right)_V = \frac{R}{V}$$

$$\therefore \left(\frac{\partial V}{\partial T} \right)_P = \frac{R}{P}$$

Put the values in above eqⁿ-

$$C_p - C_v = T \cdot \frac{R}{V} \cdot \frac{R}{P}$$

$$C_p - C_v = \frac{T \cdot R \cdot R}{PV}$$

$$C_p - C_v = \frac{PV \cdot R}{PV}$$

$$\boxed{C_p - C_v = R}$$

$$C_p - C_v = \frac{\alpha^2 \cdot T \cdot V}{\beta}$$

$$\therefore \alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P$$

$$\beta = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$$

$$\therefore C_p - C_v = T \cdot \left(\frac{\partial V}{\partial T} \right)_P \left(\frac{\partial P}{\partial T} \right)_V$$

$$\therefore C_p - C_v = T \cdot \frac{\left(\frac{\partial V}{\partial T} \right)_P}{\left(\frac{\partial T}{\partial P} \right)_V} \quad \text{--- (1)}$$

By cyclic Rule

$$\left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial T}{\partial V} \right)_P \left(\frac{\partial V}{\partial P} \right)_T = -1$$

$$-\left(\frac{\partial T}{\partial V} \right)_P \left(\frac{\partial V}{\partial P} \right)_T = \left(\frac{\partial T}{\partial P} \right)_V \quad \text{--- (2)}$$

Put the value from [2] in [1st]

$$\begin{aligned} C_p - C_v &= T \cdot \frac{\left(\frac{\partial V}{\partial T} \right)_P}{-\left(\frac{\partial T}{\partial V} \right)_P \left(\frac{\partial V}{\partial P} \right)_T} \\ &= T \cdot \frac{\left(\frac{\partial V}{\partial T} \right)_P \cdot \left(\frac{\partial V}{\partial T} \right)_P}{-\left(\frac{\partial V}{\partial P} \right)_T} \end{aligned}$$

$$= \frac{T \cdot \alpha^2 \cdot V^2}{(-) \beta \cdot V}$$

$C_p - C_v = \frac{T \cdot \alpha^2 \cdot V}{\beta}$
--

$$C_p - C_v = -T \left(\frac{\partial P}{\partial T} \right)_V^2 \left(\frac{\partial V}{\partial P} \right)_T$$

from cyclic Rule

$$\left(\frac{\partial V}{\partial T} \right)_P \left(\frac{\partial T}{\partial P} \right)_V \left(\frac{\partial P}{\partial V} \right)_T = -1,$$

$$\therefore C_p - C_v = T \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial T} \right)_P$$

$$\left(\frac{\partial V}{\partial T} \right)_P = -1 \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial P} \right)_T$$

$$C_p - C_v = -T \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial P} \right)_T$$

$$C_p - C_v = -T \left(\frac{\partial P}{\partial T} \right)_V^2 \left(\frac{\partial V}{\partial P} \right)_T$$

$$C_p - C_v = R$$

$$P(V-b) = RT$$

$$\therefore C_p - C_v = T \cdot \left(\frac{\partial P}{\partial T}\right)_V \cdot \left(\frac{\partial V}{\partial T}\right)_P$$

$$P = \frac{RT}{(V-b)}$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{(V-b)} \cdot \frac{dT}{dT}$$

$$= \frac{R}{(V-b)}$$

$$\left(\frac{\partial V}{\partial T}\right)_P \Rightarrow$$

$$\square (V-b) = \frac{RT}{P}$$

$$V = \frac{RT}{P} + b$$

$$\left(\frac{dV}{dT}\right)_P = \frac{R}{P} \frac{dT}{dT} + 0$$

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P}$$

$$C_p - C_v = T \cdot \frac{R}{(V-b)} \times \frac{R}{P}$$

$$= \frac{T \cdot R \times R}{R \cdot T}$$

$$\boxed{C_p - C_v = R}$$

$$C_p - C_v = R \left[1 + \frac{qa}{RTV} \right] \quad \text{for real gas -}$$

$$\left(P + \frac{a}{V^2} \right) (V-b) = RT \quad \text{--- (i)}$$

$$\therefore C_p - C_v = T \cdot \left(\frac{dP}{dT} \right)_V \cdot \left(\frac{dV}{dT} \right)_P \quad \text{--- (ii)}$$

By (i)

$$\left(\frac{dP}{dT} \right)_V \Rightarrow$$

$$\left(P + \frac{a}{V^2} \right) (V-b) = RT$$

$$\left(P + \frac{a}{V^2} \right) = \frac{RT}{(V-b)}$$

$$P = \frac{RT}{(V-b)} - \frac{a}{V^2}$$

$$\left(\frac{dP}{dT} \right)_V = \frac{d}{dT} \left[\frac{RT}{V-b} - \frac{a}{V^2} \right]_V$$

$$\left(\frac{dP}{dT} \right)_V = \frac{R}{(V-b)} \times 1 - 0$$

$$\left(\frac{dP}{dT} \right)_V = \frac{R}{(V-b)} \quad \text{--- (ii)}$$

$$\left(\frac{dV}{dT} \right)_P \Rightarrow$$

$$\left(P + \frac{a}{V^2} \right) = \frac{RT}{(V-b)}$$

$$\left(\frac{d}{dT} \left[P + \frac{a}{V^2} \right] \right) = \frac{d}{dT} \left[\frac{RT}{(V-b)} \right]$$

$$0 + \frac{(-2a)}{v^3} \left(\frac{dv}{dT} \right)_P = R \times \frac{d}{dT} \left[\frac{T \times (v-b)}{v} \right]$$

$$= R \times \left[\frac{1 \cdot (v-b)}{v} + T \times \left(\frac{-1}{(v-b)} \right) \left(\frac{dv}{dT} \right)_P \right]$$

$$\frac{-2a}{v^3} \left(\frac{dv}{dT} \right)_P = \frac{R}{(v-b)} - \frac{RT}{(v-b)^2} \left(\frac{dv}{dT} \right)_P$$

$$\left[\frac{RT}{(v-b)^2} - \frac{2a}{v^3} \right] \left(\frac{dv}{dT} \right)_P = \frac{R}{(v-b)}$$

$$\left[\frac{RT(v-b)}{R(v-b)^2} - \frac{2a(v-b)}{R \cdot v^3} \right] \left(\frac{dv}{dT} \right)_P = 1$$

$$\left[\frac{T}{(v-b)} - \frac{2a(v-b)}{R} \right] \left(\frac{dv}{dT} \right)_P = 1$$

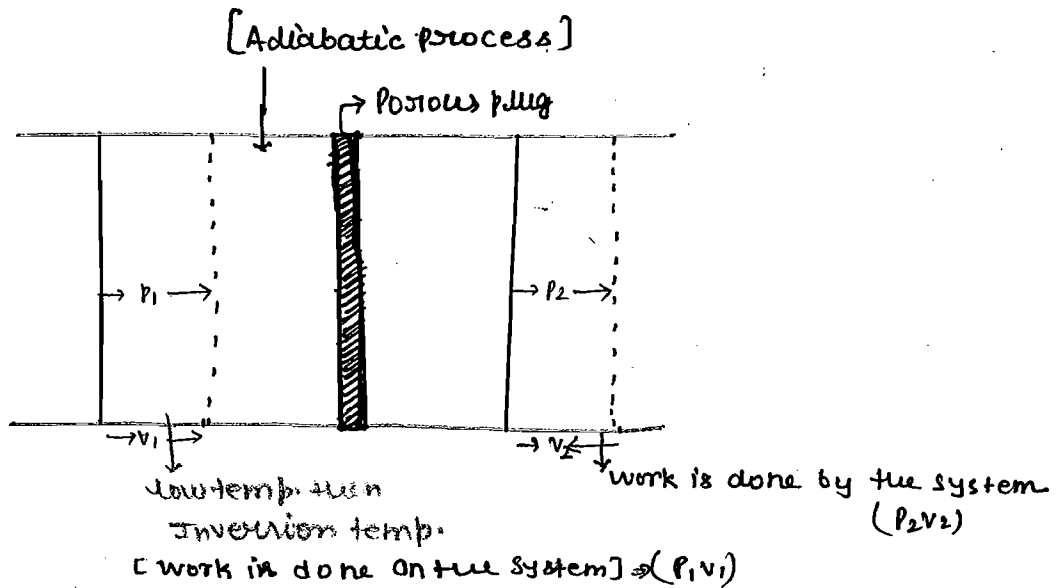
$$\left(\frac{dv}{dT} \right)_P = \frac{1}{\left[\frac{T}{(v-b)} - \frac{2a(v-b)}{v^3 \cdot R} \right]}$$

put this value in Eqnⁿ → (ii)

$$C_p - C_v = T \times \frac{R}{(v-b)} \times \left[\frac{1}{\left[\frac{T}{(v-b)} - \frac{2a(v-b)}{v^3 \cdot R} \right]} \right]$$

$$C_p - C_v = \frac{T \times R}{(v-b)} \left[\dots \right]$$

Joules-Thomson EFFECT : →



* → It is an experimental observation which says that, when a gas is allowed to expand adiabatically from high pressure to low pressure the temp. of the gas in the final state changes,

* → Joule-Thomson Effect is a Isoenthalpic process

$$\text{Net work done} = P_1 V_1 - P_2 V_2$$

Since we know that, from 1st law of thermodynamics -

$$\Delta E = q + w$$

$$q = 0 \text{ [adiabatic process]}$$

$$E_2 - E_1 = P_1 V_1 - P_2 V_2$$

$$E_2 + P_2 V_2 = E_1 + P_1 V_1$$

$$H_2 = H_1$$

$$\Delta H = 0$$

or

$$\partial H = 0$$

Inversion temp. \Rightarrow

\rightarrow The temp. below which the gas become cool down during Expansion, that temp. is called Inversion Temp.

\rightarrow All the gases show cooling effect except Hydrogen & Helium becoz inversion temp. of H₂ and He is so low it can't be achieve at R/T.
i.e they show heating effect

DERIVATION OF ISOENTHALPIC JOULE-THOMSON COEFFICIENT \Rightarrow

Suppose H is function of P and T then,

$$H = f(P, T) \quad C_p$$

$$dH = \left(\frac{\partial H}{\partial P}\right)_T dP + \left(\frac{\partial H}{\partial T}\right)_P dT$$

diff. both side with respect to dP at const. H

$$\begin{aligned} dH &= \left(\frac{\partial H}{\partial P}\right)_T dP + C_p \cdot dT \\ 0 &= \left(\frac{\partial H}{\partial P}\right)_T \times 1 + C_p \cdot \left(\frac{dT}{dP}\right)_H \end{aligned}$$

$$-C_p \left(\frac{dT}{dP}\right)_H = \left(\frac{\partial H}{\partial P}\right)_T$$

$$\boxed{\left(\frac{\partial T}{\partial P}\right)_H = \mu_{J.T.} = \frac{-1}{C_p} \left(\frac{\partial H}{\partial P}\right)_T}$$

$\mu_{J.T.} \Rightarrow$ Joule-Thomson coefficient

$$\boxed{-\left(\frac{\partial T}{\partial P}\right)_H = \mu_{J-T}}$$

→ Prove that μ_{J-T} for an Ideal Gas →

Since we know that -

$$\mu_{J-T} = -\frac{1}{C_p} \left[\frac{dH}{dP} \right]_T$$

$$\because H = E + PV$$

$$= -\frac{1}{C_p} \left[\frac{\partial [E + PV]}{\partial P} \right]_T$$

$$= -\frac{1}{C_p} \left[\left(\frac{\partial E}{\partial P} \right)_T + \left(\frac{\partial (PV)}{\partial P} \right)_T \right]$$

$$= -\frac{1}{C_p} \left[\left(\frac{\partial E}{\partial P} \right)_T + \left(\frac{\partial (RT)}{\partial P} \right)_T \right]$$

$$= -\frac{1}{C_p} \left[\left(\frac{\partial E}{\partial V} \right)_T \left(\frac{\partial V}{\partial P} \right)_T + \left(\frac{\partial (RT)}{\partial P} \right)_T \right]$$

Both are constant

↳ (Ideal Gas has fixed internal energy)

$$= -\frac{1}{C_p} [0 + 0]$$

$$= -\frac{1}{C_p} \times 0$$

$$\boxed{\mu_{J-T} = 0}$$

This Results shows that Ideal Gas does not Show Joule-Thomson Effect.

Derivation of Isothermal Joule-Thomson coefficient: \Rightarrow

$$\mu_{J.T.} = -\frac{1}{C_p} \left[\frac{dH}{dP} \right]_T$$

$$\boxed{-C_p \cdot \mu_{J.T.} = \left[\frac{dH}{dP} \right]_T}$$

↓
Isothermal Joule-Thomson coefficient

DERIVATION OF Joule-Thomson coefficient ($\mu_{J.T.}$) for A Real Gas: \Rightarrow

$$\mu_{J.T.} = -\frac{1}{C_p} \left[\frac{dH}{dP} \right]_T \quad \text{--- (A)}$$

By second thermodynamic Eqⁿ \rightarrow

$$V = T \left(\frac{dV}{dT} \right)_P + \left(\frac{\partial H}{\partial P} \right)_T$$

$$V - T \left(\frac{dV}{dT} \right)_P = \left(\frac{\partial H}{\partial P} \right)_T$$

put this value in (A)

$$\mu_{J.T.} = -\frac{1}{C_p} \left[V - T \left(\frac{dV}{dT} \right)_P \right]$$

$$\mu_{J.T.} = \frac{1}{C_p} \left[T \left(\frac{dV}{dT} \right)_P - V \right]$$

$$\therefore -\left(\frac{\partial H}{\partial P}\right)_T = \left[T \left(\frac{\partial V}{\partial T}\right)_P - V \right]$$

$$(P + \frac{a}{V^2})(V-b) = RT$$

$$PV - Pb + \frac{a}{V} - \frac{ab}{V^2} = RT$$

neglect - $\frac{ab}{V^2}$

$$PV - Pb + \frac{a}{V} = RT$$

$$PV = RT - \frac{a}{V} + Pb \quad \text{--- (3)}$$

$$V = \frac{RT}{P} - \frac{a}{PV} + b \quad \text{--- (4)}$$

Diff. Eqⁿ (4) by T at const. (P)

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{\partial}{\partial T} \left[\frac{R}{P} \times T \right] - \frac{\partial}{\partial T} \left(\frac{a}{PV} \right) + \left(\frac{\partial P}{\partial T}\right)_P$$

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P} + \frac{\partial}{\partial T} \left(\frac{a}{RT} \right) + \cancel{\left(\frac{\partial P}{\partial T}\right)_P} \rightarrow 0$$

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{V-b}{T} \frac{R}{P} + \frac{a}{RT^2} + 0$$

[$\because PV = RT$
assumption]

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P} + \frac{a}{RT^2} \quad \text{--- (5)}$$

Put the value of $\frac{R}{P}$ from Eqⁿ (4)

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{V-b}{T} + \frac{a}{RT^2} + \frac{a}{RT^2}$$

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{2a}{RT^2} + \frac{(V-b)}{T}$$

Put the value of (6) in Eqⁿ-[3]

$$\mu_{J.T.} = \frac{1}{C_p} \left[\frac{2a}{RT} + \cancel{v} - b - \cancel{v} \right]$$

$$\boxed{\mu_{J.T.} = \frac{1}{C_p} \left[\frac{2a}{RT} - b \right]}$$

conclusion of Joule-Thomson Effect: \Rightarrow

- Ideal Gas doesn't Show Joule Thomson Effect

$\mu_{J.T.}$ for an ideal gas is equal to zero

Isoenthalpic Joule-Thomson coefficient is equal to $\mu_{J.T.}$

$$\left(\frac{\partial F}{\partial P} \right)_H = \mu_{J.T.} = -\frac{1}{C_p} \left(\frac{\partial H}{\partial P} \right)_T$$

All gases at R/T shows cooling effect except Hydrogen & Helium bcoz their inversion temp. does not achivable^{I.T.} at R/T.

In the case of Real Gas the nature of $\mu_{J.T.}$

$$\mu_{J.T.} = +\frac{1}{C_p} \left[\frac{2a}{RT} - b \right]$$

\rightarrow In the case of Real Gas if the nature of $\mu_{J.T.}$ is +ive, it means that cooling takes place.

If the value of $\mu_{J.T.}$ is -ive it means that heating take place
if the value of $\mu_{J.T.}$ is zero it means that neither heating nor cooling of Gas takes place

$$\mu_{J.T.} = \left(\frac{\partial T}{\partial P} \right)_H = +ive \text{ [cooling]}$$

$$\mu_{J.T.} = \left(\frac{\partial T}{\partial P} \right)_H = -ive \text{ [heating]}$$

A temp. at which Gas shows neither heating nor cooling that temp. is called inversion temp.

$$\Delta T_i = \frac{1}{C_p} \left[\frac{2a}{RT} - b \right]$$

$$0 = \frac{1}{C_p} \left[\frac{2a}{RT} - b \right]$$

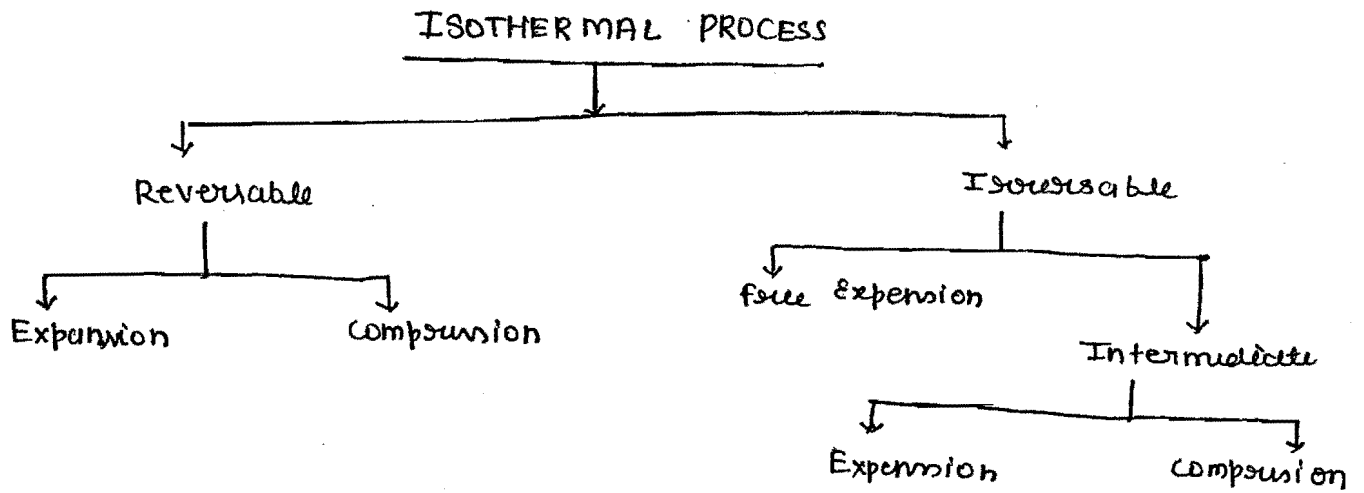
$$\frac{2a}{RT_i} = b$$

$$\boxed{\frac{2a}{Rb} = T_i}$$

$T_i \Rightarrow$ Inversion temp.

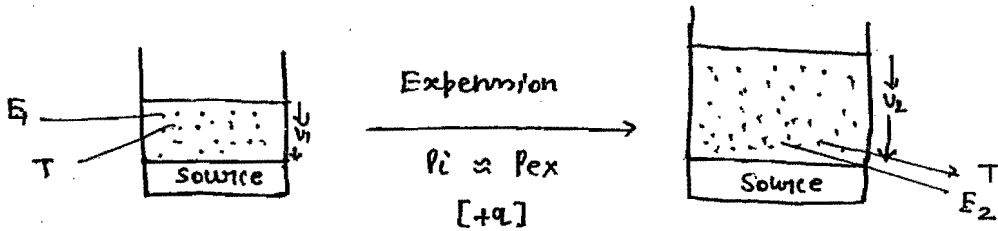
Above T_i the Gas shows heating effect and below the inversion temp. Gas shows cooling effect.

In J-T Effect, the Gas is adiabatically expand from high pressure to low pressure, there is a change is occur, the extent of change depend upon initial temp., pressure difference, nature of Gas used.



Isothermal Reversible Expansion or Compression for an Ideal Gas:

[1] Expansion: → Suppose a System which contain an Ideal Gas expand isothermally from V_1 to V_2 . In this case Temp remain same & $+q$ heat must be absorb by the Surrounding.



calculation of ΔE ⇒

In the case of Ideal Gas, E is a function of temp. As temp. remain same initial state & final state change in internal energy is equal to zero.

$\Delta E = 0$

In the adiabatic system

Temp. is same, then E does not change

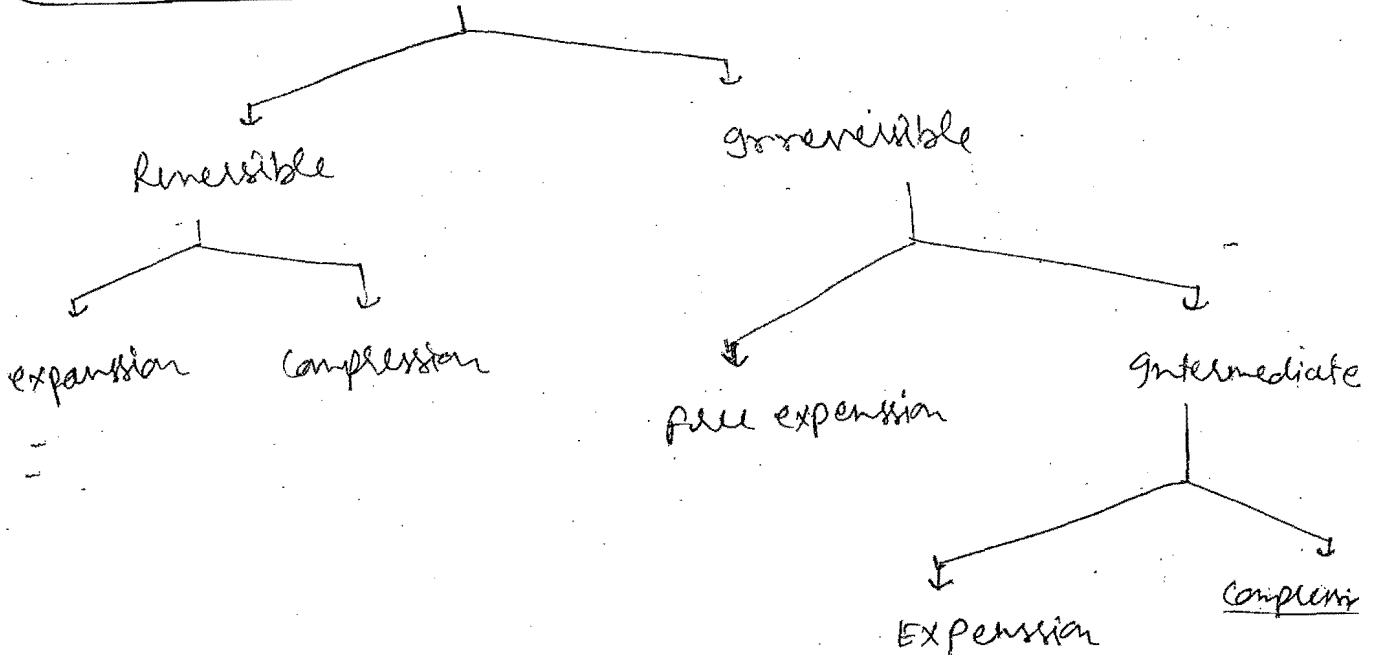
15 ①

cooling effect.

⇒ In J-T effect, the gas is adiabatically expanded from high pressure to low pressure, there is a change in temp. The extent of change depends upon initial Temp., pressure difference, nature of gas used.

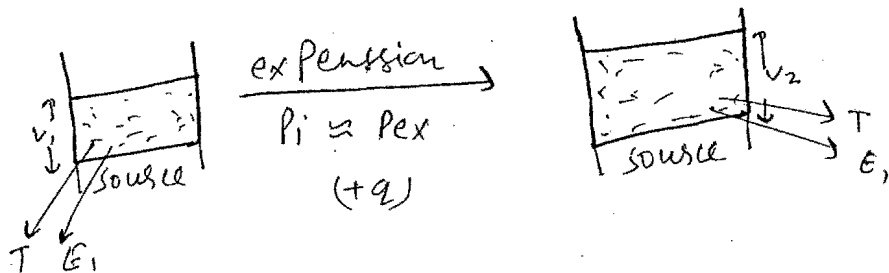
isothermal process

for a system which contains ideal gas



Isothermal reversible expansion 'R' compression for an ideal gas

① Expansion Suppose a system which contains an ideal gas, expand isothermally from v_1 to v_2 . In this case Temp. remain same so $+q$ heat must be absorbed by the surrounding.



Calculation of ΔE

In the case of ideal gas, E is a function of Temp.

As temp. remains same initial state & final state, change in internal energy is equal to zero

$$\Delta E = 0$$

In the abn. system temp. is same, then internal energy does not change.

$$E \propto T$$

$$\Delta E = 0$$

$$dE = 0$$

Calculation in change of enthalpy

$$\therefore \Delta H = \Delta E + P \Delta V$$

$$\Delta H = \Delta E + \Delta(PV)$$

$$\Delta H = \Delta E + \Delta(RT)$$

$$\Delta H = \Delta E + Q \Delta T$$

$$\Delta T = 0$$

$$\Delta H = \Delta E$$

$$\Delta H = \Delta E = 0$$

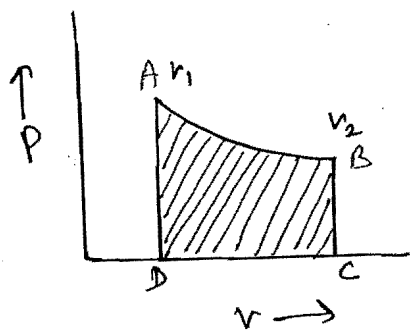
Calculation of work done

$$W = - \int_{V_1}^{V_2} P dV$$

$$= -nRT \int_{V_1}^{V_2} \frac{dV}{V}$$

$$W = -nRT \ln \frac{V_2}{V_1}$$

In the reversible case,



work done = area of ABCD

Note

In reversible work done, we always use integration

Calculation of Heat absorb

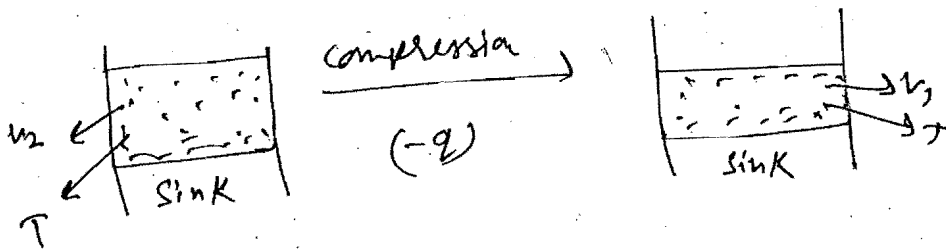
$$\Delta E = q + w$$

$$\Delta E = 0$$

$$+q = PdV = 0$$

$$+q = nRT \ln \frac{V_2}{V_1}$$

For Compression



In isothermal compression, work is done on the system, this work may increase the temp. of the system. But in the above case temp. remain same so all the heat rejected to the sink (surrounding)

Calculation of $\Delta H, \Delta E$

$$\Delta E = 0$$

$$\Delta E = 0$$

$$\Delta H = \Delta E = 0$$

$$dH = 0$$

Calculation of work done

$$W = - \int_{V_2}^{V_1} P dv$$

$$= - nRT \int_{V_2}^{V_1} \frac{dv}{v}$$

$$W = - nRT \ln \frac{V_1}{V_2}$$

Calculation of Heat Rejected

Calculation of

$$\Delta E = q + w$$

$$-q = w$$

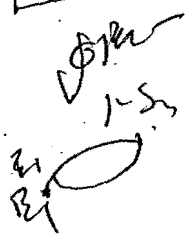
$$-q = -nRT \ln \frac{V_1}{V_2}$$

Calculation of Total work done in isothermal compression & expansion to the same extent reversible

only consider the magnitude

$$W = nRT \ln \frac{V_2}{V_1} + nRT \ln \frac{V_1}{V_2}$$

$$= nRT \ln \frac{V_2}{V_1} - nRT \ln \frac{V_2}{V_1}$$



$$W_{Total} = 0$$

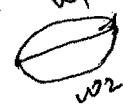
$$\oint w = 0$$

But $\oint \frac{dw}{T} \neq 0$

(final - initial) and always have a value does not zero

Irreversible when driving force & opposing force is comparable.

dw does not zero bec work is path dependent function.



$$\Delta B = 9.5 - 8.3$$

$$\Delta B = 10 - 5$$

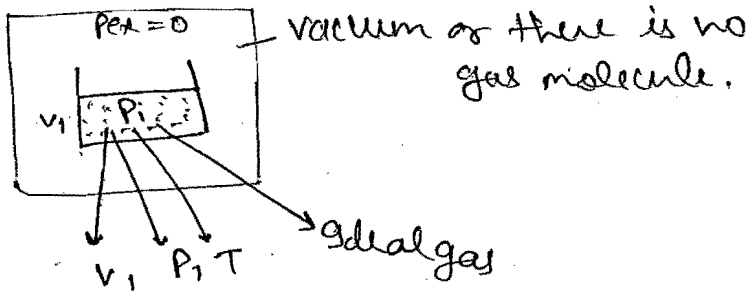
But in case of E, ΔE may be possible to zero.

Reversible process are those process in which their large diff. in driving force & opposing force.

In Irreversible their is two process possible
 (1) free expansion (2) intermediate

Expansion in vacuum or free expansion

This is a stream case of irreversible bcz opposing force is equal to zero.



Calculation of $\Delta E, \Delta H$

$$\Delta E = 0, \quad dE = 0$$

$$\Delta H = \Delta E = 0, \quad dH = 0$$

Calculation of work done

$$W = -P_{ext} (V_2 - V_1)$$

$$= 0 (V_2 - V_1)$$

$$\boxed{W = 0}$$

The external pressure is zero in a vacuum.

no work is done.

no work is done by the system.

Calculation of Heat

Since,

$$\Delta E = q + W$$

$$0 = q + 0$$

$$\boxed{q = 0}$$

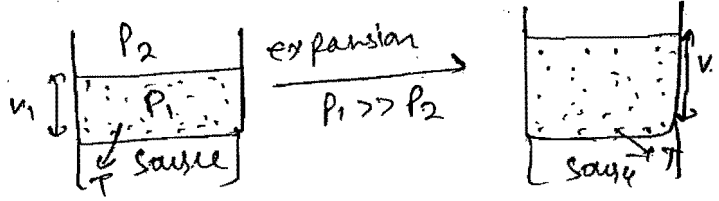
Intermediate

$P_1 > P_2$
(Expansion)

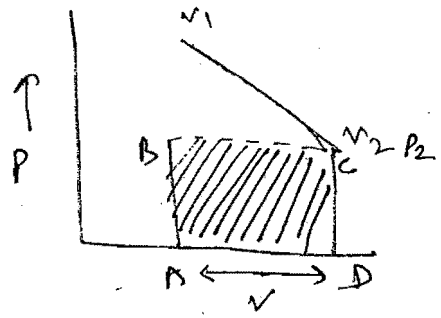
$P_2 \gg P_1$
(Compression)

Intermediate Expansion

source \rightarrow energy provided



$\Delta E = 0$
$\Delta H = 0$



Calculation of work done area of ABCD

$w = -P_2(V_2 - V_1)$

Calculation of Heat

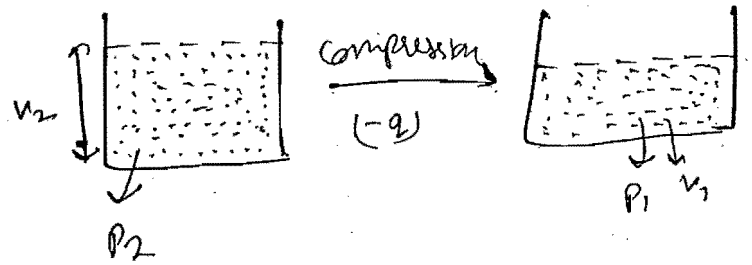
$\Delta E = q + w$

$0 = q + w$

$+q = P_2(V_2 - V_1)$

Intermediate Compression

$\Delta E = 0$
$\Delta H = 0$



$w = -P_1(V_1 - V_2)$

[Because external pressure P_1]

$\Delta E = q + w$

$0 = q + w$

$q = -w$

$q = P_1(V_1 - V_2)$

Total work done in an intermediate process

$$w = P_2(V_2 - V_1) + P_1(V_1 - V_2)$$

$$w \neq 0$$

$$\boxed{\oint w \neq 0}$$

NOTE (1) Reversible workdone or irreversible workdone can be expressed in terms of pressure -

Reversible workdone in terms of P

for expansion

$$w = -nRT \ln \frac{V_2}{V_1}$$

$$RT = \therefore P_1 V_1 = P_2 V_2 \quad (\text{ideal gas})$$

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

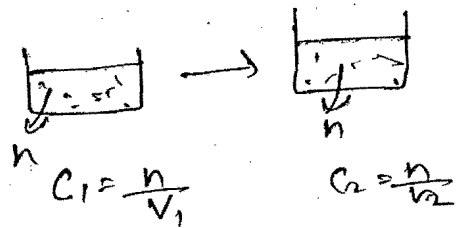
now the (1) eqnⁿ

$$\boxed{w = -nRT \ln \frac{P_1}{P_2}}$$

in terms of concⁿ

$$\frac{V_2}{V_1} = \frac{C_1}{C_2}$$

$$\boxed{w = -nRT \ln \frac{C_1}{C_2}}$$



Irreversible workdone can also be represent in terms of

P & C

$$w_{\text{irr}} = -P_2(V_2 - V_1) \quad \text{--- (1) (expansion)}$$

$$P_2 V_2 = RT$$

$$V_2 = \frac{RT}{P_2} \quad \text{--- (2)}$$

$$V_1 = \frac{RT}{P_1} \quad \text{--- (3)}$$

$$\boxed{w_{\text{irr}} = -P_2 \left[\frac{RT}{P_2} - \frac{RT}{P_1} \right]}$$

Q. Calculate ΔE , ΔH , q , w for isothermal expansion of 1 mole of an ideal gas at 27°C from a vol. of 10 dm^3 to 20 dm^3 in reversible expansion.

$$\Delta E = 0 \quad \Delta H = 0$$

$$w = -nRT \ln \frac{V_2}{V_1} \quad \begin{matrix} 27 + 273 \\ 300 \end{matrix}$$

$$= -0.314 \times 300 \ln \frac{20}{10}$$

$$= -0.314 \times 300 \times 2.303 \times 0.3010$$

$$w = -1728.99 \text{ J}$$

$$q = +1728.99 \text{ J}$$

mult
 $5006 \times 300 \text{ J mol}^{-1} \times \text{K}^{-1}$
J

$$\begin{aligned} R &= 0.082 \text{ dm}^3 \text{ atm mol}^{-1} \text{ K}^{-1} \\ &= 8.314 \text{ J K}^{-1} \text{ mol}^{-1} \\ &= 1.98 \text{ cal K}^{-1} \text{ mol}^{-1} \end{aligned}$$

Q. Calculate q , w , ΔH , ΔV for 1 mole of ideal gas which expand reversibly from 10 atm to 2 atm at 300 K .

$$\Delta E = 0, \Delta H = 0$$

$$w = -nRT \ln \frac{P_1}{P_2}$$

$$= 1 \times \cancel{0.082} \times 300 \times 2.303 \times \log \frac{10}{2}$$

$$= 1 \times 8.314 \times 300 \times 2.303 \times \log 5$$

$$= 1 \times 8.314 \times 300 \times 2.303 \times 0.698$$

$$= -3993.62$$

$$q = +3993.62 \quad = \underline{\underline{4015.15}}$$

Q. 6 mole of an ideal gas expand against a constant external pressure of 1 atm from a vol. of 10 dm³ to a vol. of 20 dm³ calculate ΔH , ΔU , q , w , ΔS at 300 K irreversibly

$\Delta H = 0$
 $\Delta U = 0$

$w = -P_1 (V_2 - V_1)$

$= -1 (20 - 10) \text{ dm}^3 \text{ atm}$

$w = -10 \times 101.39 \text{ J}$

$w = -1013.9 \text{ J}$

$q = 10$

$q = 1013.9 \text{ J}$

0.082 dm³ atm mol⁻¹ K⁻¹
 8.314 J K⁻¹ mol⁻¹
 $1 \text{ dm}^3 \text{ atm} = \frac{8.314 \text{ J}}{0.082}$
 $1 \text{ dm}^3 \text{ atm} = 101.39 \text{ J}$

Comparison b/w work of reversible expansion and work of irreversible expansion

$w_{rev} = nRT \ln \frac{P_1}{P_2}$ (expansion)

$= nRT \ln \left(1 - 1 + \frac{P_1}{P_2} \right)$

$= nRT \ln \left[1 - \left(1 - \frac{P_1}{P_2} \right) \right]$

$\therefore \ln(1-x) = -x$

$= nRT \ln \left[1 - \left(1 - \frac{P_1}{P_2} \right) \right]$
 $= nRT \ln \left[1 + \frac{P_1}{P_2} \right]$

$$\Rightarrow -nRT \left(1 - \frac{P_1}{P_2}\right) \quad \text{--- (1)}$$

$$P_2 = nRT$$

$$\begin{aligned} W_{\text{irr}} &= P_2 (V_2 - V_1) \\ \text{(expansion)} &= P_2 \left(\frac{nRT}{P_2} - \frac{nRT}{P_1} \right) \\ &= nRT \left[1 - \frac{P_2}{P_1} \right] \quad \text{--- (2)} \end{aligned}$$

Subtract (1) - (2)

$$\begin{aligned} W_{\text{rev}} - W_{\text{irr}} &= -nRT \left(1 - \frac{P_1}{P_2}\right) - nRT \left(1 - \frac{P_2}{P_1}\right) \\ &= nRT \left(\frac{P_1}{P_2} - 1\right) + nRT \left[\frac{P_2}{P_1} - 1\right] \\ &= nRT \left[\frac{P_1 - P_2}{P_2}\right] + nRT \left[\frac{P_2 - P_1}{P_1}\right] \\ &= nRT \left[\frac{P_1}{P_2} - 1 + \frac{P_2}{P_1} - 1\right] \\ &= nRT \left[\frac{P_1}{P_2} + \frac{P_2}{P_1} - 2\right] \\ &= \frac{nRT}{P_1 P_2} [P_1^2 + P_2^2 - 2P_1 P_2] \\ &= \frac{nRT}{P_1 P_2} (P_1 - P_2)^2 \end{aligned}$$

$$\therefore W_{\text{rev}} > W_{\text{irr}}$$

$$W_{\text{rev}} - W_{\text{irr}} = +ve$$

You get the square of the name, which shows that work of reversible expansion is always greater than work of irreversible expansion.

S T C
 V P

Summary chart

Reversible

Expansion

- 1) $\Delta E = 0$
- 2) $\Delta H = 0$
- 3) $q = w = nRT \ln \frac{V_2}{V_1}$
- 4) sign of (+q)
- 5) sign of (-w)

Compression

- 1) $\Delta E = 0$
- 2) $\Delta H = 0$
- 3) $q = w = nRT \ln \frac{V_1}{V_2}$
- 4) sign of (-q)
- 5) sign of (+w)

Irreversible

free vacuum

Expansion

- 1) $\Delta E = 0$
- 2) $\Delta H = 0$
- 3) $w = 0$
- 4) $q = 0$

intermediate

~~intermediate~~

no compression

Intermediate

Expansion

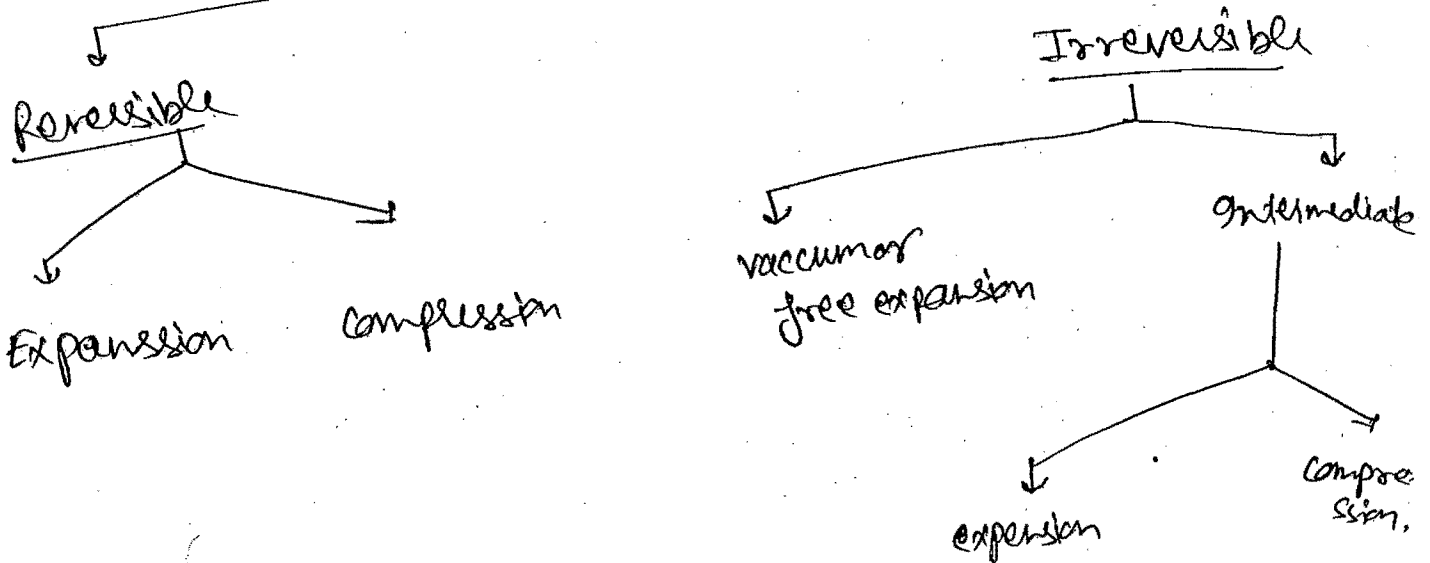
- 1) $\Delta E = 0$
- 2) $\Delta H = 0$
- 3) $w = q = P_{ext}(V_2 - V_1)$
- 4) sign of (+q)
- 5) sign of (-w)

Compression

- 1) $\Delta E = 0$
- 2) $\Delta H = 0$
- 3) $w = q = P_{ext}(V_1 - V_2)$
- 4) sign of (-q)
- 5) sign of (+w)

26 Oct

Adiabatic Process for an ideal gas

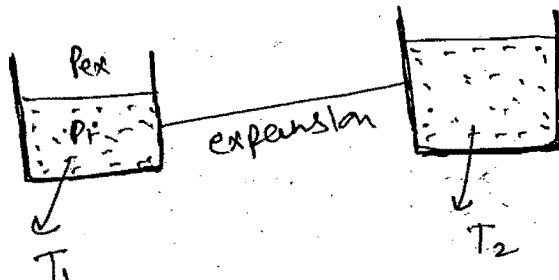


Adiabatic reversible expansion / or compression for an ideal gas

Expansion

$$\Delta E = q + w \quad q = 0$$

$$\Delta E = w$$



C_v is fixed at particular system, at particular temp.

$$T_2 < T_1$$

$$\left(\frac{\partial E}{\partial T}\right) = n C_v \quad n = \text{no. of mole}$$

$$\frac{\Delta E}{\Delta T} = n C_v$$

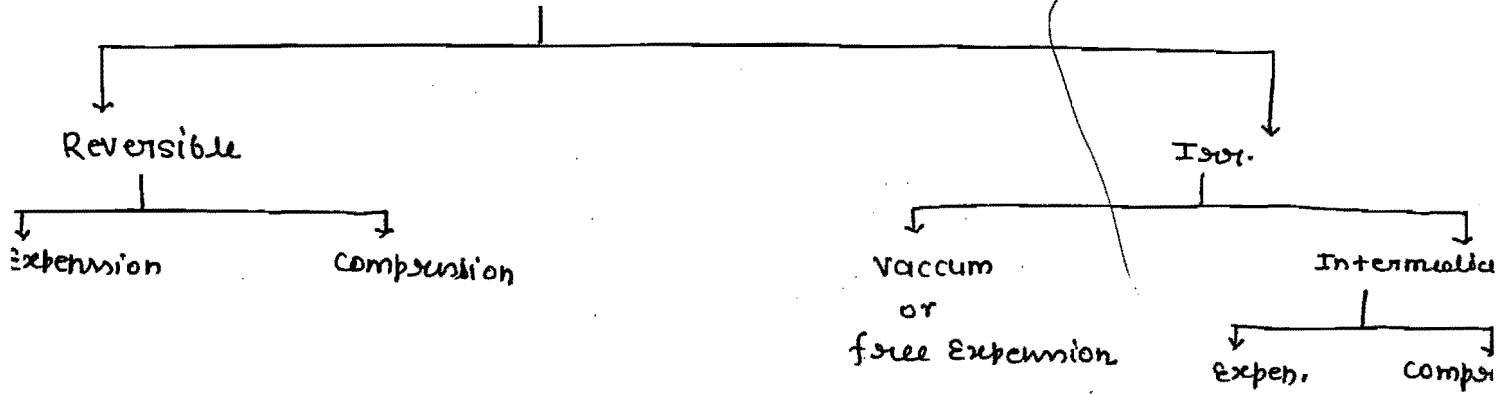
$$\Delta E = n C_v (T_2 - T_1)$$

$$\Delta E = w$$

Since, we know that an adiabatic process system expand and compress, there is change in internal energy and temp. also change.

[26, October, 2015]

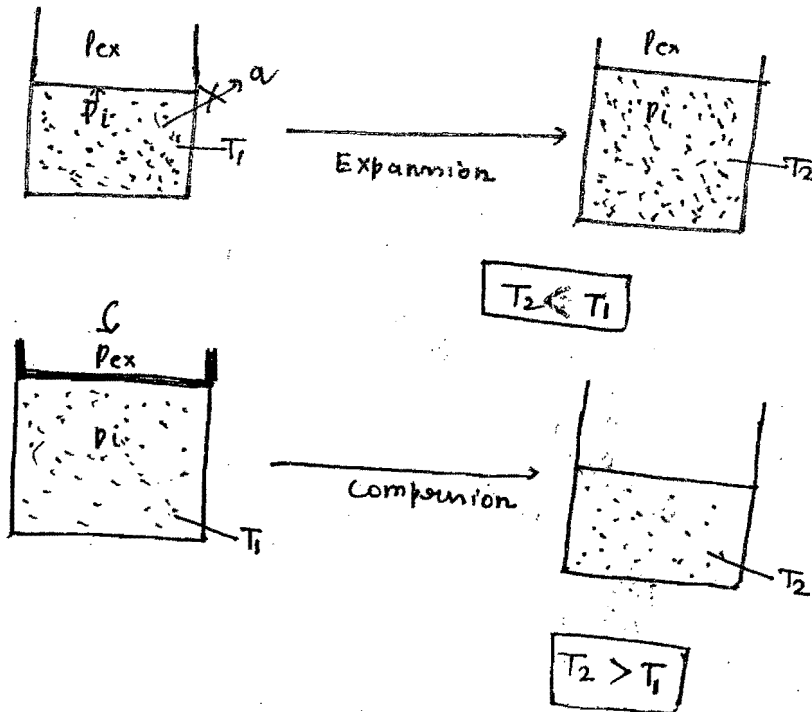
-: Adiabatic Process for an Ideal Gas :-



Adiab. I

* Adiabatic Reversible Exp. or comp. for an Ideal Gas

• calculation of ΔE



Since we know that in adiabatic process the system expand or compressed there is ΔE take place and temp. also changes

$$\text{So } \Delta E = nC_v(T_2 - T_1)$$

→ In Exp. → $T_2 < T_1$, where in comp. → $T_2 > T_1$

Calculation of change in Enthalpy (ΔH),

Enthalpy changes during the process becoz Enthalpy term include internal energy so it also changes -

$$\Delta H = n C_p (T_2 - T_1)$$

so, in Expansion $T_2 < T_1$

In compression, $T_2 > T_1$

Calculation of work done \rightarrow

in adiabatic process we represent a equation - (The Gas Eqⁿ)

since $PV^\gamma = K = \text{constant}$

$\gamma \Rightarrow$ Atomicity of Gas

and system goes from initial state to final state

$$P_1 V_1^\gamma = P_2 V_2^\gamma = K$$

Since we know that for the calculation of server. work done -

$$W = - \int_{V_1}^{V_2} P \, dV$$

$$= - \int_{V_1}^{V_2} \frac{K}{V^\gamma} \cdot dV$$

$$= -K \int_{V_1}^{V_2} V^{-\gamma} \, dV$$

$$= \frac{-K}{(-\gamma+1)} \left[V^{-\gamma+1} \right]_{V_1}^{V_2}$$

$$= \frac{-K}{[-\gamma+1]} \left[V_2^{-\gamma+1} - V_1^{-\gamma+1} \right]$$

$$= \frac{-K}{[-\gamma+1]} \left[V_2^{-\gamma+1} - V_1^{-\gamma+1} \right]$$

$$= \frac{-1}{[-\gamma+1]} [P_2 V_2 - P_1 V_1]$$

$$= \frac{-1}{[-\gamma+1]} [nRT_2 - nRT_1]$$

$$w = \frac{nR}{[\gamma-1]} [T_2 - T_1]$$

for Expan. $\rightarrow T_1 > T_2$

for Comp. $\rightarrow T_2 > T_1$

work can also be calculated in adiabatic sev. work done
can also be calculated in terms of change in
internal energy.

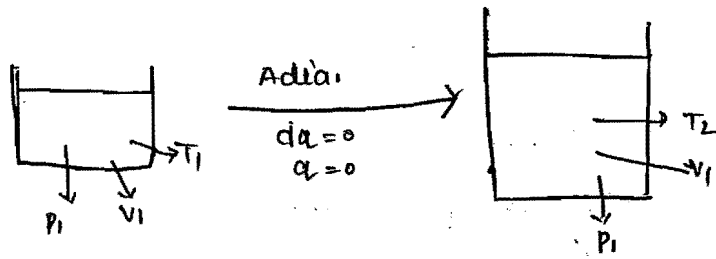
from first law of thermodynamics-

$$\Delta E = \overset{\text{[Adia. Process]}}{\cancel{Q}} + w$$

$$\Delta E = w$$

$$nC_V (T_2 - T_1) = w$$

⇐: Relation b/w Temp. & Volume in the case of Adiabatic reversible process ⇒



Since we know that from 1st law of thermodynamics

$$du = dq - pdv$$

$$du = 0 - pdv$$

$$C_v \cdot dT = - \frac{RT}{V} dv \quad [\text{for one mole of ideal gas}]$$

Inte. both side

$$C_v \int_{T_1}^{T_2} \frac{dT}{T} = -R \int_{V_1}^{V_2} \frac{dv}{V}$$

$$C_v \cdot \ln \frac{T_2}{T_1} = -R \ln \frac{V_2}{V_1}$$

$$\ln \frac{T_2}{T_1} = \frac{-(C_p - C_v)}{C_v} \ln \frac{V_2}{V_1} \quad \left(\because \frac{C_p}{C_v} = \gamma \right)$$

$$\ln \frac{T_1}{T_2} = (\gamma - 1) \ln \frac{V_2}{V_1}$$

$$\ln \frac{T_1}{T_2} = \ln \left(\frac{V_2}{V_1} \right)^{\gamma - 1}$$

taking antilog on both side

~~Imp~~

$$\boxed{\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{\gamma - 1}}$$

$$\frac{\gamma - 1}{\gamma - 1}$$

~~Relation~~

T_1 is a initial Temp. & V_1 is a initial volume

& T_2 is a final Temp. & V_2 is a final volume

⇒ and this Eqⁿ - can use for Expansion and Compression

← Relation b/w temp. and pressure ⇒

$$\therefore P_1 V_1^\gamma = P_2 V_2^\gamma = K$$

$$\left(\frac{T_1}{T_2} \right)^\gamma = \left(\frac{P_2}{P_1} \right)^{1-\gamma}$$

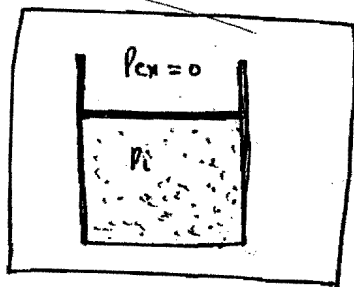
$$\boxed{T_1^\gamma \cdot P_1^{1-\gamma} = T_2^\gamma \cdot P_2^{1-\gamma}}$$

$P \propto T^\gamma \Rightarrow$

Isobaric

↓

Vacuum or free Expansion



$$W = p_{ex} \cdot dV$$

$$T_1 = T_2$$

$$\Delta E = q + W$$

$$\Delta E = 0$$

$$\Delta H = \Delta E + P \Delta V$$

$$\Delta H = \Delta E + \Delta (RT)$$

$$\Delta H = \Delta E + R \Delta T \quad [\text{Here No work No Change in temp. so } \Delta T = 0]$$

$$\Delta H = \Delta E + 0$$

$$\Delta H = \Delta E = 0$$

Calculation of thermodynamic parameter in the case of Adiabatic free Expansion or Expansion in vacuum →

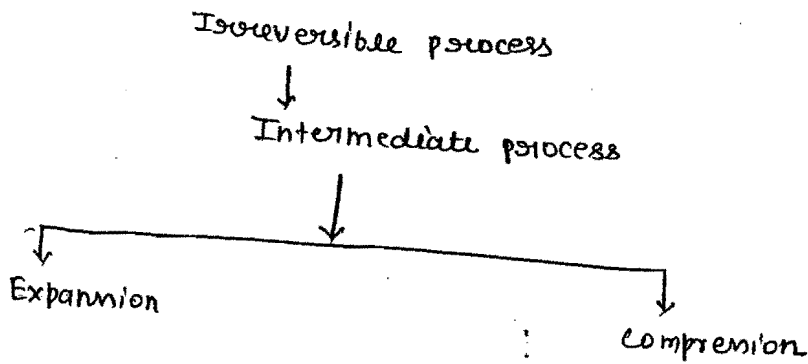
(i) calculation of ΔE ⇒ we know that in vacuum pressure is zero, so there is no change in internal energy during expansion and that is equal to $(\Delta E = 0)$

(ii) calculation of ΔH ⇒ internal energy is a part of enthalpy, so H is also not changes during expansion in vacuum,

$$\text{So, } \Delta H = 0$$

(iii) calculation of work done: ⇒ In vacuum expansion outer pressure is zero or the pressure which is against to the system is zero, so finally there is no work done in vacuum expansion, mathematically, $W = -p_{ex} \cdot dV$

$$p_{ex} = 0 \quad [\text{vacuum}]$$



$$\Delta E = nC_V (T_2 - T_1)$$

$$\Delta H = nC_P (T_2 - T_1)$$

$$W = -P_{ex} (V_2 - V_1)$$

$$= -P_{ex} \left[\frac{nRT_2}{P_2} - \frac{nRT_1}{P_1} \right]$$

$$W = -P_2 \left[\frac{nRT_2}{P_2} - \frac{nRT_1}{P_1} \right]$$

$T_2 < T_1 \rightarrow$ for Expansion

$T_2 > T_1 \rightarrow$ for Compression

(i) calculation of $\Delta E \Rightarrow$

$$\Delta E = nC_V (T_2 - T_1)$$

In exp, $T_2 < T_1$

In com, $T_2 > T_1$

(ii) calculation of $\Delta H \Rightarrow$

$$\Delta H = nC_P (T_2 - T_1)$$

(iii) calculation of work done \Rightarrow

$$W = -P_2 \left[\frac{nRT_2}{P_2} - \frac{nRT_1}{P_1} \right]$$

\therefore In case of Expansion,

$$P_2 < P_1$$

\neq In case of compression

Ques: \Rightarrow One mole of an ^{Monatomic} ideal gas at 27°C expands revers. & Adiabatically from a volume 10 dm^3 to 20 dm^3 , calculate, q , ΔU , ΔH , w and assume that $\left[C_p = \frac{5}{2}R\right]$

$$\therefore C_p - C_v = R$$

$$C_p - \frac{3}{2}R = R$$

$$C_p = \frac{5}{2}R$$

$$\Delta E = \cancel{q} + w = w$$

$$\Delta E = n C_v (T_2 - T_1)$$

$$1 \times \frac{3}{2} R (T_2 - 300)$$

$$\Delta E = 1 \cdot C_v (T_2 - 300\text{ K})$$

$$\Delta E = \frac{3}{2} C_v (T_2 - 300)$$

$$\therefore \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\frac{T_1}{T_2} = \left(\frac{20}{10}\right)^{\gamma-1}$$

$$\frac{T_1}{T_2} = (2)^{\gamma-1}$$

$$\gamma = \frac{C_p}{C_v} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3} = 1.6$$

$$\gamma = (2)^{1.6-1}$$

$$\gamma = (2)^{0.6}$$

$$\frac{300}{T_2} = (2)^{0.6}$$

$$\therefore [2]^{0.6} = 1.515$$

$$T_2 = \frac{300}{1.515} = 198.9 \approx 199\text{ K}$$

$$\ln 300 - \ln T_2 = 0.6 \ln 2$$

$$\Delta E = \frac{3}{2}R(189-300)$$

$$\Delta E = \frac{3}{2} \times 8.314 \times (-111)$$

$$= -\cancel{55.5} \times \cancel{8.314} \times 5$$

$$= -\cancel{277.5}$$

$$= -12.195 \times (111)$$

$$= -1384 \text{ J}$$

$$\Delta E = q + w$$

$$\Delta E = 0 + w \quad (\text{expansion})$$

$$(w = -1384 \text{ Joule})$$

$$\Delta H = n \cdot C_p (T_2 - T_1)$$

$$= 1 \times \frac{5}{2} \times 8.314 (189 - 300)$$

$$= -\frac{5}{2} \times 8.314 \times 111$$

$$= -\frac{4614.27}{2}$$

$$\Delta H = 2307.135 \text{ J}$$

Q4. one mole of an ideal gas at 27°C expands adiabatically against a constant external pressure at 1 atm. from volume of 10 cm^3 to 20 cm^3 calculate, q , w , ΔU , ΔH .

(Iso.)

$$\Rightarrow q = 0, \text{ adiab.}, C_v = \frac{3}{2}R, T_1 = 27^\circ\text{C}$$

$$V_1 = 10 \text{ cm}^3, V_2 = 20 \text{ cm}^3$$

$$\Delta E = n \cdot C_v (T_2 - T_1)$$

$$\Delta E = 1 \times \frac{3}{2} \times R (T_2 - 300)$$

$$\Delta E =$$

$$\Delta E = q + w$$

$$\Delta E = w$$

$$w = -P_{ex.} [V_2 - V_1]$$

$$w = -1 [20 - 10]$$

$$w = -10 \text{ atm} \cdot \text{dm}^3$$

$$w = \frac{-10 \times 8.314}{0.082}$$

$$w = -1012.6 \text{ Joule}$$

∴ process is isov.

$$\Delta E = w = -1012.6$$

$$\Delta E = C_V (T_2 - T_1) = -1012.6$$

$$\frac{3}{2} R (T_2 - 300 \text{ K}) = -1012.6$$

$$\frac{3}{2} \times 8.314 [T_2 - 300] = -1012.6$$

12.471

$$T_2 - 300 = \frac{-1012.6 \times 2}{8.314 \times 3}$$

$$T_2 - 300 = \frac{-1012.6 \times 2}{12.471} = -81.196$$

$$T_2 = 218.7 \text{ K}$$

$$\Delta H = n C_P (T_2 - T_1)$$

$$= n \times \frac{5}{2} \times 8.314 [218.7 - 300]$$

$$= 2.5 \times 8.314 [$$

$$= -1351.8564 \text{ Joule.}$$

1996-Gate

Ques:- One mole of an ideal gas expanded rev. & adiabatically from 1 dm^3 to 10 dm^3 , if initial temp. is 750 K

$$C_p = 29.334 \text{ J mol}^{-1} \text{ K}^{-1}$$

C_v

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{\gamma-1}$$

$$\frac{750}{T_2} = \left(\frac{10}{1} \right)^{\gamma-1}$$

$$\frac{750}{T_2} = (10)^{\gamma-1}$$

$$\frac{750}{T_2} = (10)^{1.3-1}$$

$$T_2 = \frac{750}{(10)^{0.3}}$$

$$T_2 = \frac{750}{1.995}$$

$$T_2 = 376 \text{ K} \cong \underline{300 \text{ K}}$$

$$C_p - C_v = R$$

$$29.334 - 8.314 = C_v$$

$$C_v = 20.92$$

$$\frac{C_p}{C_v} = \gamma$$

$$\frac{29.334}{20.92} = \gamma$$

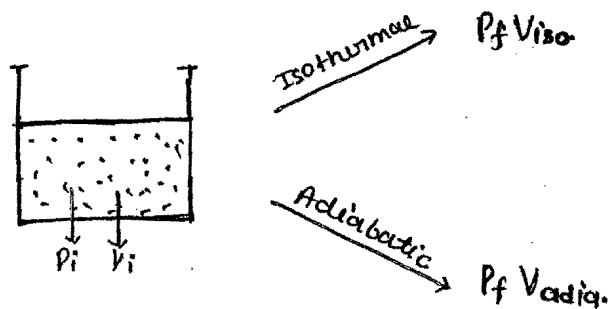
$$\gamma = 1.3$$

27, October, 2015

Comparison b/w Isothermal and adiabatic Process: →

[Comparison of Iso. & adia. Expansion]

Case - I → When a system started [Pressure (P_i) and volume (V_i)] in two different situation, one is adiabatic and another is Isothermal, during both expansion final pressure P_f is same in both the situation.



For Isothermal Expansion →

$$nRT = P_i V_i = P_f V_{iso} \quad \text{--- (I)}$$

Because temp. is same for adiabatic process

$$P_i V_i^\gamma = P_f V_{adia}^\gamma \quad \text{--- (II)}$$

from (i) →

$$\frac{P_i}{P_f} = \left(\frac{V_{iso}}{V_i} \right)^\gamma \quad \text{--- (III)}$$

from - (II)

$$\frac{P_i}{P_f} = \left(\frac{V_{adiabatic}}{V_i} \right)^\gamma \quad \text{--- (IV)}$$

from [III] and [IV]

$$\left(\frac{V_{adia.}}{V_i} \right)^\gamma = \frac{V_{iso}}{V_i}$$

taking log on both side

$$\gamma \ln \frac{V_{adia.}}{V_i} = \ln \frac{V_{iso}}{V_i}$$

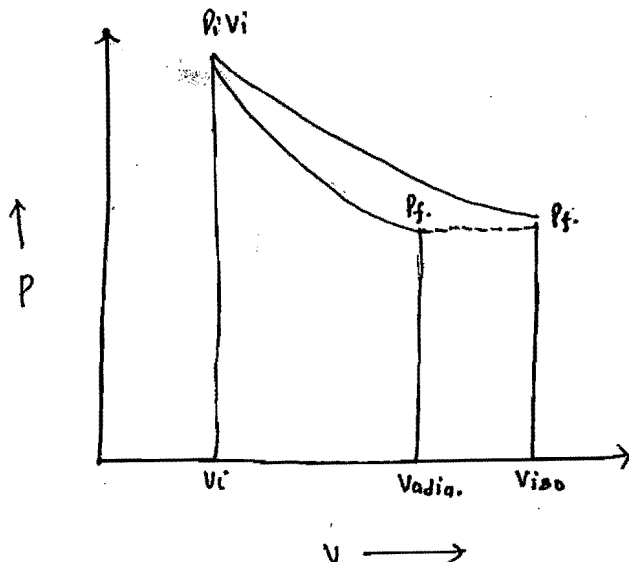
∴ γ is greater than 1

∴ we can conclude →

$$\frac{V_{iso}}{V_i} > \frac{V_{adia.}}{V_i}$$

$$\boxed{V_{iso} > V_{adia.}}$$

Graph for V_{iso} & $V_{adia.}$



The above result shows that the final volume in the case of Isothermal Expansion [V_{iso}] is greater than final volume in the case of adiabatic. This also proves that work done in the case of Isothermal Expansion is greater than work done in the case of Adiabatic Expansion.

Case - II

When final volume in both the cases of (Adia. & Isoth.) are same (V_f).

for Isothermal process

$$P_i V_i \longrightarrow P_{iso} \cdot V_f \quad \text{--- (I)}$$

for Adia. process.

$$P_i V_i^\gamma \longrightarrow P_{adia} \cdot V_f^\gamma \quad \text{--- (II)}$$

from (I)

$$\frac{P_i}{P_{iso}} = \left(\frac{V_f}{V_i} \right) \quad \text{--- (III)}$$

from (II)

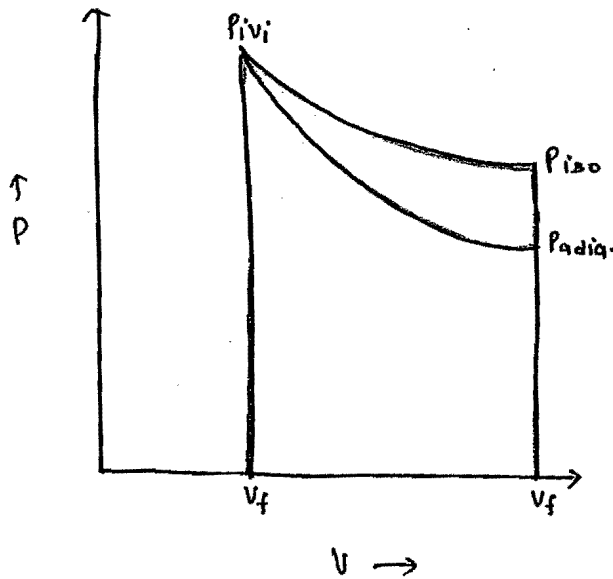
$$\frac{P_i}{P_{adia}} = \left(\frac{V_f}{V_i} \right)^\gamma \quad \text{--- (IV)}$$

from [3] and [4]

$$\left(\frac{V_f}{V_i}\right)^\gamma > \left(\frac{V_f}{V_i}\right)$$

$$\frac{P_i}{P_{\text{adia.}}} > \frac{P_i}{P_{\text{iso}}}$$

$$P_{\text{iso.}} > P_{\text{adia.}}$$



the above results shows that the final pressure in the case of Isothermal expansion is greater than final pressure in the case of adiabatic expansion.

- Summary -

$$V_{\text{isothermal}} > V_{\text{adiabatic}}$$

$$\Delta V_{\text{iso.}} > \Delta V_{\text{adia.}}$$

$$P_{\text{iso.}} > P_{\text{adia.}}$$

$$\Delta P_{\text{iso.}} > \Delta P_{\text{adia.}}$$

$$W_{\text{iso}} > W_{\text{adia.}}$$

→ CALCULATION OF THERMODYNAMIC PARAMETER IN CASE OF ISOTHERMAL REVERSIBLE PROCESS FOR AN REAL GAS :-

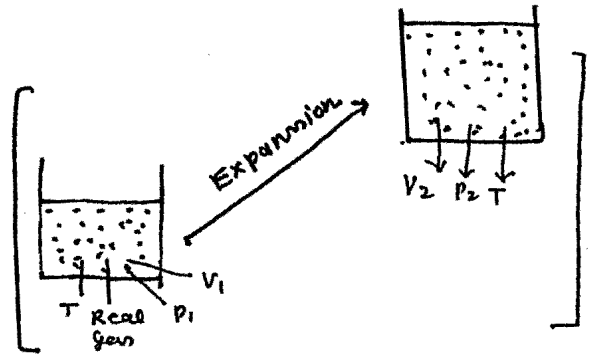
[1] Calculation of work done -

We know that -

for n mole →

$$\left(P + \frac{an^2}{V^2} \right) (V - nb) = nRT$$

$$P = \frac{nRT}{(V - nb)} - \frac{an^2}{V^2}$$



$$\therefore W = - \int_{V_1}^{V_2} P \cdot dV$$

$$W = - \int_V \left[\frac{nRT}{(V - nb)} - \frac{an^2}{V^2} \right] dV$$

$$W = - \left[nRT \int_{V_1}^{V_2} \frac{dV}{(V - nb)} - an^2 \int_{V_1}^{V_2} \frac{dV}{V^2} \right]$$

$$W = - \left[nRT \ln \frac{(V_2 - nb)}{(V_1 - nb)} + an^2 \left(\frac{1}{V_2} - \frac{1}{V_1} \right) \right]$$

Calculation of Change in Internal Energy

We know that Internal Pressure for an
real gas -

$$\left(\frac{\partial E}{\partial V}\right)_T = \frac{an^2}{V^2}$$

real Gas Expand reversibly

$$V_1 \longrightarrow V_2$$

$$dE = \frac{an^2}{V^2} \cdot dV$$

Inte. both side

$$\int dE = an^2 \int_{V_1}^{V_2} \frac{dV}{V^2}$$

$$\Delta E = -an^2 \cdot \left[\frac{1}{V}\right]_{V_1}^{V_2}$$

$$\Delta E = -an^2 \left[\frac{1}{V_2} - \frac{1}{V_1}\right]$$

Calculation of ΔH

Since we know that

∴ During initial state

$$H_1 = E_1 + P_1 V_1 \quad \text{--- (I)}$$

Final state-

$$H_2 = E_2 + P_2 V_2 \quad \text{--- (II)}$$

$$\text{(II)} - \text{(I)}$$

$$H_2 - H_1 = E_2 - E_1 + P_2 V_2 - P_1 V_1$$

$$\Delta H = \Delta E + P_2 V_2 - P_1 V_1$$

$$\therefore P = \frac{nRT}{(V-nb)} - \frac{an^2}{V^2}$$

$$\therefore P_1 = \frac{nRT}{(V_1-nb)} - \frac{an^2}{V_1^2}$$

$$\therefore P_2 = \frac{nRT}{(V_2-nb)} - \frac{an^2}{V_2^2}$$

then

$$\Delta H = \Delta E + \left[\frac{nRT}{(V_2-nb)} - \frac{an^2}{V_2^2} \right] V_2 - \left[\frac{nRT}{(V_1-nb)} - \frac{an^2}{V_1^2} \right] V_1$$

Note \Rightarrow { compression में V_2 less हो जायेगा } }

Calculation of q : \rightarrow

By First Law of thermodynamics

$$\Delta E = q + w$$

$$\Delta E = q + nRT \ln \frac{V_2 - nb}{V_1 - nb} + cn^2 \left[\frac{1}{V_2} - \frac{1}{V_1} \right]$$

$$\cancel{cn^2 \left[\frac{1}{V_2} - \frac{1}{V_1} \right]} = q + nRT \ln \frac{V_2 - nb}{V_1 - nb} + \cancel{cn^2 \left[\frac{1}{V_2} - \frac{1}{V_1} \right]}$$

$$q = nRT \ln \frac{V_2 - nb}{V_1 - nb}$$

Comparison b/w work done in the case of reversible process for an Ideal Gas and Real Gas: - (Isothermal process)

∴ we know that in a real gas -

$$W_{\text{Real}} = nRT \ln \frac{V_2 - nb}{V_1 - nb} + an^2 \left[\frac{1}{V_2} - \frac{1}{V_1} \right] \quad \text{--- (i)}$$

→ work done in ideal gas →

$$W_{\text{Ideal}} = nRT \ln \frac{V_2}{V_1} \quad \text{--- (ii)}$$

[∴ in Expansion $V_2 > V_1$]

then (ii) - (i)

$$W_{\text{Ideal}} - W_{\text{Real}} = nRT \ln \frac{V_2}{V_1} - nRT \frac{V_2 - nb}{V_1 - nb} - an^2 \left[\frac{1}{V_2} - \frac{1}{V_1} \right] \quad \begin{array}{l} nb \rightarrow \text{neglect} \\ \text{[beoz this is const.]} \\ \& \text{very small} \end{array}$$

$$W_{\text{Ideal}} - W_{\text{Real}} = nRT \frac{V_2}{V_1} - nRT \frac{V_2}{V_1} - an^2 \left[\frac{1}{V_2} - \frac{1}{V_1} \right]$$

$$W_{\text{Ideal}} - W_{\text{Real}} = -an^2 \left[\frac{1}{V_2} - \frac{1}{V_1} \right]$$

$$W_{\text{Ideal}} - W_{\text{Real}} = an^2 \left[\frac{1}{V_1} - \frac{1}{V_2} \right]$$

[∴ in Expansion V_2 is always Greater than V_1]

∴ $(W_{\text{Ideal}} - W_{\text{Real}})$ comes +ive

and we also intercept in terms of intercept

⇒ In case of Ideal Gas whole energy is converted into work done.

⇒ In the case of real gas some part of energy is wasted in terms of overcome the ^{the} attraction

CARNOT-CYCLE

[Ideal of Human being]

(maxi. Heat into work)

(Not Total Heat into work)

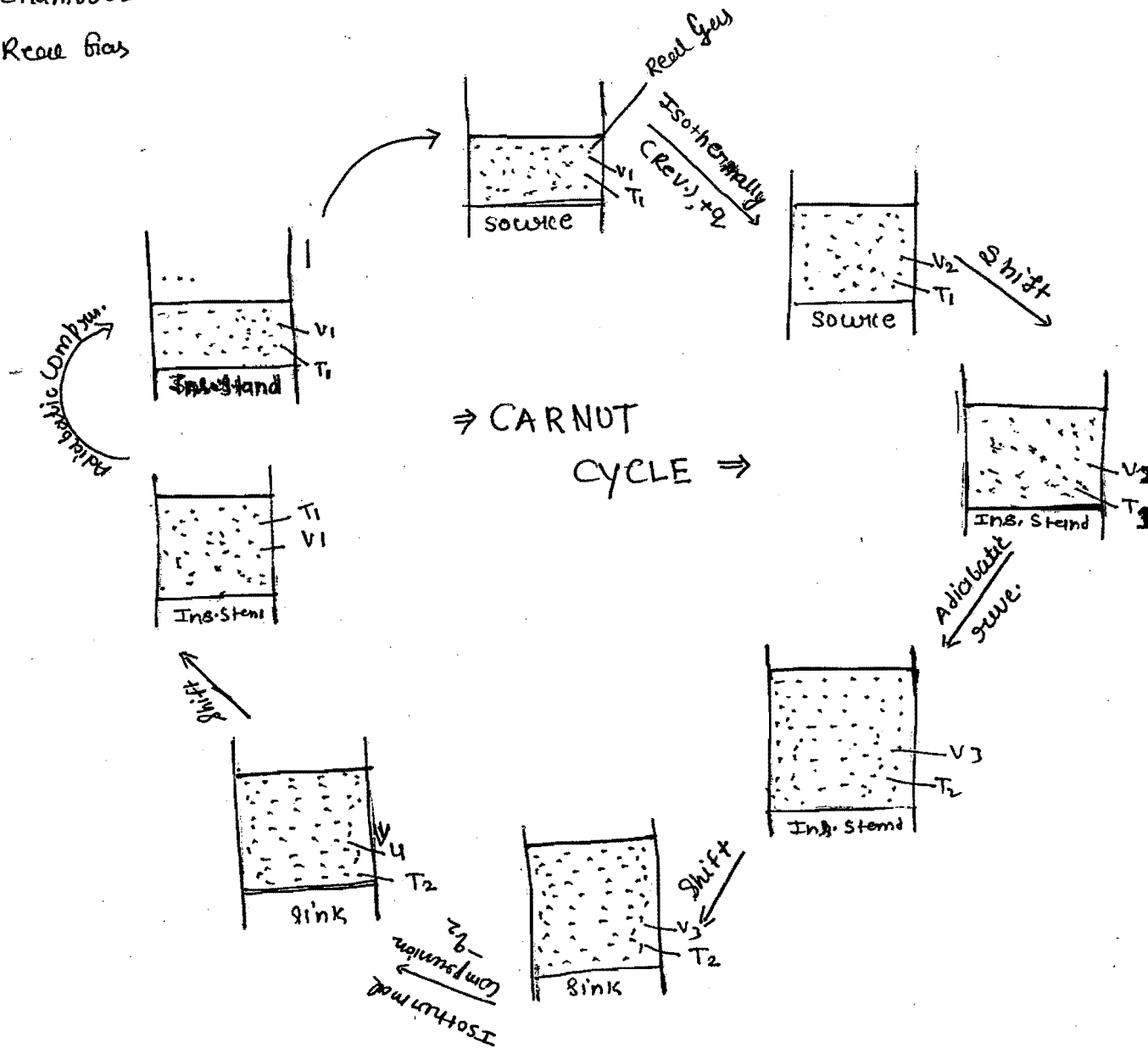
Source (T_1)

Sink (T_2)

Insulating stand

Chamber

Real Gas



$$\eta = \frac{q_1 - q_2}{q_1}$$

$$\eta = -1 - \frac{q_2}{q_1}$$

$\therefore \frac{q_2}{q_1}$ is always less than 1

[28.10.2015]

Total
work

$$W = \left[R \ln \left(\frac{V_2}{V_1} \right) \right] (T_2 - T_1)$$

It is a Imaginary, Ideal, Hypothetical Engine which are going to demonstrate maximum convertibility heat into work.

In order to showing maximum work, Heat taking all the process reversible and considering Ideal Gas as a working substance.

Carnot's reversible Engine operating through four process

All the four process are reversible

The Efficiency of Carnot Engine only depend upon operating temp.

In a Carnot Rev. Engine inspite of taking all the process are reversible taking ideal^{Gas} as working substance but we get efficiency 100%.

Some amount of Heat ($-q_2$) is rejected to the surrounding.

Efficiency of C.E. Independent of working substance but only depend upon operating temp.

⇒ Source:-

which is maintained at high temp. T_1 and which also provide Energy during Isothermal Expansion.

⇒ Sink:-

which is maintained at lower temp. T_2 which act as observer as heat ($-q_2$) that is part of surrounding

⇒ Insulating Stand:-

it is used during adiabatic Expansion or Compression.

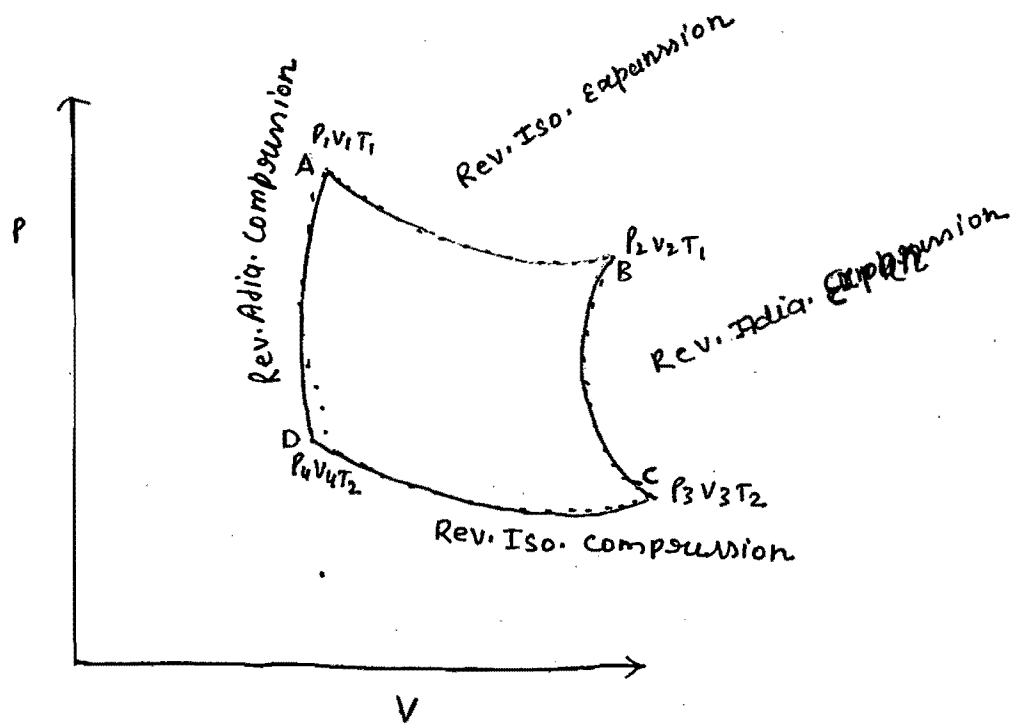
⇒ Working Substances

- [1] Ideal Gas:- we taking ideal Gas as a working substance bcoz there is no interaction b/w Gas molecule so there is not wasting of Energy take place during Expansion or Compression of ideal Gas.

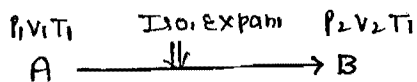
⇒ calculation of thermodynamic parameter during various process of Carnot Engine.

[1] Reversible Isothermal Expansion: →

{ 28, October, 2015 }



[1] Isothermal Expansion



$$\Delta V > 0$$

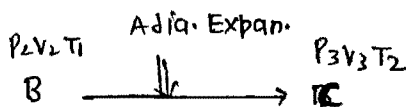
$$\Delta E = 0 \quad \because \Delta E = q_1 + w$$

$$\Delta H = 0$$

$$q_1 = w = nRT_1 \ln \frac{V_2}{V_1} \quad (n = 1 \text{ mole})$$

$$\Delta T = 0$$

[2] Adiabatic Expansion



$$\Delta E = q + w$$

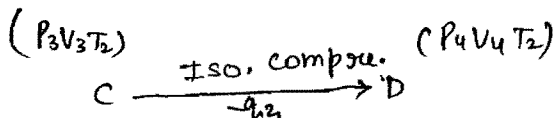
$$\Delta V > 0$$

$$\Delta E = C_V (T_2 - T_1) = w$$

$$\Delta T < 0$$

$$\Delta H = C_P (T_2 - T_1)$$

[3] Isothermal compression



$$\Delta E = 0$$

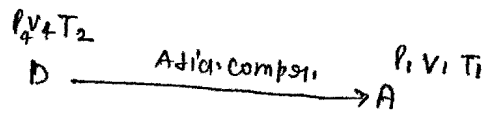
$$\Delta V < 0$$

$$\Delta H = 0$$

$$\Delta T = 0$$

$$\Delta E = q_2 + w$$

[4] Adiabatic compression



$$\Delta E = C_v (T_1 - T_2)$$

$$\Delta H = C_p (T_2 - T_1)$$

$$\Delta V < 0$$

$$\Delta T > 0$$

⇒ Total work done in Reversible Carnot Engine

$$W_{\text{Total}} = W_{\text{Isotherm. Expan.}} + W_{\text{Adiab. Expan.}} + W_{\text{Iso. Comp.}} + W_{\text{Adiab. Com.}}$$

$$W = RT_1 \ln \frac{V_2}{V_1} + C_v (T_2 - T_1) + C_v (T_1 - T_2) + RT_2 \ln \frac{V_4}{V_3}$$

$$W = RT_1 \ln \frac{V_2}{V_1} + RT_2 \ln \frac{V_4}{V_3}$$

$$T_1 \cdot V_2^{\gamma-1} = T_2 \cdot V_3^{\gamma-1} \quad [B \rightarrow C]$$

$$\frac{T_1}{T_2} = \left(\frac{V_3}{V_2} \right)^{\gamma-1}$$

$$T_2 \cdot V_4^{\gamma-1} = T_1 \cdot V_1^{\gamma-1} \quad [D \rightarrow A]$$

$$\frac{T_1}{T_2} = \left(\frac{V_4}{V_1} \right)^{\gamma-1}$$

$$\left(\frac{V_3}{V_2} \right)^{\gamma-1} = \left(\frac{V_4}{V_1} \right)^{\gamma-1}$$

$$\frac{V_3}{V_2} = \frac{V_4}{V_1}$$

$$\frac{V_1}{V_2} = \frac{V_4}{V_3}$$

$$W = RT_1 \ln \frac{V_2}{V_1} + RT_2 \ln \frac{V_1}{V_2}$$

$$W = RT_1 \ln \frac{V_2}{V_1} - RT_2 \ln \frac{V_2}{V_1}$$

$$W_{\text{(Total)}} = R (T_2 - T_1) \ln \frac{V_2}{V_1}$$

Calculation of Efficiency of Carnot Engine :->

$$\eta_{\text{(Efficiency)}} = \frac{W_{\text{Total}}}{q_1} = \frac{q_1 - q_2}{q_1} = \frac{\cancel{R} \left(\ln \frac{V_2}{V_1} \right) (T_1 - T_2)}{\cancel{R} T_1 \left(\ln \frac{V_2}{V_1} \right)}$$

$$\eta = \frac{W_{\text{Total}}}{q_1} = \frac{q_1 - q_2}{q_1} = \frac{T_1 - T_2}{T_1}$$

Coefficient of Performance of Carnot heat Engine :-

Reciprocal of Efficiency is called Coefficient of Performance—

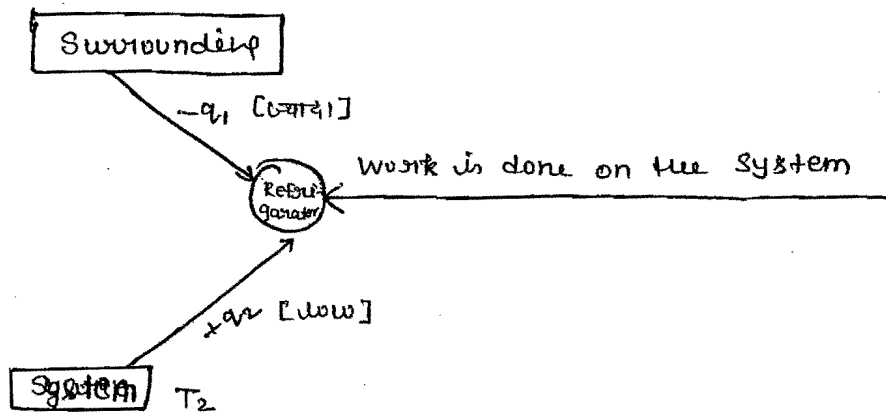
$$\text{COP} = \frac{1}{\eta} = \frac{1}{\frac{T_2 - T_1}{T_1}} = \frac{T_1}{T_2 - T_1} = \frac{q_1}{q_1 - q_2} = \frac{\text{Heat absorb.}}{\text{Total work done}}$$

Efficiency of Carnot Refrigerator

$$\eta = \frac{T_1 - T_2}{T_2} = \frac{q_1 - q_2}{q_2}$$

$$\text{COP} = \frac{T_2}{T_1 - T_2} = \frac{q_2}{q_1 - q_2}$$

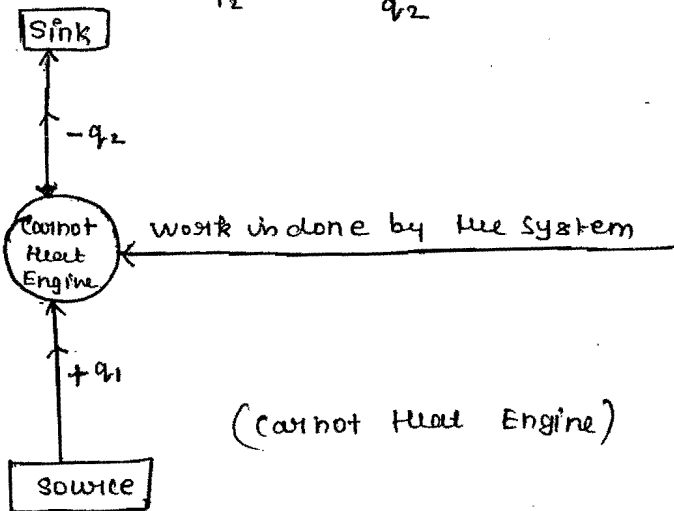
⇒ Pictorial Representation of Carnot Heat Engine and Carnot Refrigerator. ⇒



Carnot - Refrigerator

[low temp. ←→ High temp.]

$$\text{Efficiency} = \frac{T_1 - T_2}{T_2} = \frac{q_1 - q_2}{q_2}$$



$$\eta = \frac{T_1 - T_2}{T_1} = \frac{q_1 - q_2}{q_1}$$

Note: \Rightarrow

CARNOT - THEOREM

"All Periodic machine working reversibly b/w the same two temp. have same Efficiency."

conclusion: \Rightarrow

- Efficiency can never be Equal to 100%
- Carnot cycle work both as a heat Engine and refrigerator.
- All the process of Carnot is reversible, so work done is maximum but never be 100%.
- Carnot cycle is a Hypothetical cycle bcoz it is perfectly reversible process.
- Efficiency of Carnot Engine is never be 100%. this means that complete convertibility of Heat into work is impossible.

Q4. 25

Q4. 25

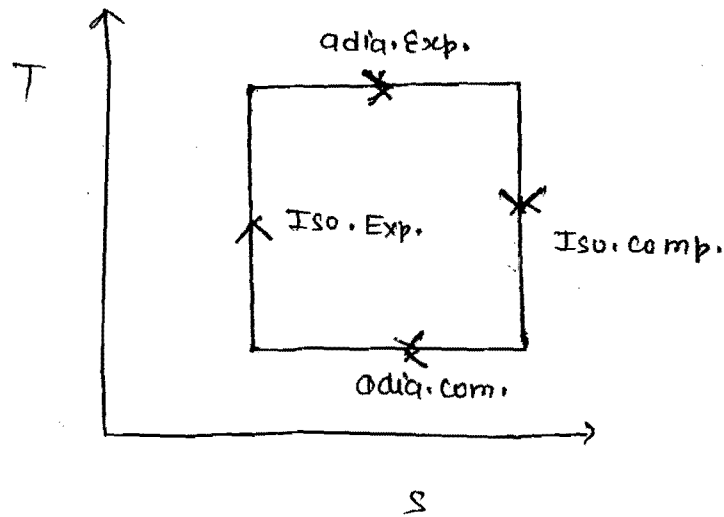
$$\Delta S = \frac{q}{T} = 0/T = 0$$

A → B [$\Delta S = +ive$] ^{ISO} → order me (jyada se kam dhalne par +ive)

B → C [$\Delta S = 0$] adiabatic

C → D [$\Delta S = -ive$] → disorder me jata hai ISO

D → A [$\Delta S = 0$] → (adiabatic)



{ T-s - diagram of Carnot Engine }

(Tepigram)

(S-T graph)

Q1.28

$$T_1 - T_2 = 200 \text{ K}, T_1 = 200 + T_2$$

$$W = 400 \text{ J}$$

$$\therefore q_2 = \frac{q_1}{3}$$

$$\frac{T_1 - T_2}{200 + T_2} = \frac{W}{q_1}$$

$$\frac{q_1 - q_2}{q_1}$$

$$\frac{q_1 - \frac{q_1}{3}}{q_1}$$

$$\frac{2q_1}{3q_1} = \frac{400}{q_1}$$

$$q_1 = \frac{1200}{2} = 600 \text{ J}$$

$$q_1 = 600 \text{ J}$$

$$\frac{200}{T_2 + 200} = \frac{2400}{3600}$$

$$200 \times 3 = 2T_2 + 400$$

$$600 - 200 = 2T_2$$

$$400 = 2T_2$$

$$T_2 = 200$$

$$\frac{600 - q_2}{-600} = \frac{400}{-600}$$

$$600 - q_2 = 400$$

$$q_2 = 200$$

$$T_1 - T_2 = 200$$

$$T_1 - 100 = 200$$

$$T_1 = 300$$

29, October 2015

Entropy: \Rightarrow

- * It is a State function.
- * It is an Extensive property.
- * It's unit is Joule/Kelvin.
- * If a System undergoing change, then the Surrounding must be affected by this.

So, While calc. ΔS_{Total} , we always calculate $\Delta S_{\text{sys.}}$ + $\Delta S_{\text{sur.}}$

* Mathematically ^{we can} Suppose derive Entropy change which is equal to -

$$\Delta S = \frac{q}{T}$$

$q \Rightarrow$ Heat absorb

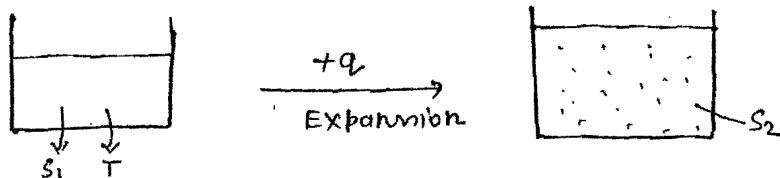
eg. Suppose a system which having temp, T
if they absorb heat (q) from the surrounding

$$\text{then } \Delta S(\text{System}) = +\frac{q_{\text{rev.}}}{T}$$



$$S_2 - S_1 = \Delta S_{\text{sys}} = \frac{q_{\text{rev.}}}{T}$$

Calculation of Total Entropy change for Rev. Process \rightarrow

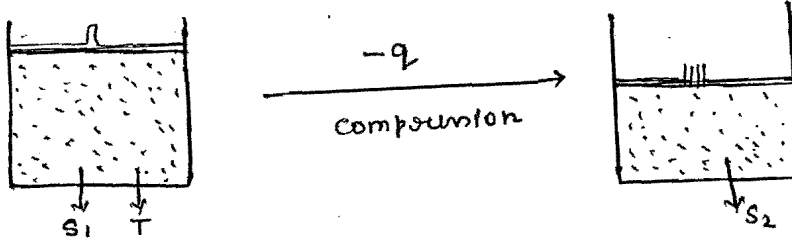


$$S_2 - S_1 = \Delta S_{\text{sys.}} = +\frac{q}{T}$$

$$\Delta S_{\text{surround.}} = -\frac{q}{T}$$

$$\Delta S_{\text{universe}} = \Delta S_{\text{total}} = 0$$

fast compression \rightarrow



$$S_2 - S_1 = \Delta S_{\text{sys.}} = -\frac{q}{T}$$

$$\Delta S_{\text{surround.}} = +\frac{q}{T}$$

$$\Delta S_{\text{total}} = \Delta S_{\text{uni.}} = 0$$

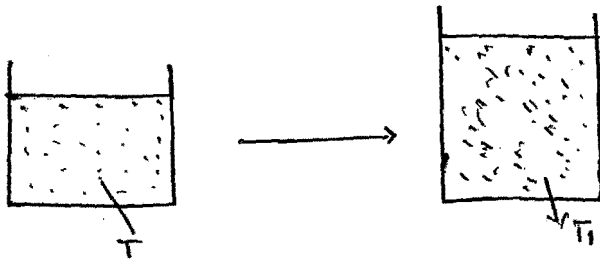
$$\Delta S_{\text{swor.}} = \frac{q}{T} = 0$$

$$\Delta S_{\text{Total}} = \Delta S_{\text{sys.}} + \Delta S_{\text{swor.}}$$

Since there is no Heat Exchange b/w System and Surrounding so that $q = 0$. [$\Delta S_{\text{swor.}}$]

$$\Delta S_{\text{Total}} = 0$$

Adiabatic Irreversible Process: \Rightarrow



$$\Delta S_{\text{sys.}} = \text{+ive}$$

The Extent of this Order is not compensated by ordered arrangement. disorder is take place due to expansion and order is take place due to lowering in temp.

$$\Delta S_{\text{swor.}} = 0/T = 0$$

$$\Delta S_{\text{Total}} > 0$$

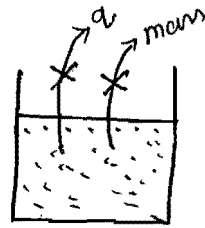
Calculation of ΔS_{Total} For Isolated Process : \Rightarrow

Reversible \Rightarrow

$$\Delta S_{\text{sys.}} = 0$$

$$\Delta S_{\text{surround.}} = 0$$

$$\Delta S_{\text{Total}} = 0$$



[No contact with
Surrounding]
 \Downarrow
Isolated

Irreversible : \Rightarrow

$$\Delta S_{\text{sys.}} = 0$$

$$\Delta S_{\text{surround.}} = +\text{ive} \Rightarrow (\text{can't predict}) \text{ so that } +\text{ive}$$

$$\Delta S_{\text{Total}} = +\text{ive or } > 0$$

$$\Delta S_{\text{Total}} > 0$$

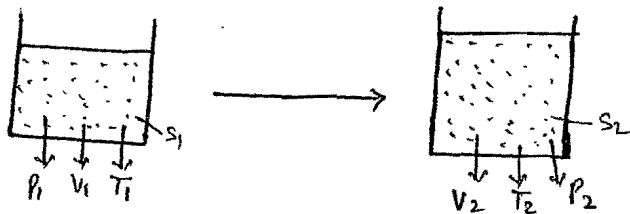
Calculation of Entropy Change of a System where temp. and volume undergo simultaneous change: \Rightarrow

for an ideal gas \Rightarrow

$$dE = dq - PdV \quad [\text{From 1st Law of thermodynamics}]$$

$$\therefore ds = \frac{dq}{T}$$

$$dq = Tds$$



$$\therefore PV = nRT$$

$$\therefore P = \frac{nRT}{V}$$

$$nC_v dT = Tds - \frac{nRT}{V} dV$$

$$nC_v \int_{T_1}^{T_2} \frac{dT}{T} + nR \int_{V_1}^{V_2} \frac{dV}{V} = \int_{S_1}^{S_2} ds$$

$$nC_v \ln \frac{T_2}{T_1} + nR \ln \frac{V_2}{V_1} = \Delta S$$

Volume change with constant temp. [isothermal]

$$dT = 0$$

$$\Delta S_{\text{isothermal}} = nR \ln \frac{V_2}{V_1}$$

For iso-choric process [dv=0], then,

$$\Delta S_{\text{isochoric}} = n C_v \ln \frac{T_2}{T_1}$$

Calculation of ΔS of a system when temp and pressure undergo simultaneous change

$$\therefore n C_v \ln \frac{T_2}{T_1} + n R \ln \frac{V_2}{V_1} = \Delta S$$

[from above relation]

we know that, from Ideal Gas \rightarrow

$$\frac{PV}{T} = \text{constant}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\left(\frac{P_1}{P_2}\right) \left(\frac{T_2}{T_1}\right) = \left(\frac{V_2}{V_1}\right)$$

$$n C_v \ln \frac{T_2}{T_1} + n R \ln \left(\frac{T_2}{T_1}\right) \times \left(\frac{P_1}{P_2}\right) = \Delta S$$

$$n C_v \ln \frac{T_2}{T_1} + n R \ln \left(\frac{T_2}{T_1}\right) + n R \ln \frac{P_1}{P_2} = \Delta S$$

$$(n C_v + n R) \ln \frac{T_2}{T_1} + n R \ln \frac{P_1}{P_2} = \Delta S$$

$$\therefore n C_p - n C_v = n R$$

$$n C_p = n R + n C_v$$

$$n C_p \ln \frac{T_2}{T_1} + n R \ln \frac{P_1}{P_2} = \Delta S$$

at constt Pressure \rightarrow

For Isobaric process $\rightarrow [dp=0]$

$$\Delta S_{\text{Isobaric}} = n c_p \ln \frac{T_2}{T_1}$$

at constt. temp. [Isothermal process]

$$\Delta S_{\text{isothermal}} = n R \ln \frac{P_1}{P_2}$$

Entropy change during Phase transformation

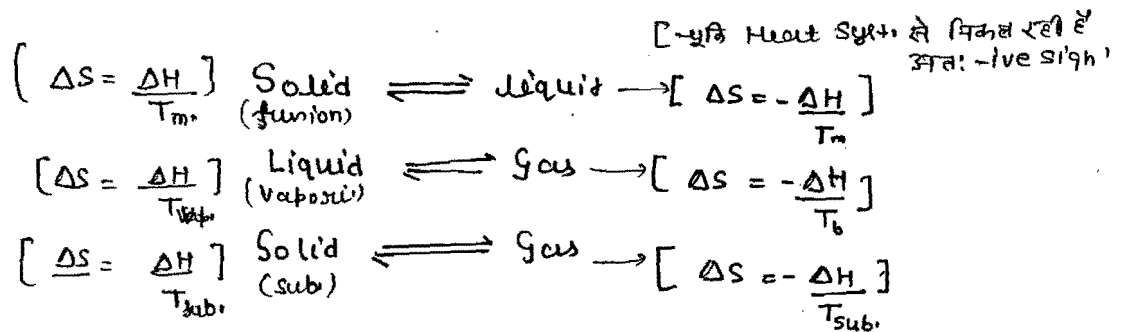
during Phase transformation temp., pressure, conc. etc. remain constt.,

e.g. • Solid changes into liquid

• liquid changes " Gas

• Solid " " Gas

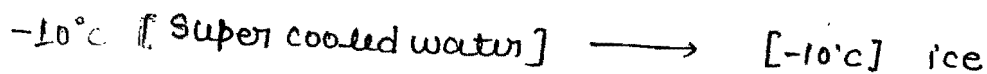
eg [Dry $\text{CO}_2 \rightarrow$ as a Refrigerant] etc.



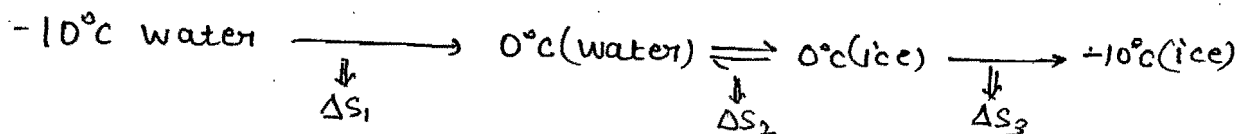
e.g.

Q. Calculate the Total Entropy change when -10°C of water at Particular amount is changes to -10°C of ice at Particular amount.

Solⁿ→



$$\Delta S = ?$$



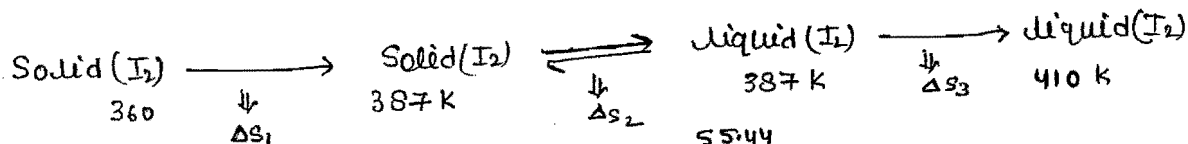
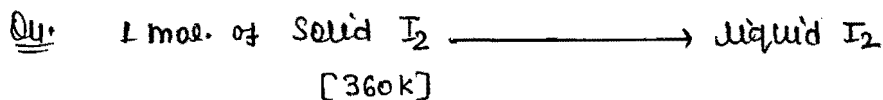
$$\Delta S_1 = n c_p \ln \frac{273}{263}$$

$$\Delta S_2 = \frac{-\Delta H}{273}$$

$$\Delta S_3 = n c_p \ln \frac{263}{273}$$

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3$$

e.g.



$$T_m = 387$$

$$(\text{Solid } I_2) C_p = 54.44 \text{ k}^{-1} \text{ mol}^{-1}$$

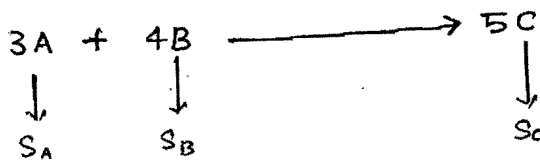
$$(\text{Liquid } I_2) C_p = 80.67 \text{ k}^{-1} \text{ mol}^{-1}$$

$$\Delta S_1 = 1 \times C_p \times \ln \frac{387}{360}$$

$$\Delta S_2 = \frac{7.97}{1} = \frac{7870}{1000} \text{ J mol}^{-1}$$

$$\begin{aligned} \Delta S_{\text{Total}} &= 54.44 \times \ln \frac{387}{360} + \frac{7870}{387} + 80.67 \ln \frac{410}{387} \\ &= 54.44 \times 0.072 + 20.33 + 80.67 \times 0.057 \\ &= 28.84 \text{ J K}^{-1} \text{ mol}^{-1} \\ &= 28.9 \text{ J K}^{-1} \text{ mol}^{-1} \quad [b] \end{aligned}$$

Entropy change during a reaction



$$\Delta S = 5S_C - (4S_B + 3S_A)$$

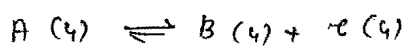
S_A = absolute Entropy at temp. above then 0K

$S_A = 0$ at 0K \Rightarrow (this is convention)

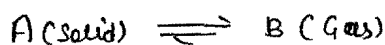
e.g. Gold-Particle-
1 mol (Au)
 $S_A = 0$ at 0K
Absolute Entropy at 0K any Element
is zero [convention] (No disorder)



we can't say

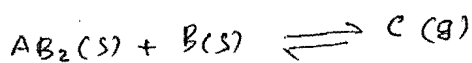


$\Delta S = +ive$



S_A S_B

$$\Delta S = +ive$$



$(\Delta S = +ive)$

* According to convention the Entropy of any Element at absolute 0 K is zero.

* The Entropy appears in the Element above then zero kelvin is called absolute Entropy of any Element at any particular temp.

29, October, 2015

Residual Entropy

According to the third law "THE ENTROPY OF PERFECTLY CRYSTALLINE SUBSTANCE AT ABSOLUTE 0K is zero"

But this law has a exception also.

e.g. \Rightarrow CO crystalline structure at absolute 0 K is not zero. bcoz CO molecule have two different orientation.

CO	CO	CO	CO
OEC	CEO	CO	CO
CO	CO	OC	CO
CO	CO	OC	CO

mathematical formula of Residual Entropy

$$S = R \ln w$$

$R \Rightarrow$ Gas const.

$w \Rightarrow$ orientation

e.g. \Rightarrow CO molecule, NO, N₂O, H₂O etc. they all have two orientation at absolute 0 K.

ENTROPY CHANGES WITH SIMULTANEOUS CHANGE OF TEMP. AND VOLUME
 IN CASE OF IDEAL GAS: \Rightarrow

$$dS = n C_v \ln \frac{T_2}{T_1} + n R \ln \frac{V_2 - nb}{V_1 - nb}$$

$$\therefore E = f(T, V)$$

$$dE = \left(\frac{dE}{dT} \right)_V dT + \left(\frac{dE}{dV} \right)_T dV$$

$$dE = \left(\frac{dE}{dT} \right)_V dT + \left(\frac{dE}{dV} \right)_T dV$$

\downarrow
 C_v

$$dE = C_v dT + \left(\frac{dE}{dV} \right)_T dV$$

$$\frac{dq}{T} - \frac{p dV}{T} = C_v \frac{dT}{T} + \left(\frac{dE}{dV} \right)_T \frac{dV}{T} \quad \therefore \frac{dq}{T} = ds$$

$ds =$

$$\therefore p = \frac{RT}{(V-b)} - \frac{a}{V^2}$$

$$\therefore \left(\frac{dE}{dV} \right)_T = \frac{a}{V^2}$$

(Gate 2010)

Q4. The Molar Entropy of a Crystalline CO at absolute 0K.

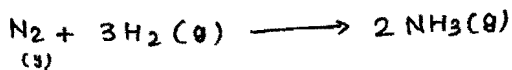
~~(i)~~ $R \ln 2$ (ii) $-R \ln 2$ (iii) $2 \ln R^2$ (iv) -2

Q4. Standard Entropy of Cryst. CO at 0 K

- | | |
|---------------|------------------------------------|
| (i) $R \ln 2$ | (i) 0.03 J/mole Kelvin |
| Rx | (ii) 2.5 J/mole Kelvin |
| | (iii) zero |
| | (iv) 5.76 J/mole Kelvin |

(Gate-2003)

Q4.12



$$= 2 \times (192.3) - 3 \times (130.6) - 191.5$$

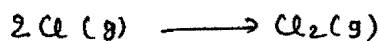
$$= 2 \times 192.3 - 3 \times 130.6 - 191.5$$

$$= -198.7 \text{ J/K mol}$$

negative sign indicates that there is a decrease in the gaseous species during the reaction.

[Syst. की Entropy घट जा रही है But Surrounding की Entropy बढ़ते ही +ive हो जाएगी]

Q4.14



[spontaneous process]

$$\Delta G = -ive$$

$$\Delta H = -ive$$

$$\Delta S = -ive$$

Q4.17

One mole of Ideal Gas compressed

V_1
 P_1

$\frac{V_1}{4}$

$2P_1$

$$\Delta S = n C_v \ln \frac{T_2}{T_1} + n R \cdot \ln \frac{V_2}{V_1}$$

$$\Delta S = 1 \times C_v \ln \frac{T_2}{T_1} + 1 \times R \cdot \ln \frac{V_1}{\frac{V_1}{4}}$$

$$\Delta S = C_v \ln \frac{T_2}{T_1} + R \ln \frac{4}{1}$$

$$\Delta S = C_v \cdot \ln 2 + R \ln \frac{4}{1}$$

$$\Delta S = C_v \cdot \ln 2 + R \ln (2)^2$$

$$\Delta S = C_v \cdot \ln 2 + 2R \ln 2$$

$$\Delta S = \ln 2 (C_v + 2R)$$

Q4.41

2 mole of Argon Gas

$$T_1 = 300K$$

$$T_2 = 500K$$

$$\Delta S = n C_{v,h} \frac{T_2}{T_1}$$

$$\Delta S = 2 \times C_v \times \ln \frac{500}{300}$$

$$\Delta S = 2 \times \frac{3}{2} \times R \times \ln \frac{5}{3}$$

$$\Delta S = 3 \times 8.314 \times \ln \frac{5}{3}$$

$$(2 \text{ mole}) \Delta S = 24.942 \ln \frac{5}{3}$$

$$(1 \text{ mole}) \Delta S = \frac{24.942 \ln \frac{5}{3}}{2} = 6.37 \text{ JK}^{-1} \text{ mol}^{-1}$$



$$T = 298\text{K}$$

$$\Delta S_{\text{sys}}^{\circ} = -197 \text{ J K}^{-1}$$

$$\Delta H_{\text{sys}}^{\circ} = -91.8 \text{ KJ} = -91.8 \times 10^3 \text{ J}$$

const. Temp. & pressure

[जितनी Heat System से गयी
अतः Surround. में जायेगी
तो +ive होगी]

$$\begin{aligned} \Delta S_{\text{Total}} &= \Delta S_{\text{sys}} + \Delta S_{\text{surround.}} \\ &= -197 \text{ J K}^{-1} + \frac{(+91.8 \times 10^3)}{298} \\ &= -197 \text{ J K}^{-1} + \frac{91.8 \times 10^3}{298} \\ &= -197 \text{ J K}^{-1} + 308.05 \\ &= 111.05 \text{ J K}^{-1} \end{aligned}$$

Q41 → $T_1 = 300 \text{ K}$

$T_2 = 400 \text{ K}$

$C_p = (5 + 0.1T)$

$\Delta S = ?$

$$\Delta S = n C_p \frac{dT}{T}$$

$$\Delta S = 1 \times \int_{300}^{400} (5 + 0.1T) \frac{dT}{T}$$

~~$$\Delta S = 5(T) + 0.1 \left(\frac{T^2}{2} \right) \Big|_{300}^{400}$$~~

~~$$\Delta S = 5(400 - 300) + \frac{0.1}{2} \times [(400)^2 - (300)^2]$$~~

~~$$\Delta S = 500 + \frac{1}{20} [160000 - 90000]$$~~

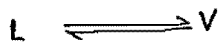
$$\begin{aligned}
\Delta S &= \int_{300}^{400} \frac{5}{T} dT + \int_{300}^{400} 0.1 dT \\
&= 5 \left[\ln T \right]_{300}^{400} + \left[0.1 T \right]_{300}^{400} \\
&= 5 \left[\ln 400 - \ln 300 \right] + 0.1 \left[400 - 300 \right] \\
&= 5 \left[\ln(20) - \ln 300 \right] + 0.1 \times 100 \\
&= 5 \left[2.99 - 5.70 \right] + 10 \\
&= 5 \left[-2.71 \right] + 10 \\
&= -13.55 + 10 \\
&= -3.55 \text{ J K}^{-1} \text{ mol}^{-1}
\end{aligned}$$

[Que. 11] \Rightarrow

$$T = 100^\circ\text{C} + 273 = 373 \text{ K}$$

$$\Delta H_{\text{vap.}} \approx 41 \text{ kJ mol}^{-1} = 41 \times 10^3 \text{ J mol}^{-1} \Rightarrow 41000 \text{ J mol}^{-1}$$

$$P = 1 \text{ atm}$$



$$\begin{aligned}
\Delta S &= \frac{\Delta H}{T} = \frac{41000}{373} \\
&= 110 \text{ J K}^{-1} \text{ mol}^{-1}
\end{aligned}$$

Q17

$$n = 10 \text{ mole}$$

$$C_p = 300 \text{ J K}^{-1}$$

$$C_v = ?$$

$$n C_p - n C_v = n R$$

$$10 \times 300 - 10 \times C_v = 8.314 \times 10$$

$$300 - C_v = 8.314 \times 10$$

$$300 - 8.314 \times 10 = C_v$$

$$300 - 83.14 = C_v$$

$$C_v = 300 - 83.14$$

$$C_v = 216.86 \text{ J K}^{-1}$$

$$C_v \approx 217 \text{ J K}^{-1} \quad [b]$$

Ass. 2nd
Q1.25 Oct 2011

1 Atm (300K, 1 bar) \longrightarrow 1 Atm (200K, 10 bar)

$$\begin{aligned} \Delta S &= n C_p \ln \frac{T_2}{T_1} + n R \ln \frac{P_1}{P_2} \\ &= 1 \times \frac{5}{2} \times R \times \ln \frac{200}{300} + 1 \times 8.314 \times \ln \frac{10}{1} \\ &= \frac{5}{2} \times 8.314 \times \ln \frac{2}{3} + 8.314 \times [2.302] \\ &= \frac{41.57}{2} \times \ln \left(\frac{2}{3} \right) + 19.143692 \\ &= \frac{41.57}{2} \times \ln \left(\frac{2}{3} \right) + 19.143692 \\ &= -\frac{41.57}{2} \times 0.405 + 19.143692 \end{aligned}$$

Q4.

Calculate ΔS , if \rightarrow

const. temp.

$$V_1 \longrightarrow V_2$$

[10 dm³ 20 dm³]

$$\Delta S = nR \ln \frac{V_2}{V_1}$$

$$\Delta S = 1 \times 8.314 \times \ln \frac{20}{10}$$

$$\Delta S = 8.314 \times \ln 2$$

~~$$\Delta S = 8.314 \times 0.693$$~~

~~$$\Delta S = 5.7616$$~~

$$\Delta S = 8.314 \times 0.6931$$

$$\Delta S = 5.7616 \text{ JK}^{-1} \text{ mol}^{-1}$$

Q4. Calculate the ΔS , when a system expand ~~isothermally~~ containing an ideal gas expand isothermally from a pressure of 10 atm to pressure of 0.1 atm.

$$\Delta S = nR \ln \frac{P_1}{P_2}$$

$$\Delta S = 1 \times 8.314 \times \ln \frac{10}{1}$$

$$\Delta S = 8.314 \times 2.303$$

$$\Delta S = 19.147 \text{ JK}^{-1} \text{ mol}^{-1}$$

⇒ GIBB'S FREE ENERGY ⇒

$G = H - TS$
at const (T, P)

- It is an Extensive property.
- It is a State function.
- It is a form of Energy which can be used in any useful work
- ↓ in free Energy is use as a non-mechanical work
- Its unit is Joule
- mathematically it can be written as

$$G = H - TS$$

where H is a total Energy of a system

where TS is Unavailable Energy.

[or u can say that disorder can't be used as a work, coz u used this energy the substance change their phase]

- Derivation of Maxwell Equation in term of G.

$$G = H - TS$$

$$G = U + PV - TS \quad [\because H = E + PV]$$

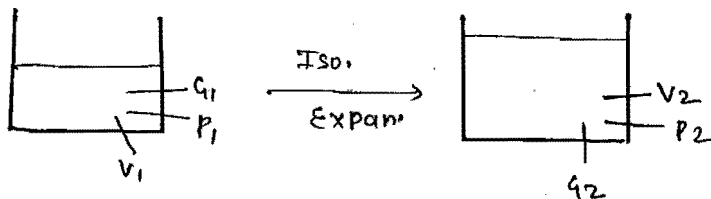
Diff. both side

$$dG = dE + PdV - Tds - sdT + VdP$$

$$\therefore dE = dG - PdV$$

$$dE = Tds - PdV$$

→ Calculation of change in free energy for an isothermal process in the case of ideal gas →



We know that -

from Maxwell Eqnⁿ

$$\left(\frac{dG}{dP}\right)_T = V$$

$$dG = V dP$$

$$[\because PV = RT]$$

$$V = \frac{RT}{P}$$

$$\Delta G = RT \int_{P_1}^{P_2} \frac{dP}{P}$$

$$\Delta G = RT \ln \frac{P_2}{P_1}$$

(Gaut-wag)

Ans. ①

Q. 32

ΔG

$n = 1$

$P_1 = 1 \text{ atm}$

$P_2 = 2 \text{ atm}$

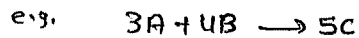
$$\Delta G = RT \ln \frac{2}{1}$$

$$\Delta G = RT \ln 2$$

⇒ calculation of ΔG for a reaction.

Imp!

$$\Delta G = \sum G_{\text{PRODUCT}} - \sum G_{\text{REACTANT}}$$



A $G_1 \Rightarrow J/\text{mole}$

B $G_2 \Rightarrow J/\text{mole}$

C $G_3 \Rightarrow J/\text{mole}$

$$\Delta G = 5G_3 - 4G_2 - 3G_1$$

⇒ For Spontaneous process ΔG always be -ive -

$$\Delta G = \Delta H - T\Delta S$$

∴ ΔS always be positive

† ΔH ~~always~~ may be positive or negative

∴ $\Delta G \rightarrow$ {always Negative}

[ΔG को -iv लाने के लिये

ΔH की value $T\Delta S$ से

छोटी होनी चाहिए

H

* Prove that for Spontaneous process ΔG is less than zero
(Isobaric proc.)

We know that the Heat change in a reversible process is more than the heat change in a irreversible process.

$$dq_{rev.} > dq_{isov.}$$

$$T ds > dq_{isov.}$$

$$\therefore \left(\Delta S = \frac{q_{rev.}}{T} \right)$$

$$0 > dq_{isov.} - T ds \quad \text{--- (1)}$$

$$(dq_{isov.})_p = dH \quad \text{--- (2)}$$

put the value of (2) in (1)

$$0 > dH - T ds$$

$$0 > (dG)_{T, p}$$

Gibbs Helmholtz- Equation

we know that

$$G = H - TS$$

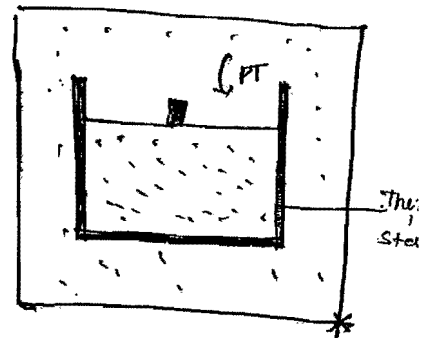
for initial \rightarrow

$$G_1 = H_1 - TS_1 \quad \text{--- (i)}$$

$$\text{for final, } G_2 = H_2 - TS_2 \quad \text{--- (ii)}$$

at constant temp. p

$$\Delta G = \Delta H - T \Delta S \quad \text{--- (3)}$$



$$dG = V dp - S dT$$

$$\left(\frac{dG}{dT}\right)_P = -S$$

for initial

$$\left(\frac{dG_1}{dT}\right)_P = -S_1 \quad (4)$$

for final

$$\left(\frac{dG_2}{dT}\right)_P = -S_2 \quad (5)$$

Substrate \rightarrow (5) - (4)

$$\left(\frac{d(\Delta G)}{dT}\right)_P = -\Delta S \quad (6)$$

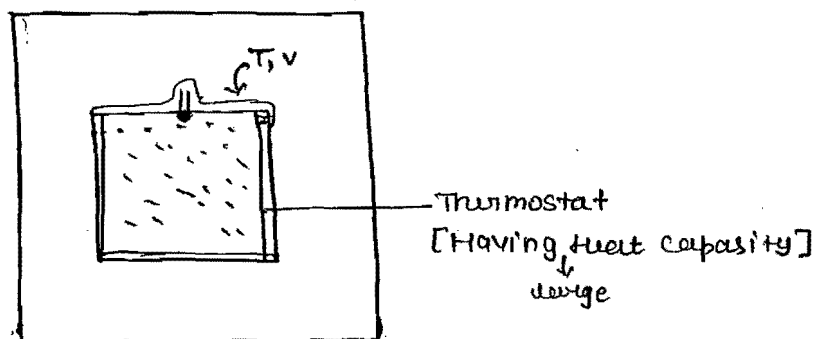
Put the value of (3) in (6)

$$\left(\frac{d(\Delta G)}{dT}\right)_P = \frac{\Delta G - \Delta H}{T}$$

$$\boxed{T \cdot \left(\frac{d(\Delta G)}{dT}\right)_P + \Delta H = \Delta G}$$

\leftarrow Gibbs' Helmholtz Equation \Rightarrow

Helmholtz function



$$f_{031} - (T, V)$$

Work-function

or

Helmholtz Function or 'A'

$$A = \underbrace{E}_{\text{Energy}} - \underbrace{TS}_{\text{disorder}}$$

- ⇒ It is a Energy function
- ⇒ It is also Extensive property
- ⇒ Its Unit Expresses in Joule
- ⇒ It is also called work function and represented by 'A'
- ⇒ When a system undergoing change at a constant volume then their Energy Express in term in internal Energy & when you subtract disorder from total Energy, we get Helmholtz free Energy or work function.

⇒ for Spontaneous $(dA)_{T, V} < 0$

⇒ Comparison b/w Helmholtz free Energy and Gibbs free Energy

* For an Isothermal process, for an ideal gas ($\Delta G = \Delta A$)

$$G = H - TS$$

$$G = E + PV - TS$$

$$dG = dE + PdV + VdP - Tds - SdT \quad [∵ dT=0]$$

$$dG = dA + \cancel{Tds} + \cancel{SdT} + PdV - VdT - \cancel{Tds} - \cancel{SdT} \quad ∵ A = E - TS$$

$$∴ dA = dE - Tds - SdT$$

$$dG = dA + \partial(PV)$$

$$[∵ PV = RT]$$

$$\partial G = dA + \partial CRT$$

$$\partial G = dA + R \partial T \quad [∵ \partial T = 0]$$

$$\boxed{dG = dA}$$

To prove ⇒

$$\boxed{\Delta A = \Delta E + T \left(\frac{\partial(\Delta A)}{\partial T} \right)_V} *$$

Partial Molar Property \Rightarrow

Suppose Z is a function of \rightarrow

$$Z = f(T, P, n_1, n_2, n_3, \dots, n_i)$$

= where system having different mole of gases
($n_1, n_2, n_3, \dots, n_i$)

\rightarrow The Total diff. of this \rightarrow

$$dz = \left(\frac{dz}{dT} \right)_{P, n_1, n_2, n_3, \dots, n_i} dT + \left(\frac{dz}{dP} \right)_{T, n_1, n_2, n_3, \dots, n_i} dP$$

$$+ \left(\frac{dz}{dn_1} \right)_{P, T, n_2, \dots, n_i} dn_1 + \left(\frac{dz}{dn_2} \right)_{T, P, n_1, n_3, \dots, n_i} dn_2$$

at const. T, P

$$dz = \left(\frac{dz}{dn_1} \right)_{T, P, n_2, n_3, \dots, n_i} dn_1 + \left(\frac{dz}{dn_2} \right)_{T, P, n_1, n_3, \dots, n_i} dn_2 + \left(\frac{dz}{dn_3} \right)_{T, P, n_1, n_2, n_4, \dots, n_i} dn_3$$

$$dz = \left(\frac{dz}{dn_i} \right)_{T, P, n_1, n_2, \dots, (n_{i-1})}$$

\Downarrow
Partial molar property

this is called P.M.P. $\left(\frac{dz}{dn_i} \right)_{T, P, n_1, n_2, \dots, n_{i-1}}$ ^(with change in i^{th}) component of the system where all other component are constant.

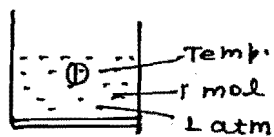
Note: \Rightarrow [1] The properties which is occur in the pure state of the system is different for the impure system

(Pure state)
e.g. When we adding one mole of water 100 cm^3 of pure water
When we get $\Delta V = 18 \text{ cm}^3$. bcz (one mole of water = 18 cm^3)

... in the impure state when we adding a 1 mole of H_2O in

2) the change in property in impure state is arises due to bcz of different interaction take place.

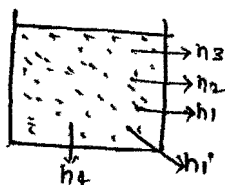
CHEMICAL POTENTIAL



$$\mu_i^0 = \frac{G}{n}$$

$\mu_i \Rightarrow$ (Temp. dependent)

$$Z = G$$



$$\mu_i = \left(\frac{\partial G}{\partial n_i} \right)_{T, P, n_1, n_2, \dots, (n_i-1)}$$

$$\mu_1 = \left(\frac{\partial G}{\partial n_1} \right)_{T, P, n_2, \dots, n_i}$$

$$\mu_2 = \left(\frac{\partial G}{\partial n_2} \right)_{T, P, n_1, n_3, \dots, n_i}$$

$$\mu_3 = \left(\frac{\partial G}{\partial n_3} \right)_{T, P, n_1, n_2, \dots, n_i}$$

⇒ change in free energy with change in particular component in a system which containing diff. no. of moles where all other moles are constant

⇒ Gibbs free energy changes with the change in particular component as constant T & P where all other component are constant that is substituted by chemical potential.

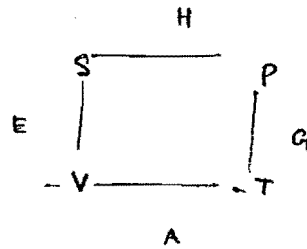
⇒ for a pure state free energy per mole is called chemical potential. Standard chemical potential can be defined for a pure state of a compound, amount 1 mole, 1 atm pressure at any temp. then it is called standard chemical potential.

$$[\mu^\circ] = G/n$$

⇒ Standard chemical potential only depends upon temp. where chemical potential of a particular component in a pure state depends upon T & P both.

$\mu = \mu^\circ + RT \ln P_i$	⇒ in terms of partial pressure
$\mu = \mu^\circ + RT \ln X_i$	⇒ in terms of mole fraction

Chemical potential can also be represented in different thermodynamically.



$$\left(\frac{dA}{dn_i} \right)_{V,T} = \mu_i$$

$$\left(\frac{dH}{dn_i} \right)_{S,P} = \mu_i$$

$$\left(\frac{dG}{dn_i} \right)_{P,T} = \mu_i$$

$$\left(\frac{dE}{dn_i} \right)_{S,V} = \mu_i$$

⇒ Variation of chemical potential with temperature

{ ∴ G decreases with temp. } $\left[\frac{\Delta}{T} \right]$

∴ μ also be ↓ with temp.

⇒ [एकै phase transfer वई μ_i same रहेगा]

we know that ⇒

$$\left(\frac{dG}{dn_i} \right)_{T,P} = \mu_i$$

$$\frac{d}{dT} \left(\frac{\partial G}{\partial n_i} \right) = \frac{\partial \mu_i}{\partial T}$$

$$\frac{\partial^2 G}{\partial T \partial n_i} = \frac{\partial \mu_i}{\partial T} \quad \text{--- (1)}$$

$$\therefore dG = vdp - SdT$$

$$\left(\frac{\partial G}{\partial T}\right)_P = -S$$

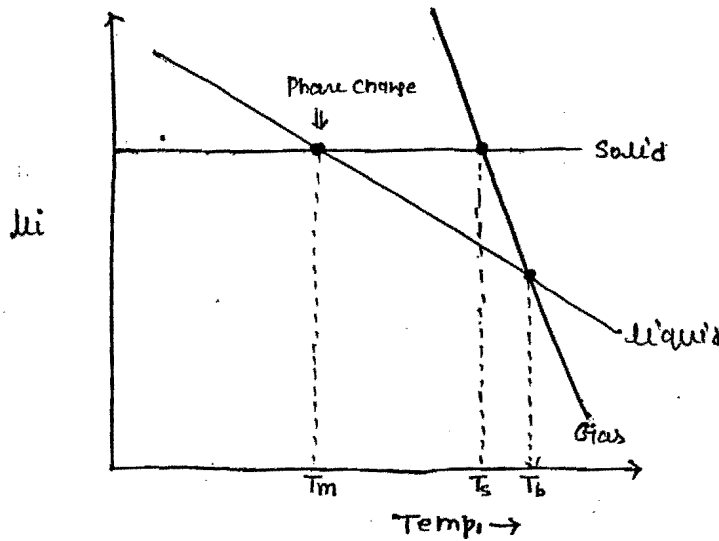
by (I) and (II)

$$\frac{\partial^2 G}{\partial n_i \partial T} = -\left(\frac{\partial S}{\partial n_i}\right)$$

$$\frac{\partial^2 G}{\partial n_i \partial T} = -\bar{S}_i \quad \text{--- (II)}$$

$$\boxed{\left(\frac{\partial \mu_i}{\partial T}\right)_P = -\bar{S}_i}$$

Calc-2010



T_s = Sublimation temp.

T_b = Boiling temp.

T_m = melting temp.

Graph: \Rightarrow Variation of chemical potential with change in temp.

\Rightarrow during phase transition, chemical potential remains same for both the phases.

[where T, v, ϕ all other component are constant],

\Rightarrow Since we know that G is state function so it must follow

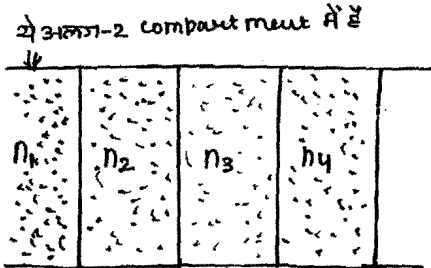
Euler's theorem.

$$\boxed{\left(\frac{\partial (\mu_i)}{\partial T}\right)_P = -\bar{S}_i} *$$

VIMP

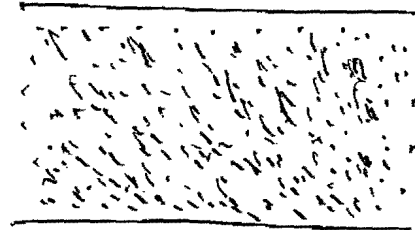
Calculation of thermodynamic parameter [ΔH^{mixing} , ΔV^{mix} , ΔS^{mix} , ΔA^{mix}]

In the case of ideal gases



(Before mixing)

(After removing compartment)
[सारे compartment को open कर दिया जाएगा]



(After mixing)

n_1 [Pure state] *

$$\mu_1^0 = \frac{G_1}{n_1}$$

$$n_2, \mu_2^0 = \frac{G_2}{n_2}$$

$$n_3, \mu_3^0 = \frac{G_3}{n_3}$$

$$n_4, \mu_4^0 = \frac{G_4}{n_4}$$

$$\mu_1 = \mu_1^0 + RT \ln X_1$$

$$\mu_2 = \mu_2^0 + RT \ln X_2$$

$$\mu_3 = \mu_3^0 + RT \ln X_3$$

$$\mu_4 = \mu_4^0 + RT \ln X_4$$

$$G_{Total} = n_1 \mu_1 + n_2 \mu_2 + n_3 \mu_3 + n_4 \mu_4$$

[After mixing]

$$G_{Total} = n_1 \mu_1^0 + n_2 \mu_2^0 + n_3 \mu_3^0 + n_4 \mu_4^0$$

[Before mixing]

$$\Delta G_{mixing} = G_{Total} \text{ (After mixing)} - G_{Total} \text{ (Before mixing)}$$

$$= (n_1 \mu_1 + n_2 \mu_2 + n_3 \mu_3 + \dots) - (n_1 \mu_1^0 + n_2 \mu_2^0 + n_3 \mu_3^0 + \dots)$$

$$= n_1 (\mu_1^0 + RT \ln X_1) + n_2 (\mu_2^0 + RT \ln X_2) + n_3 (\mu_3^0 + RT \ln X_3) + \dots - (n_1 \mu_1^0 + n_2 \mu_2^0 + \dots)$$

$$= n_1 RT \ln X_1 + n_2 RT \ln X_2 + \dots$$

2

$$\Delta G_{mix} = RT \sum n_i \ln x_i$$

$$\frac{\Delta G_{mix}}{T} = R \sum n_i \ln x_i$$

$$\frac{\partial}{\partial T} \left[\frac{\Delta G_{mix}}{T} \right]_P = R \sum \left(n_i \ln x_i \right)_P$$

$$-\frac{\Delta G_{mix}}{T^2} + \frac{1}{T} \left(\frac{\partial (\Delta G_{mix})}{\partial T} \right)_P = 0 \quad \text{--- (1)}$$

$$\therefore \Delta G_{mix} = \Delta H_{mix} + T \left(\frac{\partial (\Delta G_{mix})}{\partial T} \right)_P$$

Divide both side by T^2 then,

$$\frac{\Delta G_{mix}}{T^2} = \frac{\Delta H_{mix}}{T^2} + \frac{1}{T} \left(\frac{\partial (\Delta G_{mix})}{\partial T} \right)_P$$

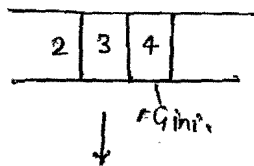
$$-\frac{\Delta H_{mix}}{T^2} = -\frac{\Delta G_{mix}}{T^2} + \frac{1}{T} \left(\frac{\partial (\Delta G_{mix})}{\partial T} \right)_P \quad \text{--- (2)}$$

By Eqn- (1) + (2)

$$-\frac{\Delta H_{mix}}{T^2} = 0$$

$$\boxed{\Delta H_{mix} = 0}^*$$

e.g. 2 mol O_2 , 3 mol N_2 , 4 mol H_2



$$n_{\text{Total}} = 9$$

$$\Delta G$$

$$\Delta G = G_f - G_i$$

$$\Delta G_{\text{mix.}} = RT \left(2 \ln \frac{2}{9} + 3 \ln \frac{3}{9} + 4 \ln \frac{4}{9} \right)$$

(per mole)

$$= RT \left[\left(\frac{2}{9} \right) \ln \left(\frac{2}{9} \right) + \frac{3}{9} \ln \left(\frac{3}{9} \right) + \frac{4}{9} \ln \left(\frac{4}{9} \right) \right]$$

(Rxn of kinetic energy ΔG_{mix}) \rightarrow \uparrow temp, \uparrow $K.E.$, $P.E. \Rightarrow$

$$\Delta G_{\text{mix}_1} = \frac{RT \sum n_i \ln x_i}{n_{\text{total}}}$$

$$\Delta G_{\text{mix}_1} = RT \sum x_i \ln x_i$$

(per mole)

Q4.8

2 mole of N_2

3 mole of H_2

2 mole of NH_3

$$\begin{aligned}\Delta S_{\text{mixing}} &= 8.314 \times \left[2 \ln\left(\frac{2}{7}\right) + 3 \ln\left(\frac{3}{7}\right) + 2 \ln\left(\frac{2}{7}\right) \right] \\ &= 8.314 \times 2.303 \left[2 \log\frac{2}{7} + 3 \log\frac{3}{7} + 2 \log\left(\frac{2}{7}\right) \right] \\ &= 8.314 \times 2.303 \times \left[4 \log\frac{2}{7} + 3 \log\frac{3}{7} \right] \\ &= -8.314 \times 2.303 \times \left[-2.4854 + (-0.36 \times 3) \right] \\ &= -8.314 \times 2.303 \left[-2.4854 - 1.08 \right] \\ &= -8.314 \times 2.303 \times (-3.5654) \\ &= 62.80 \text{ J K}^{-1}\end{aligned}$$

Q4.9

□ Monatomic $\xrightarrow[\text{Expan.}]{\text{adia.}}$ \Rightarrow loose energy very much
□ diatomic $\xrightarrow[\text{Expan.}]{\text{adia.}}$ \Rightarrow tight bond

No. of atom \propto Rotation, & vibration
No. of atom \propto Heat capacity *

$$\Rightarrow T_m < T_p < T_i$$

Monatomic \Rightarrow Temp. independent.

3

$$\Delta G_{\text{mixing}} = \Delta H_{\text{mixing}} - T \Delta S_{\text{mixing}}$$

$$\Delta G_{\text{mixing}} = - T \Delta S_{\text{mixing}}$$

$$\Delta S_{\text{mixing}} = - \frac{\Delta G_{\text{mixing}}}{T}$$

$$\Delta S_{\text{mixing}} = - R \sum n_i \ln x_i$$

$$\Delta S_{\text{mixing, permole}} = - R \sum x_i \ln x_i$$

$$\Delta G_{\text{mixing}} = \Delta A_{\text{mixing}}$$

[beoz (sothunma)] \Rightarrow ideal working substance

\Rightarrow When ideal gases mixing with each other since there is no interaction b/w the molecules so there is no absorption or rejection of heat take place so that $\Delta H = 0$

$$\Delta H_{\text{mixing}} = 0$$

(किसी भी आणविक नहीं है
No interaction
No work energy)

Ques One mole of CO_2 , one mole of NO_2 & two mole of O_2
 when mixed at 300K calculate the Entropy of mixi,
 ΔH_{mixing} , ΔV_{mixi} , ΔA_{mixi} &

1 mole of CO_2
 1 mole of NO_2
 2 mole of O_2

$$\Delta G = RT \left[1 \ln \frac{1}{4} + 1 \ln \frac{1}{4} + 2 \ln \frac{2}{4} \right]$$

$$\Delta G = RT \left[2 \ln \frac{1}{4} + 2 \ln \frac{1}{2} \right]$$

$$\Delta G = RT \times 2.303 \left[2 \log \frac{1}{4} + 2 \log \frac{1}{2} \right]$$

$$\Delta G = 8.314 \times 300 \times 2.303 \times [-2 \times (-0.60) - 2 \times (0.30)]$$

$$= 8.314 \times 300 \times 2.303 \times [-1.2 - 0.6]$$

$$= 8.314 \times 300 \times 2.303 \times [-1.8]$$

$$= -10,374$$

$$\boxed{\Delta H_{\text{mix}} = 0}$$

$$\Delta G = \Delta A = -10,374$$

$$= -10.374 \text{ KJ}$$

$$\Delta S = -\frac{\Delta G}{T} = +34.58 \text{ J}$$

$$\boxed{\Delta V_{\text{mixing}} = 0}$$

OR

$$\Delta S = 6R \ln 2$$

Ans

$$\ln \frac{k_2}{k_1} = \frac{E_a}{R} \left[\frac{T_2 - T_1}{T_1 \cdot T_2} \right] \quad \text{Arrhenius Eq}^n$$

[in term of rate constant]

$$\ln \frac{k_{p1}}{k_{p2}} = \frac{\Delta H^\circ}{R} \left[\frac{T_2 - T_1}{T_1 \cdot T_2} \right]$$

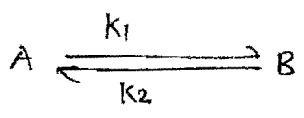
in term of k_p [Equi. constt]

$$\ln \frac{k_{c1}}{k_{c2}} = \frac{\Delta F^\circ}{R} \left[\frac{T_2 - T_1}{T_1 \cdot T_2} \right]$$

[in term of k_c]

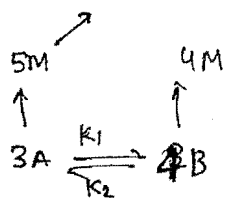
Vant-Hoff Relation

⇒ For Chemical Equilibrium first conditions system must be close. [It is Dynamic in Nature]



- $k \Rightarrow$
- $k_x \Rightarrow$
- $k_p \Rightarrow$
- $k_c \Rightarrow$

$$\alpha = \frac{5M}{1M} = 5$$



unitless (mole fraction)
(always)

$$K_x = \frac{[X_B]^4}{[X_A]^3}$$

(k_p & k_c may or may not unitless)
means depends upon
Stoichiometry)

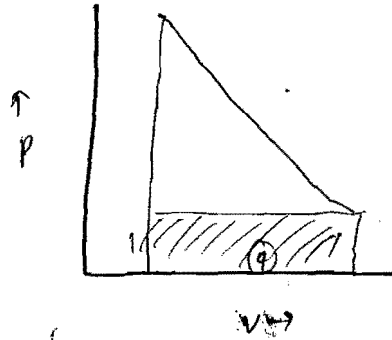
$$\frac{k_1}{k_2} = k_c = \frac{[C_B]^4}{[C_A]^3}$$

$$K_p = \frac{[P_B]^4}{[P_A]^3} \quad (\text{in mixture})$$

K = thermodynamic
equi. (unitless)
always

Isobar

Q4.21



$$\begin{aligned} \text{Rev:} &= \frac{1}{2} \times 4 \times 4 + 4 \times 1 \\ &= 8 + 4 = 12 \quad [X] \end{aligned}$$

$$\text{Isobar} = 1 \times 4 = 4 \text{ L.atm}$$

[05.01.2015]

$$\frac{\partial (\ln K_p)}{\partial T} = \frac{\Delta H^\circ}{RT^2}$$

$$\frac{\partial (\ln K_c)}{\partial T} = \frac{\Delta E^\circ}{RT^2}$$

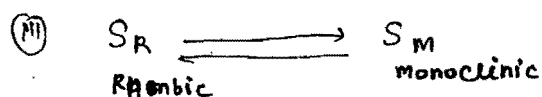
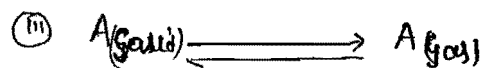
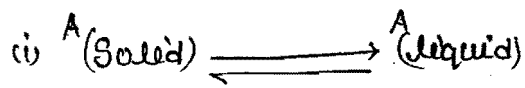
-: CLASSICAL - CLAYPERON RELATION

$\therefore \Delta G = 0$ [at Equilibrium]

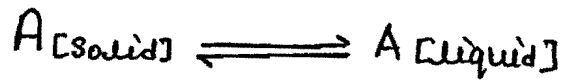
[Phase-transformation]
chemical potentials = 0]-(beoz at Equi.)
 $\Delta G = 0$

Classical Clayperon Equation is applying for any phase transformation a phase transformation which is a immediate process, during this the temp, pressure, Equilibrium conc., free Energy of two different phases Etc. are equal.

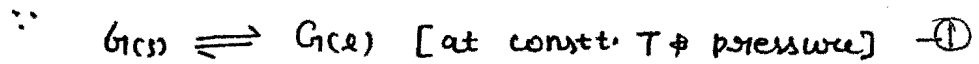
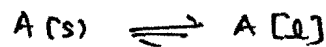
For eg. \Rightarrow There is a lot of phase transfer



for Solid \rightleftharpoons liquid Equi.



~~for~~ Solid changes into liquid at their melting point during this they absorb heat at a constt. pressure from the surrounding. during this phase transformation the sign of Heat absorb is +ive.
[अगर liquid से solid जाता तो -ive]



At a slight increase of temp. ($T+dT$, $P+dp$)



from (i) & (ii)

$$dG(s) \rightleftharpoons dG(l)$$

applying Maxwell Equation \rightarrow

~~(iii)~~

$$V_G dp - S_G dT = V_L dp - S_L dT$$

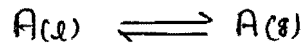
$$\frac{dp}{dT} = \frac{\Delta S}{V_L - V_G} \quad \text{--- (iii)}$$

$$\therefore \Delta S = \frac{\Delta H}{T} \quad \text{--- (iv)}$$

Here integrated form not possible coz we neglect V_G &

$$\boxed{\frac{dp}{dT} = \frac{\Delta H}{T}}$$

Differential form of



$$\therefore G(l) \rightleftharpoons G(g) \quad [\text{at const. } T, P] \quad \text{--- (I)}$$

When you slightly increases the $(T + \partial T, P + \partial P)$ then,

$$G(l) + \partial G(l) \rightleftharpoons G(g) + \partial G(g) \quad \text{--- (II)}$$

from I & II \Rightarrow

$$\partial G(l) \rightleftharpoons \partial G(g)$$

$$\therefore \partial G = V dp - S dT \quad [\text{Maxwell Equation}]$$

$$V(l) dp - S(l) dT = V(g) dp - S(g) dT$$

$$\frac{dp}{dT} = \frac{\Delta S}{V(g) - V(l)}$$

$$\therefore \Delta S = \frac{\Delta H}{T}$$

$$\boxed{\frac{dp}{dT} = \frac{\Delta H}{T} (V(g) - V(l))}$$

diff. form of Clausius-Clapeyron Eqⁿ

$$\therefore V_g \gg \gg \gg V_l$$

$$\frac{dp}{dT} = \frac{\Delta H}{T \times V(g)}$$

$$\frac{dp}{dT} = \frac{\Delta H \times P}{T \times RT} \quad [\because PV = RT], \quad V = \frac{RT}{P}$$

$$\frac{dp}{dT} = \frac{\Delta H \cdot P}{RT^2}$$

Integrated on both side

$$\int_{P_1}^{P_2} \frac{dP}{P} = \frac{\Delta H}{R} \int_{T_1}^{T_2} \frac{dT}{T^2}$$

$$\ln \frac{P_2}{P_1} = \frac{\Delta H^\circ}{R} \left[\frac{T_2 - T_1}{T_1 \cdot T_2} \right]$$

Ans.

\Leftarrow FUGACITY \Rightarrow
 (Latin) means (figure) \downarrow (corrected pressure for a real gas) = f (fugacity)
 \downarrow
 [Escaping tendency]

Ankur

- * It is a corrected pressure which is used for real gas
- * Fugacity of a real gas contain all the interaction which is take place in real gas.
- * It can also be understand as a escaping tendency of a real gas.
- * ~~It is~~ This new term is given by "G.N. Lewis"
- * at low pressure when pressure tending toward zero then the real gas behave as a ideal gas.

We know that f is a fugacity & P is pressure.

$$\therefore f/P = \gamma \text{ [fugacity coefficient]}$$

$$\lim_{P \rightarrow 0} f/P = 1 \quad [\because \gamma = 1]$$

$$\boxed{f = P}$$

The gas behaving as a ideal gas.

Mathematical Calculation of fugacity \Rightarrow

We know that,

Free Energy for an ideal gas at a particular temp. can be written as -

$$G = G^* + RT \ln P \quad \text{---(I)}$$

this expression can be used for real gas also
By introducing a new term 'f'

$$G = G^* + RT \ln f \quad \text{---(II)}$$

Free Energy for an real gas at particular temp. can be written above \Rightarrow

Eqⁿ (ii) differentiate both side w.r.t to P at constant temp

$$\left(\frac{dG}{dP}\right)_T = 0 + RT \left[\frac{d(\ln f)}{dP}\right]_P -$$

$$\left(\frac{dG}{dP}\right)_T = RT \left[\frac{d(\ln f)}{dP}\right]_T \quad \text{---(III)}$$

$$\therefore dG = VdP - SdT \quad \text{[at const. T, } dT=0]$$

$$\left(\frac{dG}{dP}\right)_T = V_m(\text{real})$$

put the value of $\left(\frac{dG}{dP}\right)_T$ in (III)

$$V_{\text{real}} = RT \left(\frac{\partial \ln f}{\partial p} \right)_T$$

$$V_{\text{real}} \cdot dp = RT (d \ln f) \quad \text{--- (IV)}$$

$$\therefore \alpha = V_{m(\text{ideal})} - V_{m(\text{real})}$$

Multiply dp both side by dp

$$\alpha \cdot dp = V_{m(\text{ideal})} dp - V_{m(\text{real})} dp$$

$$\alpha \cdot dp = \frac{RT}{p} dp - RT d(\ln f)$$

\(\therefore\) Inte. both side \(\rightarrow\)

$$\int_0^p \alpha \cdot dp = RT \int \frac{dp}{p} - RT \int d(\ln f)$$

$$\int_0^p \alpha \cdot dp = + RT \ln p - RT \ln f$$

$$\int_0^p \alpha \cdot dp = -RT [\ln f/p]$$

$$\boxed{-\frac{1}{RT} \int_0^p \alpha \cdot dp = \ln f/p}$$

$$\ln f/p = -\frac{1}{RT} \int_0^p \alpha \cdot dP$$

$$f/p = e^{-\frac{1}{RT} \int_0^p \alpha \cdot dP}$$

$$f = p \cdot e^{-\frac{1}{RT} \int_0^p \alpha \cdot dP}$$

$$\gamma = f/p = e^{-\frac{1}{RT} \int_0^p \alpha \cdot dP}$$

$\gamma \Rightarrow$ fugacity coefficient

Fugacity can also be terms of compressibility factor -

$$Z = \frac{P(V_{m,real})}{RT}$$

[$1 < Z > 1$] - real Gas

for an Ideal Gas [$Z=1$]

$$\ln f/p = -\frac{1}{RT} \int_0^p [V_{m(Ideal)} - V_{m(real)}] dP$$

$$\ln f/p = -\frac{1}{RT} \int_0^p \left[\frac{RT}{P} - \frac{ZRT}{P} \right] dP$$

$$\ln f/p = -\frac{1}{RT} \int_0^p \alpha \cdot dP$$

$$f/p = e^{-\frac{1}{RT} \int_0^p \alpha \cdot dP}$$

$$f = p \cdot e^{-\frac{1}{RT} \int_0^p \alpha \cdot dP}$$

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$$\ln f/p = \int_0^p \frac{1}{p} (Z-1) dp$$

$$\ln \gamma = \int_0^p \frac{(Z-1) dp}{p}$$

fugacity at low pressure \Rightarrow

$$\ln f/p = -\frac{1}{RT} \int_0^p \alpha \cdot dp$$

$$\ln f/p = -\frac{\alpha p}{RT}$$

$$\therefore \ln x = x-1 \text{ [Stirling's formula]}$$

$$\therefore \ln f/p = f/p - 1$$

from (i)

$$-\frac{\alpha p}{RT} = f/p - 1$$

$$1 - \frac{\alpha p}{RT} = f/p$$

$$1 - \frac{\left[\frac{RT}{p} - V_{\text{real}} \right] p}{RT} = f/p$$

[bcz No change

in pressure

\therefore Initial = Final

$$\ln f/p = \int_0^p \frac{1}{p} (Z-1) dp$$

$$\ln \gamma = \int_0^p \frac{(Z-1) dp}{p}$$

fugacity at low pressure \Rightarrow

$$\ln f/p = -\frac{1}{RT} \int_0^p \alpha \cdot dp$$

$$\ln f/p = -\frac{\alpha p}{RT}$$

$$\therefore \ln x = x-1 \text{ [Stirling's formula]}$$

$$\therefore \ln f/p = f/p - 1$$

from (i)

$$-\frac{\alpha p}{RT} = f/p - 1$$

$$1 - \frac{\alpha p}{RT} = f/p$$

$$1 - \frac{\left[\frac{RT}{p} - V_{\text{real}} \right] p}{RT} = f/p$$

[bcz No change

in pressure

\therefore Initial = final

$$1 - \frac{[RT - PV_{\text{molar}}]}{RT} = f/p$$

$$\frac{\cancel{RT} - \cancel{RT} + PV_{\text{molar}}}{RT} = f/p$$

$$\frac{PV_{\text{molar}}}{RT} = f/p$$

$$f = \frac{p^2 V_{\text{molar}}}{RT}$$

↓
This is fugacity at low pressure

[16.11.15]

$$P(V_m - b) = RT$$

$$PV - Pb = RT$$

$$\frac{PV}{RT} - \frac{Pb}{RT} = 1$$

$$Z = \frac{PV}{RT} = 1 + \frac{Pb}{RT}$$

$$\ln f/p = \int_0^p \frac{(1 + \frac{Pb}{RT})}{P} dp$$

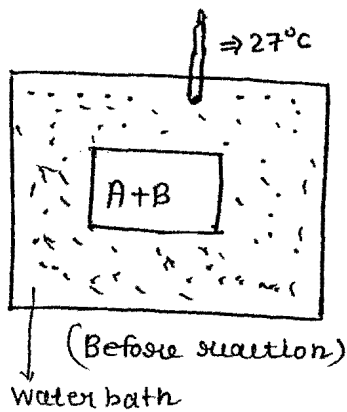
$$= \int_0^p \frac{pb}{RTp} dp$$

$$= \int_0^p \frac{b}{RT} dp$$

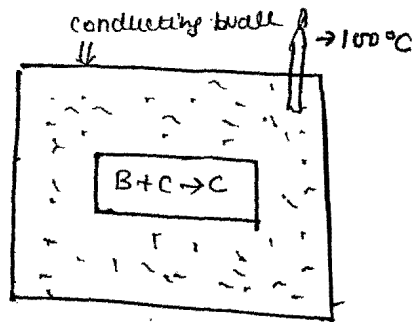
$$\ln f/p = \left(\frac{bp}{RT} \right)$$

$$f/p = e^{bp/RT}$$

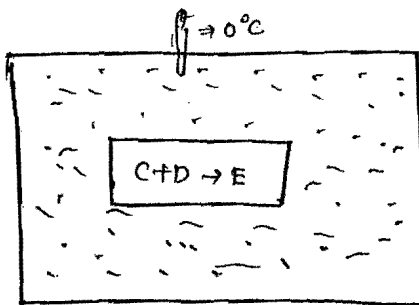
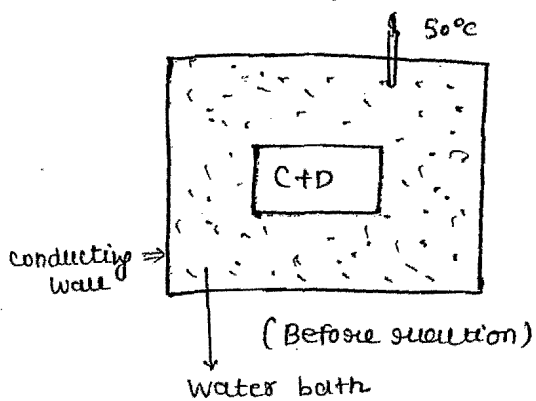
$$f = pe^{bp/RT}$$



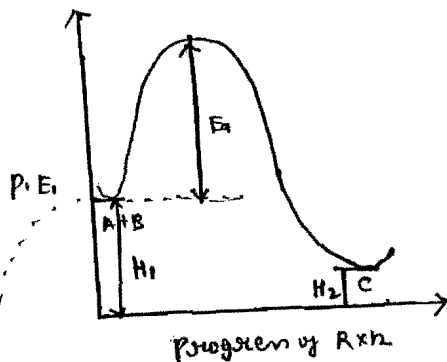
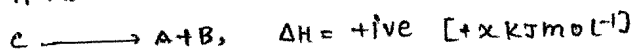
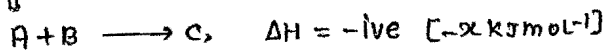
Exothermic Rxn



Endothermic Rxn



(A mole) \rightarrow small



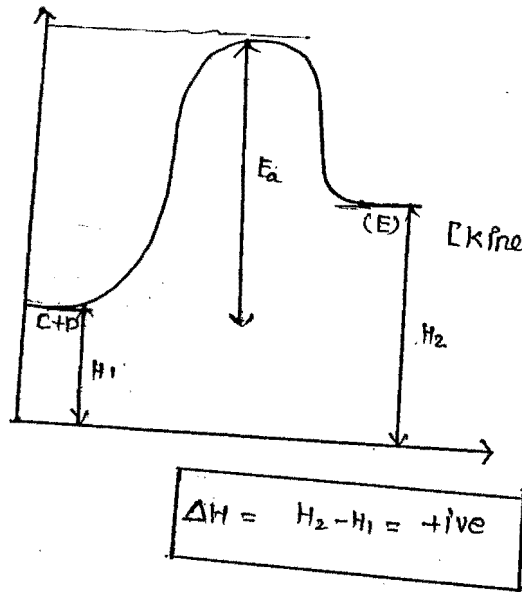
[Threshold Energy - average P.E.] = Activation E

\rightarrow Average - potential Energy of Reactant.

⇒ **Activation Energy = Threshold Energy - Average P.E. of Reactant**

⇒ The product form in Exothermic Rxn is thermodynamically stable

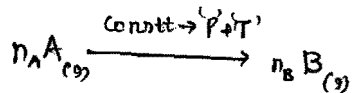
⇒ Temp only change when Heat is Evolved or absorbed



[Kinetically control product]

↓
form very easily and unstable becoz it change into reactant becoz Ea of product is very less. [most of the pressure is commt. so that change in ΔH]

Relation b/w ΔH and ΔE in the case of Ideal Gases ⇒



$$PV_A = n_A RT \text{ [Reactant]} \quad \text{--- (I)}$$

$$PV_B = n_B RT \text{ [Product]} \quad \text{--- (II)}$$

$$(II) - (I)$$

$$P(V_B - V_A) = (n_B - n_A) RT$$

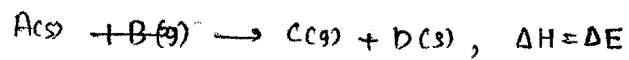
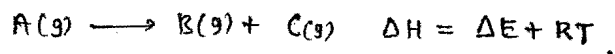
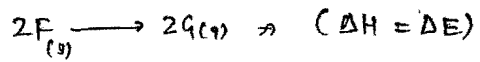
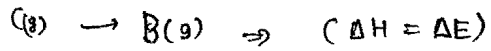
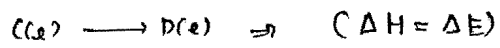
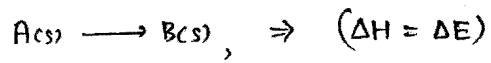
$$P \Delta V = \Delta n_g RT \quad \text{--- (3)}$$

most of the Rxn which is happen in the nature in const. temp & pressure.

$$H = E + PV$$

$$\partial H = \partial E + \partial(PV) \quad \text{--- from [3]}$$

$$\partial H = \partial E + P \partial V + V \partial P$$

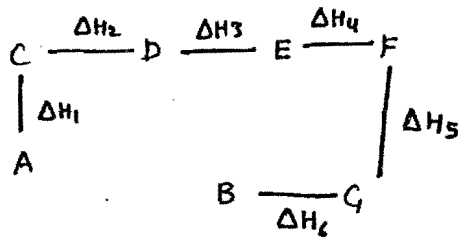
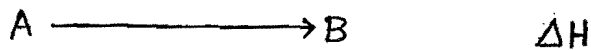


Heat change in const-volume (ΔE)

$$q_p = q_v + \Delta n_g RT$$

Heat change in const-pressure (ΔH)

⇐ Hess's Law ⇒



$$\Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3 + \Delta H_4 + \Delta H_5 + \Delta H_6$$

Conservation of Energy

Heat may be absorbed or evolved

“According to Hess's Law ⇒ the Enthalpy change during the reaction is the same whether the reaction complete one step or several steps.”

Numerical Question on Hess's Law! ⇒

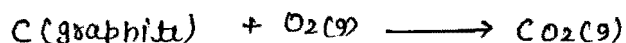
Ques:- Compute the standard heat of formation of methane



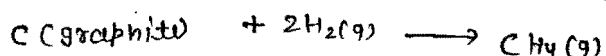
$$\Delta H^\circ(298\text{K}) = -890.35\text{KJ}$$



$$\Delta H^\circ(298\text{K}) = -285.84\text{KJ}$$



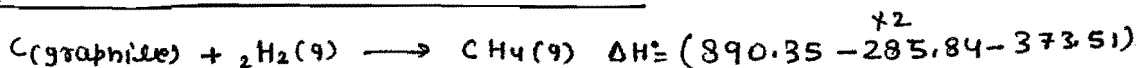
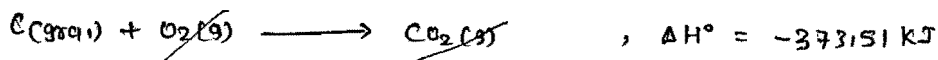
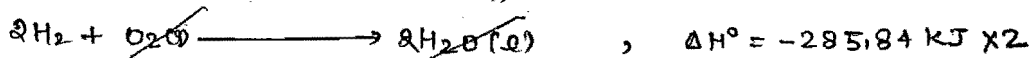
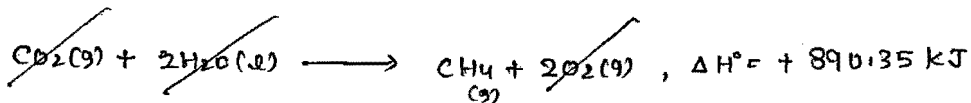
$$\Delta H^\circ(298\text{K}) = -373.51\text{KJ}$$



$$\Delta H^\circ(298\text{K}) = ?$$

⇒ Calculate the enthalpy change for above reaction.

B40



$$= \cancel{890.35 - 571.68 - 373.51}$$

$$= \cancel{121.17 - 285.84}$$

$$= -$$

$$= 890.35 - 2 \times 285.84 - 373.51$$

$$= 890.35 - 571.68 - 373.51$$

$$= 890.35 - 945.19$$

$$= -54.84\text{KJ}$$

Standard-State

Solid State (Pure Crystalline)

Substance at 1 atm or 1 bar at particular temp.

Liquid-State (Pure/molar)

1 atm pressure or 1 bar at particular temp.

$P_{\text{white}} > P_{\text{black}}$ [Phosphorus]

$S_R > S_M$ [Sulphur]
Rhombic (monoclinic)

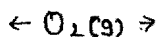
$C_{\text{graphite}} > C_{\text{diamond}}$ [Carbon]

Gas \Rightarrow 1 bar, pure
or 1 atm

$\Delta H^\circ \Rightarrow$ Standard Enthalpy change

Enthalpy of formation for an Element

⇒ The standard enthalpy of formation for any element in its stable state of aggregation at 1 bar pressure and specified temp. is assigned as zero



$$\Delta H_f^\circ [\text{O}_2(\text{g})] = 0$$

$$\Delta H_f^\circ [\text{C}_{\text{graphite}}] = 0 \text{ [stable]}$$

But $\Delta H_f^\circ [\text{C}_{\text{diamond}}] \neq 0$ [diamond is not stable state]

$$\Delta H_f^\circ [\text{Br}_2, \text{1 atm}] = 0$$

$$\Delta H_f^\circ [\text{S}_{\text{rhombic}}] = 0 \text{ [rhombic is stable state]}$$

$$\Delta H_f^\circ [\text{S}_{\text{monoclinic}}] \neq 0$$

$$\Delta H_f^\circ [\text{P}_{\text{white}}] = 0$$

$$\Delta H_f^\circ [\text{P}_{\text{black}}] \neq 0$$

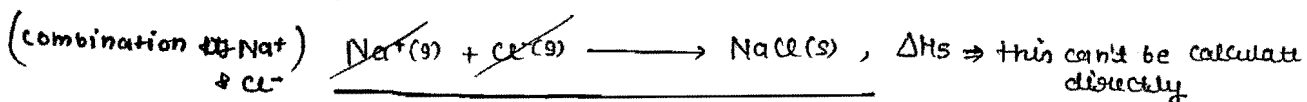
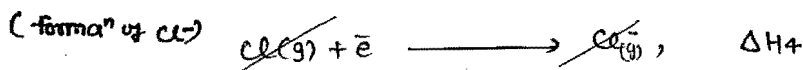
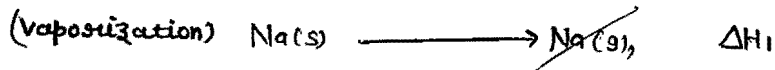
⇒ Born-Haber Cycle ⇒

Born-Haber cycle is used to calculate lattice energy, lattice energy can't be calculated experimentally but with the application of B.H. cycle we are able to calculate lattice energy.



"Lattice energy is defined as the energy required to separate one mole of solid ionic compound into gaseous ions."

Formation of NaCl crystals from Na(s) & Cl₂(g)



[Practically impossible]

from Hess's law -

$$\Delta H_6 = \Delta H_1 + \Delta H_2 + \Delta H_3 + \Delta H_4 + \Delta H_5$$

By the applⁿ of Hess's law & Born-Haber's cycle ⇒

we know, $\Delta H_1, \Delta H_2, \Delta H_3, \Delta H_4, \Delta H_6$
then,

$$\Delta H_5 = \Delta H_6 - (\Delta H_1 + \Delta H_2 + \Delta H_3 + \Delta H_4)$$

↓
(Lattice Energy)

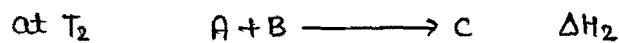
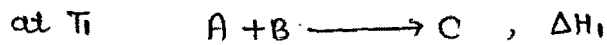
⇒ $\Delta H_f^\circ(\text{NaCl}) = -410.87 \text{ kJ mol}^{-1}$

⇐: Kirchhoff's Equation ⇒

(variation of temp., ΔH)

↘ (at diff) Rxn at ΔH given at diff. temp. T

↓
Kirchhoff Eqⁿ



$$\Delta H = H_2 - H_1$$

$$\left(\frac{\partial(\Delta H)}{\partial T} \right)_P = \left(\frac{\partial H_2}{\partial T} \right)_P - \left(\frac{\partial H_1}{\partial T} \right)_P$$

$$= (C_p)_{Prod} - (C_p)_{Reac}$$

$$\left(\frac{\partial(\Delta H)}{\partial T} \right)_P = \Delta C_p$$

$$\partial(\Delta H) = \Delta C_p \cdot \partial T$$

Inte. both side

$$\int_{H_1}^{H_2} \partial(\Delta H) = \Delta C_p \int_{T_1}^{T_2} \partial T$$

$$\boxed{\Delta H_2 - \Delta H_1 = \Delta C_p (T_2 - T_1)}$$

Variation of heat of reaction with temp.

Mathematical form of this statement is Kirchhoff Eqⁿ.

Kirchhoff Eqⁿ can be expressed either in terms of ΔH or ΔE

Kirchoff's Equation in term of ΔE .

$$\therefore \Delta E = E_2 - E_1$$

$$\left(\frac{\partial (\Delta E)}{\partial T} \right)_V = \left(\frac{\partial E_2}{\partial T} \right)_V - \left(\frac{\partial E_1}{\partial T} \right)_V$$

$$\left(\frac{\partial (\Delta E)}{\partial T} \right)_V = (C_V)_{\text{HOT}} - (C_V)_{\text{REF}}$$

$$\left[\frac{\partial (\Delta E)}{\partial T} \right]_V = \Delta C_V$$

$$\partial (\Delta E) = \Delta C_V \cdot \partial T$$

int on both side

$$\int_{\Delta E_1}^{\Delta E_2} \partial (\Delta E) = \Delta C_V \int_{T_1}^{T_2} \partial T$$

$$\Delta E_2 - \Delta E_1 = \Delta C_V (T_2 - T_1)$$

Numerical \Rightarrow



where C_p of (C \Rightarrow product) \Rightarrow 5

$$(C_p)_B = 3$$

$$(C_p)_A = 1$$

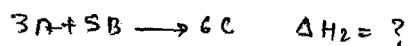
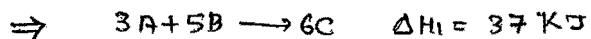
$$\Delta H_2 - 37 = (\cancel{5-3}) (500 - 298)$$

$$\Delta H_2 - 37 = 1 \times (202)$$

$$\Delta H_2 = 202 + 37$$

$$\Delta H_2 = 239 \text{ KJ}$$

$$= 23900 \text{ KJ}$$



$$(C_p)_A = 3$$

$$(C_p)_B = 5$$

$$(C_p)_C = 1$$

$$\Delta H_2 - \Delta H_1 = [3 \times 6 - 3 \times 1 - 5 \times 5] [500 - 298]$$

$$\Delta H_2 - 37000 = [18 - 28] \times 202$$

$$\Delta H_2 - 37000 = -2020$$

$$\Delta H_2 = 37000 - 2020$$

$$\Delta H_2 = 34980 \text{ J}$$

(Rxn Higher temp \uparrow is Endothermic)

Q4.6

$$\Delta H_1 = -922 \text{ kJ}$$

$$\Delta H_2 = ?$$

$$T_2 = 373$$

$$T_1 = 298$$

$$\begin{array}{r} 106.7 \\ 70.2 \\ \hline 36.5 \end{array}$$

$$\Delta H_2 - \Delta H_1 = (2 \times 35.1 - 3 \times 28.8 - 1 \times 29.1) (373 - 298)$$

$$\Delta H_2 + 92.2 = (70.2 - 77.6 - 29.1) (75)$$

$$\Delta H_2 + 92.2 = (70.2 - 106.7) 75$$

$$\Delta H_2 + 92.2 = -45.3 \times 75$$

$$\Delta H_2 = -45.3 \times 75 - 92.2 \times 1000$$

~~$$\Delta H_2 = -45.3 \times 75 - 92.2 \times 1000$$~~

~~$$\Delta H_2 = -45.3 \times 75 - 92.2 \times 1000$$~~

$$\Delta H_2 = -45.3 \times 75 - 92200$$

$$\Delta H_2 > \Delta H_1$$

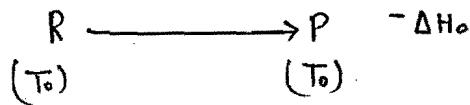
more exothermic

Adiabatic flame temp.

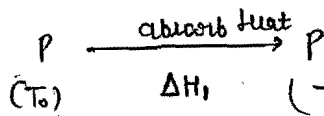
and

Adiabatic - Explosion temp.

Ist step



IInd step



at a const. pressure \Rightarrow Adiabatic-flame
at a const. volume \Rightarrow " \rightarrow explosion

$$-\Delta H_0 = \Delta H_1$$

(step 2)
complete
Combustion
 \downarrow
 $CO_2(g) + H_2O(l)$
(Only product)
(No react)

1st step में product केगा वए product final temp.
achive करेगा वए temp वए 1st step में $-\Delta H_0$ release
हुआ हुनसे heat लेगा अगर ये complete combustion
अगर const. pressure पर होगा तो adiabatic flame temp.
अगर const. volume पर होगा तो adiabatic explosion temp.

$$-\Delta H_0 = \sum n C_p \int_{T_0}^{T_f} dT$$

$$-\Delta H_0 = \sum n C_p (T_f - T_0)$$

$$\frac{-\Delta H_0}{\sum n C_p (\text{product})} + T_0 = T_f$$

$$-\frac{\Delta E^{\circ}}{\Delta C_v} + T_0 = T_f$$

\downarrow
 (Product)
 \downarrow
 at constt volume

Adiabatic flame temp'

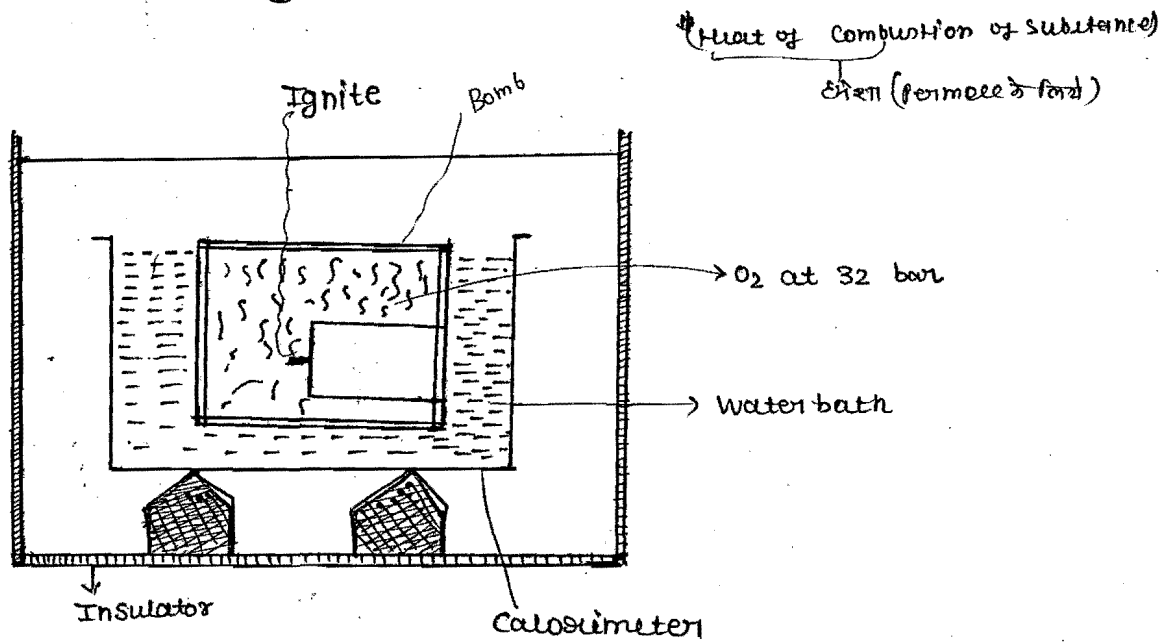
⇒

When a complete combustion of a substance take place, the final temp. which is achieved by product at constt. pressure condition is called A.F.T.

Adiabatic explosion temp'

⇒ When a complete combustion of a substance take place the final temp' which is achieved by product at constt. volume condition is called. A.E.T. Close

BOMB-CALORIMETER



$$q_{\text{water}} = m C_{\text{water}} \Delta T$$

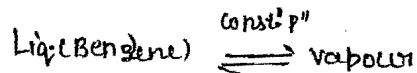
$$q_{\text{Bomb}} = m \times C_{\text{bomb}} \Delta T$$

$$q_{\text{combustion}} = q_{\text{water}} + q_{\text{Bomb}}$$

[17.11.15]

⇒ Trouton's Rule: ⇒ ^(Failed) [which undergo association or dissociation]

$$\Delta S = \frac{\Delta H_{\text{vap.}}}{T_b}$$



⇒ "For Molt of the liquid at a respective b.p.t.,
molar entropy of vaporization is fixed.
which is 88 JK⁻¹mol⁻¹ or 21 cal K⁻¹mol⁻¹."

88 JK⁻¹mol⁻¹

or

21 cal K⁻¹mol⁻¹

[Statement of Trouton's Rule]

This law is failed which liquid is association or dissociation.

[e.g. - CH₃COOH]

This law is also failed which having ^{very} High + very low b.p.t.

Gibbs - Duhem Equation: -

$$G = f(T, P, n_1, n_2, n_3, \dots, n_i)$$

$$dG = \left(\frac{\partial G}{\partial T}\right)_{P, n_1, n_2, n_3, \dots, n_i} dT + \left(\frac{\partial G}{\partial P}\right)_{T, n_1, n_2, \dots} dP + \left(\frac{\partial G}{\partial n_1}\right)_{T, P, n_2} dn_1 + \dots$$

at const - T, P

$$dG = dn_1 \mu_1 + dn_2 \mu_2 + dn_3 \mu_3 + dn_4 \mu_4 + \dots$$

Integrate both side

$$G = \mu_1 n_1 + \mu_2 n_2 + \mu_3 n_3 + \dots \quad - (2)$$

Differentiate both side

$$dG = n_1 d\mu_1 + \mu_1 dn_1 + n_2 d\mu_2 + \mu_2 dn_2 + \dots \quad - (3)$$

Substrate (3) - (2)

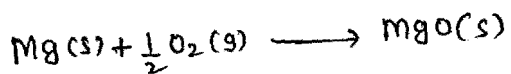
$$0 = n_1 d\mu_1 + n_2 d\mu_2 + n_3 d\mu_3 + n_4 d\mu_4 + \dots$$

$$\sum n_i d\mu_i = 0$$

↓
Gibb's Dheem Equation

TIFR Assignment

Qn. 1



$$\Delta H_{298}^{\circ} = -602 \text{ kJ/mol}$$

$$\Delta S_{298}^{\circ} = -108 \text{ kJ/mol}$$

$$\Delta G_{298}^{\circ} = \Delta H_{298}^{\circ} - T \Delta S_{298}^{\circ}$$

$$= -602 \text{ kJ/mol} - 273 \times (-108)$$

$$= -602 \text{ kJ/mol} + 29484 \text{ kJ/mol}$$

$$= 28882$$

$$\Delta G = \Delta H - T \Delta S$$

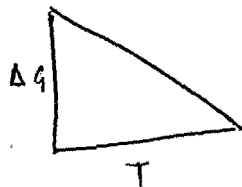
$$\gamma = n\epsilon + m\alpha$$

Qn. 3

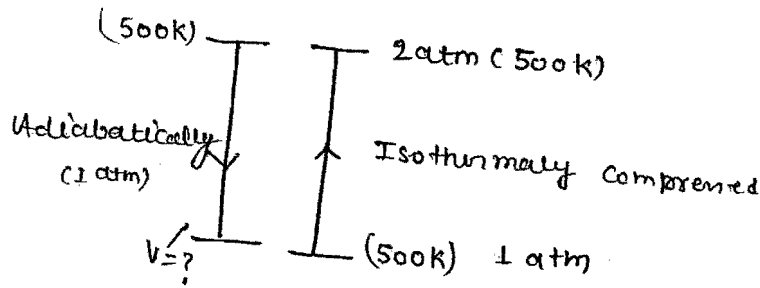
$$\text{Slope} \left\{ \tan \theta = \frac{1}{\text{Base}} \right\} =$$

$$\frac{3600 \text{ J} \text{ (P)}}{4000}$$

$$\Rightarrow 75 \text{ J mol}^{-1} \text{ K}^{-1}$$



Q.20



[∵ Ideal Gas]

$$PV = nRT$$

$$2 \times 10^5 = 1 \times 0.082 \times 500$$

$$V = \frac{0.082 \times 500}{2}$$

$$= 20.5 \text{ dm}^3$$

$$V_1 = 20.5 \text{ dm}^3$$

$$T_1 = 500 \text{ K}, T_2 = ?$$

$$\left(\frac{T_1}{T_2}\right)^\gamma = \left(\frac{P_2}{P_1}\right)^{1-\gamma}$$

$$\left(\frac{500}{T_2}\right)^{1.66} = \left(\frac{1}{2}\right)^{(1-1.66)}$$

$$\left(\frac{500}{T_2}\right)^{1.66} = \left(\frac{1}{2}\right)^{-0.66}$$

$$\left(\frac{500}{T_2}\right)^{1.66} = 1.58$$

$$\frac{500}{T_2} = (1.58)^{1/1.66}$$

$$\frac{500}{T_2} = 1.33$$

$$T_2 = \frac{500}{1.33} = 375.9$$

$$(PV)^\gamma = \text{constant}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\left(\frac{V_1}{V_2}\right)^\gamma = \frac{P_2}{P_1}$$

$$\left(\frac{20.5}{V_2}\right)^\gamma = \frac{2}{1}$$

$$\left(\frac{20.5}{V_2}\right)^{1.66} = 2$$

$$C_V = 1.5R$$

$$C_P = C_V + R$$

$$C_P - 1.5R = R$$

$$C_P = 2.5R$$

$$\frac{C_P}{C_V} = \gamma$$

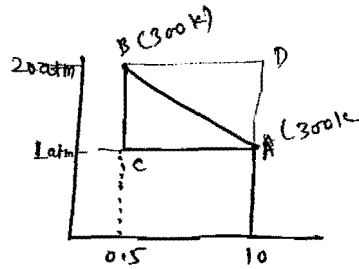
$$\frac{2.5}{1.5} = \gamma$$

$$\gamma = \frac{5}{3} = 1.66$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\frac{500}{375.9} = \frac{V_2}{20.5}$$

27



$$V_1 = 10 \text{ L}$$

$$V_2 = 0.5 \text{ L}$$

Expansion:

isothermal (rev. work done)

$$W = -nRT \ln \frac{V_2}{V_1}$$

$$= -n \times 300 \times \ln \frac{0.5}{10}$$

$$= -\frac{10}{300} \times 300 \ln \frac{0.5}{100}$$

$$= -10 \ln \frac{5}{100} = (-) -10 \ln \frac{100}{5}$$

$$= \underline{\underline{10 \ln 20}}$$

$$PV = nRT$$

$$\frac{1 \times 10}{300} = nR$$

04.30

$$T_1 = 15^\circ\text{C}$$

$$T_2 = 75^\circ\text{C}$$

$$C_p \text{ of water} = 75 \text{ J K}^{-1} \text{ mol}^{-1}$$

$$\frac{\Delta H}{\Delta T} = n c_p$$

$$n = \frac{w}{m} = \frac{54}{18} = 3 \text{ mol}$$

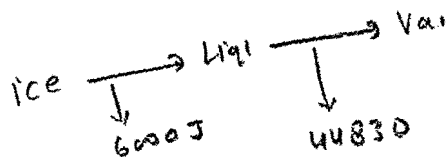
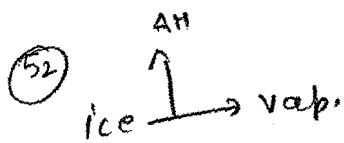
$$\underline{\Delta H} = 3 \times 75$$

~~348-288~~

$$\frac{\Delta H}{348-288} = 3 \times 75$$

$$\frac{\Delta H}{+60} = 3 \times 75$$

$$\Delta H = 13.5 \text{ KJ}$$



53

$$\ln \frac{P_2}{P_1} = \frac{\Delta H}{R} \left(\frac{T_2 - T_1}{T_1 T_2} \right)$$

variation of temp. with pressure
 \rightarrow Clausius-Clapeyron Eqn

$$P_1 = 2.02 \times 10^3 \text{ N m}^{-2}$$

$$P_2 = 1 \text{ atm} = 1 \times 10^5 \text{ N m}^{-2}$$

T_2
 364

$\Delta H = 41 \text{ kJ mol}^{-1}$ solve yourself

$$\ln \frac{1 \times 10^5}{2.02 \times 10^3} = \frac{41000}{8.314} \left(\frac{T_2 - 293}{293 T_1} \right)$$

$$\ln \left(\frac{100}{2.02} \right) =$$

$$\ln 49.50 = 4931.44 \left(\frac{T_2 - 293}{293 T_1} \right)$$

$$3.902 = 4931.44 \left(\frac{T_2 - 293}{293 T_1} \right)$$

$$\frac{3.902 \times 293}{4931.44} = \frac{(T_1 - 293)}{T_1}$$

$$0.23 = 1 - \frac{293}{T_1}$$

$$1 - 0.23 = \frac{293}{T_1}$$

$$T_1 = \frac{293}{0.77}$$

$$= 380 \text{ K}$$

Q4.57 $U = 3.5 PV + K$

$$v_1 = 0.25 \text{ m}^3$$

$$v_2 = 0.86 \text{ m}^3$$

$$P_1 = 5 \text{ Nm}^{-2}$$

$$PV^{1.3} = \text{constant}$$

where $\gamma = 1.3$

$$\frac{5}{1} = \left(\frac{0.86}{0.25} \right)^{1.3}$$

Note \Rightarrow

(Water), $\Delta H_{\text{fusion}} = 324 \text{ kJ/kg}$ or J/kg

$$\Delta H_{\text{vaporization}} = 2264.76 \text{ kJ/kg}$$

$$c_p = \text{specific heat of water} = 4.18 \text{ J/g}$$

$$c_p = \text{specific heat of ice} = 2.023 \text{ J/g}$$

(Net previous)

Q.2 ② ⇒ $PV^{\alpha} = \text{constant}$

$$T_1 V_1^{\alpha-1} = T_2 V_2^{\alpha-1} = K$$

$$\therefore T_2 > T_1$$

$$\frac{T_1 V_1^{\alpha-1}}{V_2^{\alpha-1}} = T_2$$

$$\frac{T_1 V_1^{\alpha-1}}{V_2^{\alpha-1}} > T_1$$

$$\left(\frac{V_1}{V_2}\right)^{\alpha-1} > 1$$

$$\left(\frac{V_1}{V_2}\right)^{\alpha-1} > \left(\frac{V_1}{V_2}\right)^0$$

$$\alpha - 1 < 0$$

$$\boxed{\alpha < 1}$$

Q.6

$$\frac{dP}{dT} = \frac{\Delta H}{T(V_{\text{vap.}} - V_L)}$$

Q.28

$$q = -(q_{\text{water}} + q_{\text{can}})$$

$$q = -2500 \text{ J K}^{-1} \times 4$$

$$q = -2500 \times (\Delta T)$$

$$q = -2500 \times 4$$

$$q_{\text{com.}} = -10000$$

$$n = \frac{0.5 \text{ g}}{50 \text{ g}} = \frac{1}{100} = 0.01$$

$$q_{\text{comb.}} / \text{mole} = \frac{-10000 \text{ J}}{0.01}$$

$$= -1000000$$

$$q_{\text{combun.}} = -1000 \text{ kJ}$$

per mol

Degree of freedom

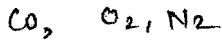
Tr, Rot, Vib

Linear

$$\downarrow$$

$$T_r = 3$$

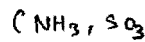
$$R_{ot} = 2$$



Non linear

$$T = 3$$

$$R = 3$$



(3N-6)

$$T + R + V = 3N \text{ (No. of atom)}$$

$$V = 3N - 5$$

$$V = 3 \times 2 - 5$$

$$= 6 - 5$$

$$= 1$$

(Contribution)



(T)

$$C_v \Rightarrow R/2$$

$$C_v \Rightarrow (R) \rightarrow R/2$$

$$C_v \Rightarrow (V) \rightarrow R \text{ at High}$$

0.2R at R/T

0 at low temp.

Monatomic



He, Ne, Ar, Kr, Xe, Rn

$$T = 3$$

$$R_{otational} = 0$$

$$V_{ibrational} = 0$$

$$(T) C_v = 3R/2$$

$$R_{otat} = 0$$

$$V_{ibrat} = 0$$

$$\text{Total } C_v = 3R/2 + 0 + 0$$

$$\boxed{C_v = 3R/2}$$

$$C_p = C_v + R$$

$$\boxed{C_p = 5R/2}$$

$$\boxed{\frac{C_p}{C_v} = \frac{5}{3} = 1.66}$$

(monatomic)

The ratio of $\frac{C_p}{C_v}$ for diatomic gas \Rightarrow
 $[O_2]$

$$\text{Transl} = 3$$

$$\text{Rotat} = 2$$

$$\begin{aligned} \nu &= 3N - 5 \Rightarrow 1 \\ &= 3 \times 2 - 5 \Rightarrow \\ &= 1 \end{aligned}$$

at high temp,

$$\begin{aligned} \text{Total } C_p &= 3 \times R/2 + 2 \times R/2 + 1 \times R \\ &= 1.5R + R + R \\ &= 3.5R \\ &= \frac{7R}{2} \end{aligned}$$

$$C_p = C_v + R \Rightarrow \frac{7R}{2} + R = \frac{9R}{2}$$

$$\frac{C_p}{C_v} = \frac{9}{7} = 1.28$$

29

$$C_v = 20 \text{ J K}^{-1} \text{ mol}^{-1}$$

$$\text{Coll. Volu.}, T_2 = 2T_1$$

$$\underline{W + q}$$

$$\Delta E = q + W$$

$$\Delta E = q + 0$$

$$C_v(T_2 - T_1) = q$$

$$20(2 \times 298 - 298) = q$$

$$q = 20 \times 298$$

$$q = 5960 \text{ J}$$

$$\boxed{q = 5.96 \text{ J}}$$

