Shivaji University, Kolhapur

Question Bank For May 2022 (Summer) Examination

B.Sc. (Part-II) (Semester- III) Examination

Subject Code: 73305

Subject Name: STATISTICS (Paper – V) (CBCS-DSC-7C)

Name of the Paper: Probability distributions – I

Multiple Choice Questions (1 Mark each)

Q. **Answer the following questions choosing the most correct alternative given below them.**

1) If X and Y are two independent Poisson random variables with parameters 1 and 1 respectively then distribution of X+Y is…

A) Poisson with parameter 2
B) Poisson with parameter 3
C) Poisson with parameter 1
D) none of these

2) If X ~ Poisson (5) then ratio of mean to the variance is?

A) 5
B) 1
C) 100
D) 25

3) If X ~ P(λ) then p.g.f. of X is…

A) $e^{-(1-S)\lambda}$
B) $e^{-(1+S)\lambda}$
C) $e^{(1-S)\lambda}$
D) $e^{-\lambda}$

4) If X ~ Poisson distribution with parameter 1 then P(X=0) is…

A) e
B) 1/e
C) 1
D) none of these

5) If X is a Poisson variate with P[X =1] = P[X=2] then mean of X is…

A) 1
B) 4
C) 3
D) 2

6) The Poisson distribution is limiting case of binomial distribution when p→ 0 and …

A) n → 0
B) n → ∞
C) n → p
D) n → 1/2

7) Which of the following distribution has lack of memory property?

A) Poisson distribution | B) Geometric distribution

C) Binomial distribution | D) none of these

8) If X ~ Geometric distribution with parameter p and $P(X > 8 / X > 3) = 0.7$ then $P(X > 5)$ is…

A) 0.7 | B) 0.3

C) 0.1 | D) 0

9) If X ~ G(p) then mean of geometric distribution is…

A) p / q | B) q

C) p | D) q /p

10) If X ~G(p) and Y ~G(p) are independent variables then X + Y ~…

A) G(p) | B) G(q)

C) NBD(2, p) | D) NBD(4, q)

11) If X ~ NBD(k, p) then mean of X is …

A) kp | B) pq

C) kp/q | D) kq/p

12) If X ~ NBD(k, p) it reduces to geometric distribution if …

A) k =1 | B) k =0

C) p =1 | D) p = 0

13) If X is number of failures before k^{th} success then X follows…….distribution.

A) Poisson | B) Geometric

C) Negative Binomial | D) None of these

14) The mgf of a random variable X is $M_X(t) = (1-2t)^{-1}$. Then E(X) is …

A) 2 | B) 5

C) 4 | D) ½

15) The cumulant generating function (c.g.f.) of a continuous r. v. X is –log(1–2t) then E(X) is …

A) 2 | B) 5

C) 4 | D) 0.5

16) The probability distribution of continuous r. v. X has measures of kurtosis (β_2) is 3 and fourth central moment(μ_4) is 12 then variance is...

A) 4 | B) 2

C) 6 | D) 8

17) If X is a continuous r.v. with p.d.f. $f(x)$ then P(X= 5) is…

A) ½ B) 0

C) infinity D) 1

18) Which of the following is true?

A) First central moment = 0

B) Second Central moment = Second cumulant

C) Third Central moment = Third cumulant

D) All of the these

19) If p.d.f of continuous random variable X is f(x) = 1/2 , if 3 < x < 5 and F(x) be the cdf of X then F(6) is…..

A) 1 B) –1

C) 0 D) infinity

20) If Q1 and Q3 are first and third quartile of a continuous r. v. X then P(X < Q1) + P(X > Q3) = …

A) 0.5 B) 0.25

C) 0.125 D) 1

21) If X is a continuous r.v. with p.d.f. $f(x)$ then $\int_{-\infty}^{\infty} f(x)dx$ = ………

A) ½ B) 0

C) ∞ D) 1

22) If X is a continuous r. v. with probability density function(p.d.f.)

$$f(x) = \begin{cases} ke^{-x} & 0 \le x < \infty \\ 0 & otherwise \end{cases}$$ then value of k is…

A) 1 B) 2

C) 3 D) ½

23) If F(x) be the distribution function of continuous random variable X then P(4 ≤ x ≤ 10) is equal to:

A) F(10) – F(4) B) F(10) + F(4)

C) F(4) – F(10) D) F(10)* F(4)

24) If X is a continuous r. v. with probability density function(p.d.f.)

$$f(x) = \begin{cases} kx^2 & 0 < x < 1 \\ 0 & otherwise \end{cases}$$ Then value of k is --------

A) 2 B) 1

C) 3 D) ½

25) A continuous r.v. X has pdf

$$f(x)=\begin{cases}2x & 0<x<1\\ 0 & otherwise\end{cases}$$

Then mean of the r.v. X is…

A) 1/2 B) 2/3

C) 1 D) none of these

26) If X is continuous random variable with pdf

$$f(x)=\begin{cases}2x, & 0<x<1\\ o, & \text{otherwise}\end{cases}$$ then median of X is…

A) $\frac{1}{2}$ B) $\frac{1}{\sqrt{2}}$

C) $\frac{1}{4}$ D) $\sqrt{2}$

27) A continuous r.v. X has pdf

$$f(x)=\begin{cases}6x(1-x) & 0<x<1\\ 0 & otherwise\end{cases}$$

Then mode of the r.v. X is…

A) 1/2 B) 6

C) 0 D) ¼

28) If p.d.f of continuous random variable X is f(x) = 0.25, if 0 < x < 4 then E(X) is…

A) 0.5 B) 2

C) 0 D) 4

29) A continuous r.v. X has mean zero. The expression E (X^2) is…

A) μ_3 B) Var(X)

C) E(X) D) None of these

30) If X and Y are two independent continuous r. v.'s with mean of X is 5 and mean of Y is 2 then E(XY) is…

A) 0 B) 7

C) 3 D) 10

31) For the following joint p.d.f. of bivariate continuous r.v. (X, Y) the value of k is…

$$f(x)=\begin{cases}4kxy & 0<x<1,\quad 0<y<1\\ 0 & otherwise\end{cases}$$

A) 4 B) 1

C) 2 D) 1/3

32) The joint cumulative distribution function of X and Y i.e. F(x,y) =…

A) $P(X = x, Y = y)$ B) $P(X \geq x, Y \geq y)$

C) $P(X \leq x, Y \leq y)$ D) $P(X \leq x, Y \geq y)$

33) If V(X) = V(Y) = Cov(X, Y) then r(X, Y) is -------

A) 1 B) V(X)

C) – 1 D) 1/(V(X)

34) If F(x, y) be the joint cumulative distribution function(c.d.f.) of X and Y then it lies in the interval…

A) [–1, 0] B) [0, 1]

C) [–1, 1] D) [0, –1]

35) For which of the following conditional expectations, the regression line of Y on X is linear ?

A) E(Y|X = x) = 1/ X B) E(Y|X = x) = 0.2 X + 3

C) E(Y|X = x) = 1/ Y D) none of these

36) If f(x) = 1, $5 < x < 6$ and f(y) = 1, $3 < x < 4$ then ------------

A) E(X) < E(Y) B) E(X) > E(Y)

C) E(X) = E(Y) D) none of these

37) If m.g.f. of independent continuous r.v.s X and Y is same and it is M(t) then m.g.f. of a r.v. X+Y is…

A) 2M(t) B) 0

C) 1 D) M(t).M(t)

38) If X and Y are two independent continuous r. v.'s then …

A) Covariance (X,Y) = 0 B) Correlation(X,Y) = 0

C) E(XY) = E(X).E(Y) D) All of these

39) For bivariate continuous r.v. (X,Y) which of the following is not true ?

A) Cov(X,Y) = Cov(Y, X) B) Cov(–X, –X) = Cov(X, X)

C) Cov(X,3) = Cov(3, Y) D) Cov(–X, –Y) = – Cov(X, Y)

40) For the following joint p.d.f. of bivariate continuous r.v. (X,Y) the value of c is -----

$$f(x) = \begin{cases} c & 0 < x < y < 1 \\ 0 & otherwise \end{cases}$$

A) 1 B) 4

C) 2 D) 1/3

41) If Var(X) = 1, Var(Y) = 9 and Cov(X, Y) = 1 then r(X, Y) is…

A) 1/3 B) 0

C) – 1 D) – 1/3

42) If E(Y|X = x) = X , then regression coefficient of Y on X is…

A) 0.5 B) 0.1

C) 1 D) 0

43) If $E[E(X/Y)]$ = 5 then …

A) E(X) = 5 B) E(Y) = 5

C) V(Y) = 5 D) V(X) = 5

44) If Joint p.d.f. of X and Y is f(x,y) = 3 – x – y; 0 < x < 1 ; 0 < y < 1 then marginal distribution of y is….

A) f(y) = 2.5 – y B) f(y) = y – 2.5

C) f(y) = 3 – y D) f(y) = 3

45) For the following joint p.d.f. of bivariate continuous r.v. (X, Y) the value of k is…

$$f(x) = \begin{cases} 8kxy & 0 < x < 1, \quad 0 < y < 1 \\ 0 & otherwise \end{cases}$$

A) 1 B) 4

C) 2 D) 1/2

46) Let (X, Y) be the bivariate continuous random variable has following joint pdf

f (x, y) = 1 ; 0 < x < 1; 0 < y < 1

= 0 ; Otherwise

Then P {0 < X < 0.5; 0.5 < Y < 1} will be

A) 0.25 B) 0.50

C) 0.75 D) 1

47) IF X and Y are independent continuous r.v.s, then…

A) E (Y|X) = E(X) B) E(X|Y) =E(Y)

C) V(Y|X) = V(X) D) E(Y|X) =E(Y)

48) If r.v. X has p.d.f. f(x) = $3x^2$, 0 < x < 1 then range of variable Y = 2X+3 is…….

A) (0,2) B) (2, 3)

C) (3,5) D) (0, 1)

49) If r.v. X has p.d.f. f(x) = $3x^2$ / 2, – 1< x < 1 then range of variable Y = X^2 is…….

A) (0,2) B) (–1, 1)

C) (0,1) D) (0, 3/2)

50) If (X,Y) be the bivariate continuous r.v.s with joint p.d.f. f(x,y) then joint p.d.f. of $U=g_1(x,y)$ and $V=g_2(x,y)$ is g(u,v) =……..

A) f(x).f(y) where x and y are in terms of u and v

B) f(x,y) where x and y are in terms of u and v

C) f(x,y)|J| where x and y are in terms of u and v

D) None of these.

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Long Answer Questions (10 Mark each)

1) Define Poisson distribution and find its mean and variance.

2) Define Poisson distribution. Find it's p. g. f., mean and variance.

3) Show that under certain conditions to be stated, Poisson distribution is limiting case of Binomial distribution

4) Define Negative binomial distribution and find its mean and variance.

5) Define Geometric distribution and find its mean and variance

6) For a univariate continuous r.v. X. Define

i) Mean ii) Mode iii) First Quartile

iv) Moment generating function v) r^{th} central moment

7) For a univariate continuous r.v. X. Define

i) Probability density function ii) Median iii) Harmonic mean

iv) Cumulant generating function v) r^{th} order raw moment

8) A continuous r. v. X has following p.d.f.

$$f(x)=\begin{cases} C.x(2-x) & 0<x<2 \\ 0 & otherwise \end{cases}$$

Find i) C ii) mean iii) variance iv) Mode

9) For a bivariate continuous r. v. (X, Y) show that

i) E(X +Y) = E(X) +E(Y)

ii) If X and Y are independent then E(XY) = E(X)E(Y)

10) A bivariate r. v. (X, Y) has joint pdf

$$f(x,y)=\begin{cases} 3-x-y & 0<x<1,\ 0<y<1 \\ 0 & otherwise \end{cases}$$

i) Find marginal distribution of X and Y

ii) Find E(X), E(Y) **iii)** E(XY) and Cov(X,Y)

11) A bivariate r. v. (X, Y) has joint pdf

$$f(x,y)=\begin{cases} xe^{-x(1+y)} & x\geq 0,\ y\geq 0 \\ 0 & otherwise \end{cases}$$

i) Find marginal p.d.f. of X and Y

ii) Find conditional p.d.f. of Y given X and E(Y / X)

iii) State whether the regression of Y on X is linear or not.

12) A bivariate r. v. (X, Y) has joint pdf

$$f(x,y)=\begin{cases} 4x(1-y) & 0<x<1,\ 0<y<1 \\ 0 & otherwise \end{cases}$$

i) Find marginal p.d.f. of X and Y

ii) Find conditional p.d.f. of X given Y and E(X / Y)

iii) Are X and Y are independent ?

13) The joint p. d. f of (X,Y) is

$$\text{f(x, y)} = \begin{cases} K\text{xy} & 0<x<1,\ 0<y<1 \\ 0 & o.w. \end{cases}$$

i) Find the value of constant K.

ii) Check whether X and Y are independent?

iv) Obtain E(X) and V(Y).

14) A bivariate r. v. (X, Y) has joint pdf

$$f(x,y)=\begin{cases} 1 & 0\leq x\leq 1,\ 0\leq y\leq 1 \\ 0 & otherwise \end{cases}$$

Find the distribution of U = XY

15) A bivariate r. v. (X, Y) has joint pdf

$$f(x,y)=\begin{cases} e^{-(x+y)} & 0\leq x<\infty,\ 0\leq y<\infty \\ 0 & otherwise \end{cases}$$

Find p.d.f. of U = (X +Y)/2

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Short Answer Questions (5 Mark each)

1) Define Poisson distribution and find its recurrence relation for probabilities.

2) State and prove additive property of Poisson distribution

3) If X and Y are two independent Poisson variates with parameter 2 and 3 respectively, Find P(X+Y < 2)

4) Define Geometric and find its recurrence relation for probabilities.

5) State and prove lack of memory property of geometric distribution.

6) Define Geometric distribution and find its p.g.f.

7) Define Geometric distribution and find its cumulative distribution function(c.d.f.)

8) Define Negative binomial distribution and find its recurrence relation for probabilities.

9) Find mean of Negative binomial distribution

10) Define moment generating function (m. g. f.) for univariate continuous r. v. and explain how to obtain raw moments from m.g.f.

11) Define cumulative distribution function (c.d.f.) of continuous univariate random variable and state its properties.

12) Define central moments and cumulants for continuous univariate random variable and state relation between them up to order four

13) Define moment generating function (m.g.f.) and state their properties

14) For a univariate continuous r.v. X. Define

i) Mean ii) Mode iii) First and Third Quartile

15) The following is the p.d.f of continuous r.v. X

$$f(x) = \begin{cases} \frac{3}{4} x(2-x) & 0 \le x \le 2 \\ 0 & otherwise \end{cases}$$

Find Mode of the r.v. X

16) Define probability density function (p.d.f) of continuous r.v. X and find mean for the following pdf.

$$f(x) = \begin{cases} \frac{3}{4} x(2-x) & 0 \le x \le 2 \\ 0 & otherwise \end{cases}$$

17) Define probability density function (p.d.f) of continuous r.v. X and verify following function is pdf.

$$f(x)=\begin{cases} x^2 & 0<x\le 1 \\ x(2-x) & 1\le x<2 \\ 0 & otherwise \end{cases}$$

18) For a bivariate continuous r.v.(X, Y), Show that E(X–Y)= E(X) – E(Y)

19) Define joint cumulative distribution function for a bivariate continuous r.v. (X,Y) and state their properties.

20) For a bivariate continuous r.v.(X, Y).

Define i) Conditional expectation ii) Conditional variance

21) For Bivariate continuous r. v. s (X, Y) define the terms

i) Marginal distribution of X.

ii) Conditional distribution of X given Y=y.

22) Let (X, Y) be continuous r. v., prove that E{E(X/Y)}=E(X).

23) The joint p.d.f of (X,Y) is

$$f(x,y)=\begin{cases} 1/4 & -1<x,y<1 \\ 0 & o.w \end{cases}$$

Then find i) P(X > 0 , Y > 0) ii) P(X >1/2)

24) A bivariate r. v. (X, Y) has joint pdf

$$f(x,y)=\begin{cases} 1 & 0\le x\le 1,\ \ 0\le y\le 1 \\ 0 & otherwise \end{cases}$$

Find marginal p.d.f. of X and Y

25) A continuous r. v. X has p.d.f.

$$f(x)=\begin{cases} \frac{x}{6} & 2\le x\le 4 \\ 0 & otherwise \end{cases}$$

Find the probability distribution of Y = X/2

26) A continuous r. v. X has p.d.f.

$$f(x)=\begin{cases} 2x & 0\le x\le 1 \\ 0 & otherwise \end{cases}$$

Find the probability distribution of Y = 3X+1

27) Let X be a r. v having p. d. f.

$$f(x)=\begin{cases} \frac{1}{2} & -1 \leq x \leq 1 \\ 0 & o.w. \end{cases}$$

If $Y=X^2$ then obtain i) p. d. f of Y ii) E(Y)

28) Let X be a r.v having p.d.f.

$$f(x)=\begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & o.w. \end{cases}$$

If Y = 1 – X then obtain i) p. d. f of Y ii) E(Y)

29) A continuous r. v. X has p.d.f.

$$f(x)=\begin{cases} \frac{1}{6} & -3 \leq x \leq 3 \\ 0 & otherwise \end{cases}$$

Find the probability distribution of $Y = X^2$

30) A continuous r. v. X has p.d.f.

$$f(x)=\begin{cases} 1/2 & -1 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

Find the probability distribution of $Y = X^2$

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