SHIVAJI UNIVERSITY, KOLHAPUR

Question Bank For Mar 2022 (Summer) Examination

Subject Code : 78912 Subject Name : Statistics Paper VII

Q.1. Select the correct alternative from the following: [10 x1]

1. If X U (a,b) then mean of the distribution is ……………

   a) $\frac{(b-a)}{2}$ b) $\frac{(a-b)}{2}$ c) $\frac{ab}{2}$ d) $\frac{(a+b)}{2}$

2. X ~ U (a,b) with mean 3 and variance 3 respectively then -------------.

   a) a=8, b=2 b) a=10, b=6 c) a=2, b=8 d) a=10, b=3

3. X ~ U (a,b) such that $P[|X| > 1] = \frac{6}{7}$ then value of a is ----------------.

   a) 7 b) 6 c) 14 d) 12

4. If X ~ U (0,1) then distribution of $Y = -\frac{1}{\theta}\log X$ is -------------------.

   a) uniform b) exponential c) gamma d) none of these.

5. X ~ U (-1,1) then value of P (X ≤ 0) is ----------------.

   a) 0.5 b) 1 c) 1/3 d) 0

6. If X ~ U (a,b) then distribution of $Y = \frac{X-a}{b-a}$ follows -------------------.

   a) uniform b) exponential c) gamma d) none of these.

7. Let X has U (0, 1) distribution. Then E (1/X) is equal to

   a) 1 b) 1 / 2 c) log (1/2) d) Does not exist.

8. Let X ~ U (0, 1) then distribution of $Y = -\lambda^{-1}\log(1-X)$, for λ> o is--------

   a) Exp. With Mean (1/λ) b) Exp. With mean λ
   c) Uniform over [0, 1/λ] d) Uniform over [0, λ]

9. X ~ U(-3, 2) then P { | X| ≤ 2}

   a) 2/5 b) 1/2 c) 4/5 d) 1

10. Let X~U(5, 10) , Y_1 = (X-5) / 5 and Y_2 = (10-X) / 5 then which of the following sentence is false ?

    a) Y_1 and Y_2 are identically distributed.
    b) Y_1~U (-1, 0) and Y_2~U(0, 1)
    c) Y_1 and Y_2 have both U (0, 1)
    d) Distribution of Y_1 and Y_2 is symmetric.

11. If X~F(x), any continuous distribution function then distribution of F(X) is ----------

    a) U(0, 1) b) Degenerated at 0
    c) Degenerated at 1 d) Discrete uniform over {0, 1}

12. Let E(Y/X=x) = 2x, V(Y/X=x) = $4x^2$. Suppose X ~ U (0, 1), what is V(Y)?

    a) 7/3 b) 5/3 c) 4/3 d) 1/3

13. If Y ~ Exp (θ) then distribution of Z = $e^{-\theta y}$ is ……..

    a) U (0, 1) b) U (-1, 1) c) Exp (1) d) Exp (θ)

14. The exponential distribution is ………

    a) Positively skewed and leptokurtic b) Positively slewed and platykurtic
    c) Negatively platyokurtic d) Negatively skewed and leptokurtic

15. Let X and Y be i.i.d exponential random variables with mean θ.

Let α : (X + Y) /2 is exponential r.v with θ.

β : (X + Y) is exponential r.v with θ.

a) Only α is true b) Only β is true c) Both α &β true d) Both α &β false

16. A r. v. X has an exponential distribution with mean 6, then P [X > 9 / X > 3] is----

a) $e^{-1.5}$ b) $e^{-0.5}$ c) $e^{-1.0}$ d) $e^{-6.0}$

17. If X follows exponential distribution with mean β then $E(e^{-(X/\beta)})$ is---------

a. a) 1 b) 1/2 c) 0 d) 2

18. If X ~Exp(θ) then P_r\{ X > (t+s) / X > t \} depends on ----------

a) all s, t & θ b) only on s & t c) only on s & θ d) only on t & θ

19. If X follows exponential distribution with mean $(1/\theta)$ then mgf of X is--------------.

a) $(1- t/\theta)^{-1}$ b) $(1- t/\theta)$ c) $(1-\theta /t)^{-1}$ d) $(1-\theta /t)$

20. If X ~ exp(1) then the distribution of $Y= e^{-X}$ is -----------.

a) exp (1) b) exp (1/2) c) U(0,1) d) U(0,2)

21. If X and Y are i.i.d. exponential variate with mean α then X+Y has -----------.

a) $G\left(\frac{1}{\alpha},2\right)$ b) $G\left(\frac{2}{\alpha},1\right)$ c) $G(\alpha,2)$ d) $G\left(\alpha,\frac{1}{2}\right)$

22. For standard exponential distribution, variance of the distribution is -----------.

a) 1 b) 0 c) 2 d) none of these

23. If X follows exponential distribution with mean $(1/\theta)$ then mgf of X is--------------.

a) $(1- t/\theta)^{-1}$ b) $(1- t/\theta)$ c) $(1-\theta /t)^{-1}$ d) $(1-\theta /t)$

24. If X ~ G(3, 8) and Y ~ G(3, 6) are independent gamma variates then the probability distribution of X/X+Y is-----------

a) β_1 (8, 6) b) β_1 (6, 8) c) β_2 (8, 6) d) β_2 (6, 8)

25. Let X is Gamma distributed with scale parameter α and shape parameter λ. For which of the following choice X follows an exponential distribution.

a) $\alpha = \lambda = 1$ b) $\alpha = 2$, $\lambda =1$ c) $\lambda = 1$ d) All of above

26. Let X ~ Gamma (3, 5), Y ~ Gamma (3, 6) are independently distributed. Then distribution of $\frac{X}{X+Y}$ is ---------

a) β_1 (5, 6) b) β_2 (5, 6) c) β_1 (6, 5) d) β_2 (6, 5)

27. Let X_1 ~ G (8, 2), X_2 ~ G (10, 5) be independent variables then distribution of $\frac{5X_2}{4X_1+5X_2}$ is ----

a) β_2 (2, 5) b) β_2(5, 2) c) β_1 (5, 2) d) β_1 (4, 5)

28. If independent random variables X and Y are distributed as G(1, 5) and G(1, 10) respectively. Define U = (X+Y) and V = X/(X+Y) then----------

a) U and V are also Independent
b) U ~G (1, 15) and V ~β_1(5, 10)
c) Only b) is true
d) Both a) and b) are true.

29. If X ~ G(1, θ) then it --------

a) is always positively skewed and laptykurtic
b) has positive skewness&/or leptykurtic that depending on value of θ.
c) is always negatively skewed and platykurtic
d) is always symmetric but its kurtosis depends on value of θ

30. If X ~ G (θ,n) then measure of kurtosis β_2 of X is --------------.
a) function of n only b) function of θ only
c)function of both n and θ d) independent both θ and n

31. Assume that X_1 and X_2 are independent random variable such that X_1 ~ G (6,9), X_2 ~ G (6,11) then distribution of X_1+X_2 is ------------.
a) G (10,6) b) G (6,20) c) G (20,6) d) G (12,20)

32. X $\sim G\left(\frac{1}{2},1\right)$ then probability distribution of $\frac{X}{2}$ is ------------.
a) $G\left(\frac{1}{2},\frac{1}{2}\right)$ b) $G\left(\frac{1}{2},1\right)$ c) $G\left(1,\frac{1}{2}\right)$ d) $G(1,1)$

33. If X ~ β_1 (m, n) then which of the following has β_1 (n, m) distribution?
a) 1/X b) 1-X c) X/ (1-X) d) None of these

34. If X ~ β_1(m,n) with mean 0.25 and variance =1/8 then values of m and n are respectively
a) 1/4, 3/8 b) 1/8, 3/8 c) 1/8, 3/4 d)1/2, 3/8.

35. Let X ~β_1(2, 3) and Y = (1 – X) then-----------
a) Y ~β_1(2, 3) b) Y ~β_2(2, 3) c) Y ~β_2(3, 2) d) Y ~β_1(3, 2)

36. If X ~ β_2 (3, 2) then E(1/X) is----------
a) 1 b) 3/2 c) 2/3 d) 1/E(X)

37. If X ~ β_1 (1,2), then mean of X is --------------.
a) 1/3 b) 2/3 c) 3/4 d) none of these

38. If X ~β_2 (m,n), then distribution of $\frac{1}{1+X}$ is ----------
a) $\beta_1(m,n)$ b) $\beta_2(m,n)$ c) $\beta_1(n,m)$ d) $\beta_2(n,m)$

39. If X ~ β_2 (m,n), then mode of X is --------------.
a) $\frac{m}{n+1}$ b) $\frac{m-1}{n+1}$ c) $\frac{n}{m+1}$ d) none of these

40. If X ~ β_1 (m,n), then it is symmetric at m = n = -------------.
a) 0 b) 1 c) 0.5 d) none of these

41. If the random variables X ~ N(0,1/2) and Y ~ N(0,1/2) are independently distributed. Then Z= $(X-Y)^2 + (X+Y)^2$ is
a) G (1/2, 1) b) G (1, 1) c) N(0,1) d) None of these.

42. If random variable X has N (a, b) then P(X<a) is-----------
a) 1 b) 0 c) 1/2 d) a/b

43. If X is a random variable having normal distribution with mean 1 and variance 4, then $P(X^2 \leq 2)$ is given by
a) $\emptyset\left(\frac{\sqrt{2}-1}{2}\right)-\emptyset\left(\frac{-\sqrt{2}-1}{2}\right)$ b) $\emptyset(\sqrt{2}-1)-\emptyset(-\sqrt{2}-1)$
c) $\emptyset(\sqrt{2})-\emptyset(-\sqrt{2})$ d) 1 - $[\emptyset(\sqrt{2})-\emptyset(-\sqrt{2})]$

44. Let X_i ~ N (i , 1), i = 1, 2. Define the events A_i = [i< X_i< 4]. Which of the following statements is correct?
a) $P(A_1) = P(A_2)$ b) $P(A_1) < P(A_2)$ c) $P(A_1) > P(A_2)$ d) None of these

45. If X~ N (a, b) then P [X < a] is equal to-----------
a) 1 b) 0 c) 0.5 d) a / b

46. If X and Y are independently distributed as N(0, 1/2) and N(0, 1/2) respectively then distribution of $Z = 2(X-Y)^2$ is -----------

a) Gamma (1/2, 1) b) Gamma (1, 1) c) N(0, 1) d) None of these

47. If X~N(5, 9) then it is--------------

a) Mesokurtic and positively skewed
b) Laptykurtic and negatively skewed
c) Platykurtic and symmetric
d) Mesokurtic and symmetric

48. Marks on a Statistics test follow a normal distribution with a mean of 65 and a standard deviation of 5. Approximately what percentage of the students have scores below 50?

a) 0 b) 3% c) 50% d) 97%

49. The mean of normal distribution is 50 then its mode is ----------

a) 25 b) 40 c) 50 d)) none of these

50. Suppose X_1 and X_2 are independent standard normal random variates. Pdf of $X_1 - X_2$ is -----

a) N(0,1) b) N(1,0) c) N(1,1) d) N(0,2)

51. If X and Y are i.i.d. N (μ, σ^2) then distribution of X-Y is ------------

a) Chi-square b) F c) Normal d) None of these

52. If X ~N (μ, σ^2) with $\mu_4 = 12$ then the value of σ is -----------

a) 4 b) 2 c) $\sqrt{2}$ d) 1

53. For normal distribution Q.D., M.D. & S.D. are in the ratio -------------.

a) 10:12:15 b) 15:12:10 c) 12:15:10 d) 10:15:12

54. If X has chi-square distribution with 8 d.f. then its mode is -------------

a) 3 b) 4 c) 5 d) 6

55. If X has chi-square distribution with 10 d.f. then its variance is -------------

a) 10 b) 20 c) 100 d) $\sqrt{10}$

56. Let X_i ~ N (0 , 1), i = 1, 2. Then Pdf of $\dfrac{(X_1 - X_2)^2}{2}$ is -----

a) $\chi^2_{(1)}$ b) $\chi^2_{(2)}$ c) N(0,1) d) N(0,2)

57. If X ~N (0,1) then the distribution of X^2 is -----------

a) χ^2 with 1 d.f. b) $G\left(\frac{1}{2}, \frac{1}{2}\right)$ c) both (a) and (b) d) none of these

58. If X has chi- square distribution with mode 12 then its m.g.f. $M_X(t)$ is ------------

a) $(1-2t)^{-6}$ b) $(1-2t)^{-12}$ c) $(1-2t)^{-7}$ d) $(1-2t)^{-10}$

59. If X has chi- square distribution with mode 13 then its variance is -------

a) 15 b) 30 c) 26 d) 22

60. If all odd ordered central moments are zero then the distribution may be..............

a) normal b) t c) normal or t d) none of these

61. The mean of t - distribution is -----------

a) 1 b) 0 c) n d) 2n

62. The coefficient of kurtosis γ_2 for t - distribution with 10 d.f. is -----

a) 1 b) 2 c) 4 d) 0

63. If X follows F-Distribution with (6, n) d.f. with mean 2 then the value of n is----------

a) 2 b) 3 c) 6 d) 4

64. If X ~F (n_1, n_2). If $n_2 \to \infty$, then $n_1 X$ has the--------------- distribution.

a) t(n_1) b) F (n_1, n_2) c) $\chi^2_{n_1}$ d) F (n_2, n_1)

65. If X ~F (n_1, n_2) then E (X) is..........

a) $\frac{n_1}{n_2-2}$ b) $\frac{n_2}{n_1-2}$ c) $\frac{n_2}{n_2-2}$ d) $\frac{n_1}{n_1+n_2}$

66. If X ~F (n_1, n_2). then 1/X follows --------------- distribution.

a) t(n_1) b) F (n_1, n_2) c) $\chi^2_{n_1}$ d) F (n_2, n_1)

67. F distribution is invented by --------------------

a) G. W. Snedecor b) R.A. Fisher c) W.S.Gosset d) None of these

68. If X ~$t_{(n)}$ then X^2 follows --------------- distribution.

a) $t_{(n)}$ b) F(1,n) c) F(n,1) d) F(1,1)

69. If X ~ N (0,1) and Y follows χ^2 with 4 d.f. then $\frac{X}{\sqrt{Y/4}}$ follows --------------- distribution.

a) t b) F c) χ^2 d) Normal

70. If X ~ N (0,1) and Y follows χ^2 with 4 d.f. then $\frac{X^2}{Y/4}$ follows --------------- distribution.

a) t b) F c) χ^2 d) Normal

Q2) Attempt any TWO of the following. **[2 x 10]**

1. Define Uniform distribution. Also find its γ_1 and γ_2.
2. If X ~ U (a,b) then find the distributions of i) (X - a) / (b - a) & ii) (b -X) / (b - a).
3. Define exponential distribution with parameter ɵ. Also find its mean and quartiles.
4. Define exponential distribution. Also state and prove its lack of memory property.
5. If X ~ Exp (θ) then find its first four cumulant and hence find the γ_1 and γ_2.
6. Define gamma distribution. Also find its mean and variance.
7. If X ~ β_1 (m,n), then find its mean and variance.
8. If X ~ β_2 (m,n), then find its mean and variance.
9. Define beta distribution of first kind and beta distribution of second kind. Also State and prove relation between these two distributions.
10. Find distribution of X/ (X+Y) if X and Y are independent gamma variates.
11. Find distribution of X/ Y if X and Y are independent gamma variates.
12. Find mgf and cgf of normal distribution. Also find first four cumulants.
13. Define Normal distribution also show that its mean and median are equal.
14. Find recurrence relation in terms of even order central moments for Normal distribution.
15. Define Normal distribution. Also find distribution of linear combination of normal variates.
16. Derive pdf of Chi-square distribution.
17. Define χ^2 distribution and find its coefficient of skewness (β_1) & coefficient of kurtosis (β_2).
18. Derive pdf of t- distribution.
19. Find mean and variance of t-distribution.
20. Define t- distribution. Also find its mode.
21. Define t- distribution and find its coefficient of skewness (β_1) & coefficient of kurtosis (β_2).
22. Derive pdf of F- distribution.
23. Find mean and variance of F-distribution.
24. Define F- distribution. Also find its mode.

Q3) Attempt any Four of the following. **[4 x 5]**

1. Find mean and variance of uniform distribution.
2. If X ~ U (a,b) then find its mgf.
3. If X ~ U (a,10), such that P(3 < X < 7) = 0.5 then find 'a'.
4. Find distribution of (-1/ɵ)logX if X ~U(0,1).
5. If X ~ Exp (θ) then find its median.
6. Find distribution of sum of i.i.d. exponential variates.
7. Define gamma distribution. Also find its mode.
8. State and prove additive property of Gamma distribution.
9. If X_i ~ G (2, i), i = 1,2,3,4,5,6 are i.i.d random variates then find E(Y) & V(Y), where $Y = \sum_{i=1}^{4} X_i$.
10. If X ~ β_1 (m,n), then find its mode.
11. If X ~ β_1 (m,n) then show that it is symmetric distribution at m = n.
12. If X ~ β_2 (m,n), then find its mode.
13. If X ~ β_1 (m,n) then show that $X/(1-X) \sim \beta_2$ (m,n).
14. If X ~ β_2 (m,n) then show that $1/(1+X) \sim \beta_1$ (n,m).
15. Define standard normal distribution and also state properties of normal curve.
16. Define standard normal distribution and also state its mean, variance, median, Q.D., μ_5 & γ_1.
17. Find M.D. about mean of normal distribution.
18. Find Q.D. of normal distribution.
19. Find mgf and cgf of normal distribution.
20. State and prove additive property of normal distribution.
21. If X ~N(0,1) then find distribution of X^2.
22. If X_i ~ N (2, i), i = 1,2,3,4,5,6 are independent random variates then find mode and variance of Y, where $Y = 4X_2 + 2X_3 - X_6 + 3$.
23. Find mean and variance of Chi-square distribution.
24. Define Chi-square distribution. Also find its mode.
25. Find mgf and cgf of Chi-square distribution.
26. State and prove additive property of Chi-square distribution.
27. X ~$t_{(n)}$, then with usual notations show that $\mu'_{2r+1} = \mu_{2r+1} = 0 \qquad r = 0,1,2,...,n$
28. If X ~F(n_1, n_2), then show that 1/X follows F(n_2, n_1).
29. If X_i ~ N (0, 1), i = 1,2,3,4,5 are independent random variates then find distribution of Y, where $Y = \left(5X_2^2\right) \Big/ \left(\sum_{i=1}^{5} X_i^2\right)$.
30. State the inter relation between normal, t, F and χ^2 distributions.

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