Faraday's and Lenz's Law

1 Induction

In the early 19th Century, Michael Faraday discovered that a *changing* magnetic field could induce an emf (and hence a current) in a loop. Specifically, he found that a current was generated when *the magnetic flux,* Φ_B changed. This is now known as *Faraday's Law:*

$$\mathcal{E} = \frac{-d\Phi_B}{dt}$$

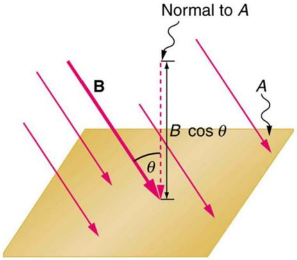
If we were "vector ninjas" that minus sign would help give a direction. The derivative term describes doing something with the magnet to change the "Magnetic Flux" in time. Like any B field effect, N loops multiplies the effect N times.

$$\mathcal{E} = \frac{-N d\Phi_B}{dt}$$

We've known for ages that you rub two sticks together to make fire – here Faraday's Law is telling you what you have to do to a coil of wire and a magnet to make electricity.

2 Magnetic Flux

Magnetic flux is just like our earlier definition of Electric flux: $\Phi_B = \int \vec{B} \cdot d\vec{A}$ For *only the special case when B is uniform throughout the area of the loop,* this integral simplifies to *this*



$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

1. B is the magnetic field in Tesla.
2. A is the area of the loop.
3. θ is the angle between the two vectors.

Magnetic flux is measured in a unit called a *weber (Wb)*, $1\,\mathrm{Wb} = 1\,\mathrm{T\,m^2}$

3 How does Flux change in Time?

The chain rule is used, considering B, A, θ depending on the problem might be functions of time.

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{d(BA\cos\theta)}{dt}$$

$$\mathcal{E} = -N(A\cos\theta\frac{dB}{dt} + B\cos\theta\frac{dA}{dt} - BA\sin\theta\frac{d\theta}{dt})$$

The trick to each problem is identifying which variables are changing!

1. $\frac{dB}{dt}$ EMF if B changes in time (like EMP)
2. $\frac{dA}{dt}$ EMF if A changes in time (like a rail gun)
3. $\frac{d\theta}{dt}$ EMF with rotation (like a generator)

4 Lenz's Law

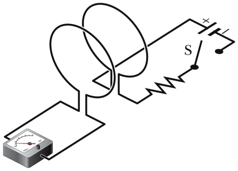
Remember that negative sign in Faraday's Law? If we are careful about how the dot product is taken it would tell us the direction of the current in the loop. However, Lenz's Law is a good way to know the answer without math. It tells us the direction of the induced current.

The induced current is in the direction such that the field produced by the induced current opposes the change in flux which produced it.

Steps to using Lenz's law:

1. Which way is the existing B field?
2. Is the flux increasing or decreasing?
3. If decreasing, induced current moves to make B field in the same direction as the existing field.
4. If increasing, induced current moves to make B field in the opposite direction from the existing field.

Exercise 1: A circular wire loop of radius 1.0 cm is oriented so that its normal makes a 60° angle with a uniform 1.0 mT magnetic field. (a) What is the magnetic flux through the loop? (b) If the loop is rotated so that its normal makes a 45° angle with the magnetic field, does the flux increase, decrease, or remain the same?



Exercise 2: Suppose that when the switch is closed, the current in the right-hand loop takes 1.0 µs to build to its steady-state value, and that during this time, the magnetic flux through the left-hand loop increases steadily from zero to 1.0×10^{-8} Wb.

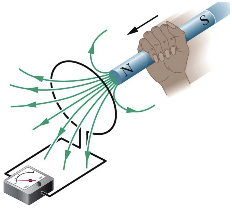
(a) While the flux is increasing, what are the magnitude and direction of the induced emf in the left-hand loop?

(b) During this time, what is the direction of the current through the ammeter?

(c) Once the current through the right-hand loop stabilizes, the flux through the loops remains constant. Now how much emf is induced in the left-hand loop?

(d) When the switch is opened, the current in the right-hand loop decreases steadily to zero in 1.0×10^{-7} s. During this time, what are the magnitude and direction of the induced emf in the left-hand loop?

(e) After the current is stopped, what is the emf in the left-hand loop?



Exercise 3: What is the direction (left, right, or zero) of the current through the ammeter in each case?

(a) The magnet's north pole is moved toward the loop as shown.

(b) The magnet is held still while its north pole is inside the loop.

(c) The magnet is moved away from the loop.

(d) The magnet is turned around and moved toward the loop, south end first.

(e) The magnet is held still with the south end in the loop.

(f) With its south pole in the loop, the magnet is moved away from the loop.