Shivaji University, Kolhapur
Question Bank For Mar 2022 (Summer) Examination

Subject Code: 79699 **Subject Name: Statistics Paper X**

(Statistical Inference – I)

Q1) Choose the most correct alternative.

1. The process of making estimates about the population parameter from a sample is called......
   A) Statistical independence B) Statistical inference
   C) Statistical hypothesis D) Statistical decision

2. An estimator is a random variable because it varies from......
   A) Population to sample B) Population to population
   C) Sample to sample D) Sample to population

3. The point estimator of population mean μ is......
   A) Sample mean B) Sample variance
   C) Sample standard deviation D) Sample size

4. Standard error is the standard deviation of the sampling distribution of ......
   A) Estimate B) Estimation
   C) Estimator D) Error of estimation

5. A one-dimensional statistic that best estimates the parameter of distribution is -----------
   A) Interval estimator B) Point estimator
   C) constant D) None of the above

6. If a random variable X has N (θ, θ^2) distribution then the parameter space of the distribution is -----------
   A) $\{\theta \mid \theta < 0\}$ B) $\{\theta \mid -\infty < \theta < \infty\}$
   C) $\{\theta \mid \theta > 0\}$ D) None of the above

7. An estimator $\hat{\theta}$ is a negatively biased estimator of a parameter θ if -----------
   A) $E(\hat{\theta}) = 0$ B) $E(\hat{\theta}) < \theta$.
   C) $E(\hat{\theta}) > \theta$ D) None of the above

8. If $X_1, X_2, X_3, \ldots, X_n$ is a random sample of size 'n' from U (0, θ) then with usual notations, unbiased estimator of θ is -----------
   A) $\bar{X}$ B) $\bar{X}/2$
   C) $2\bar{X}$ D) Maximum of $\{X_1, X_2, X_3, \ldots, X_n\}$

9. If $X_1, X_2, X_3, \ldots, X_n$ is a random sample of size 'n' from Bernoulli Distribution with parameter p with $T=\Sigma X_i$ then the unbiased estimator of p(1-p) is -----------
   A) T(n-T)/(n(n-1)) B) T/n
   C) T/[n(n-1)] D) T(T-1)/(n-1)

10. With usual notations, an unbiased estimator for the parameter θ based on a random sample of size ‘n’ from Exponential Distribution with mean θ is -----------

A) Sample mean B) $nX_{(1)}$

C) both A) and B) D) None of the above

11. If $\hat{\theta}$ is the estimator of the parameter θ, then $\hat{\theta}$ is called unbiased if......

A) $E(\hat{\theta}) < \theta$ B) $E(\hat{\theta}) > \theta$

C) $E(\hat{\theta}) \neq \theta$ D) $E(\hat{\theta}) = \theta$

12. Which of the following is biased estimator?

A) $\frac{\sum X}{n}$ B) $\frac{X}{n}$

C) $\frac{\sum(X-\bar{X})^2}{n-1}$ D) $\frac{\sum(X-\bar{X})^2}{n}$

13. If $T = t(X_1, X_2, \dots, X_n)$ is an unbiased estimator of θ, then $\varphi(T)$ is an unbiased estimator of $\varphi(\theta)$ iff ……

A) $\varphi(.)$ is a cubic function. B) $\varphi(.)$ is a quadratic function.

C) $\varphi(.)$ is a linear function. D) $\varphi(.)$ is a many to one function.

14. If T is a sample mean based on a random sample $X_1, X_2, \dots, X_n$ of size n from Binomial Distribution with parameters m and θ, then …… is an unbiased estimator of θ^2 for known m.

A) $\frac{T(T-1)}{m(mn-1)}$ B) $\frac{T(nT-1)}{m(n-1)}$

C) $\frac{T(nT-1)}{m(mn-1)}$ D) $\frac{T(nT-1)}{m(m-1)}$

15. If T is a sample mean based on a random sample $X_1, X_2, \dots, X_n$ of size n from Bernoulli Distribution with parameter θ, then ……is an unbiased estimator of $\theta(1-\theta)$.

A) $\frac{T(1-T)}{n-1}$ B) $\frac{nT(1-T)}{n-1}$

C) $\frac{n(1-T)}{n-1}$ D) $\frac{1-T}{n-1}$

16. ……is an unbiased estimator of σ when a random sample $X_1, X_2, \dots, X_n$ of size n is drawn from Normal Distribution with parameters μ and σ^2.

A) $\sqrt{\pi/2}\ \frac{\sum|X_i-\mu|}{n}$ when μ is known. B) $\sqrt{2/\pi}\ \frac{\sum|X_i-\mu|}{n}$ when μ is known.

C) $\sqrt{\pi}\ \frac{\sum|X_i-\mu|}{n}$ when μ is known. D) $\sqrt{\pi/2}\ \frac{\sum|X_i|}{n}$.

17. If $X_1, X_2, \dots, X_n$ is a random sample of size n from Normal Distribution with parameters μ and σ^2, then unbiased estimators of μ and σ^2 are……

A) sample mean and sample mean square respectively.

B) sample median and sample mean square respectively.

C) sample mean and sample median respectively.

D) sample mode and sample mean square respectively.

18. If $X_1, X_2, \ldots., X_n$ is a random sample of size n from Poisson Distribution with parameter θ, then unbiased estimator of θ is……

A) sample mean.
B) sample mean square.
C) linear combination of sample mean and sample mean square.
D) all of the above.

19. If T is a sample mean based on a random sample $X_1, X_2, \ldots., X_n$ of size n from Exponential Distribution with parameter θ, then ……is an unbiased estimator of $1/\theta^2$.

A) $\frac{nT^2}{n-1}$
B) $\frac{nT^2}{n+1}$
C) nT^2
D) $\frac{T^2}{n+1}$

20. If T is a sample mean based on a random sample $X_1, X_2, \ldots., X_n$ of size n from Normal Distribution with mean μ and variance 1, then unbiased estimator of μ^2 is…….

A) T^2
B) $T^2 + \frac{1}{n}$
C) $T^2 - \frac{1}{n}$
D) $T^2 - \frac{2}{n}$

21. If T is a sample mean based on a random sample $X_1, X_2, \ldots., X_n$ of size n from Exponential Distribution with mean θ, then ……is an unbiased estimator of $\theta(1-\theta)$.

A) $\frac{T}{n+1}$
B) $\frac{nT}{n+1}$
C) $T(n - \frac{nT}{n+1})$
D) $T(1 - \frac{(n+1)T}{n})$

22. With usual notations, which one of the following is correct for a biased estimator $\hat{\theta}$ of the parameter θ?

A) MSE($\hat{\theta}$) = SD($\hat{\theta}$) + Bias
B) MSE($\hat{\theta}$) = Var($\hat{\theta}$) + Bias^2
C) MSE($\hat{\theta}$) = Var($\hat{\theta}$) + Bias
D) MSE($\hat{\theta}$) = SD($\hat{\theta}$) + Bias^2

23. If $\{T_n\}_{n \geq 1}$ is a consistent sequence of estimators based on a random sample of size 'n', for parameter θ, then which of the following is true for large n?

A) $P(|T_n - \theta| > \varepsilon) \rightarrow 0$
B) $P(|T_n - \theta| \leq \varepsilon) \rightarrow 1$
C) $T_n \rightarrow \theta$
D) All of the above

24. Sample mean is always ----------- estimator of population mean.

A) Unbiased
B) Consistent
C) Unbiased and consistent
D) None of the above

25. If $\hat{\theta}$ is an unbiased estimator based on a random sample of size 'n', for the parameter θ, with Var($\hat{\theta}$)→0 as n→∞, then $\hat{\theta}$ is said to be -----------

A) Unbiased
B) Sufficient
C) Efficient
D) Consistent

26. If T_n is unbiased and consistent estimator based on a random sample of size 'n', for the parameter θ, then T_n^2 is ----------- for θ^2.

A) unbiased and consistent
B) biased and consistent
C) unbiased and inconsistent
D) biased and inconsistent

27. With usual notations, if $E(\hat{\theta}) \to \theta$ and $Var(\hat{\theta}) \to 0$ as $n \to \infty$, then $\hat{\theta}$ is …… for θ.

A) Unbiased
B) Consistent
C) Sufficient
D) Efficient

28. If $T = t(X_1, X_2, \ldots, X_n)$ is a consistent estimator of θ, then $\varphi(T)$ is consistent estimator of $\varphi(\theta)$ if …...

A) $\varphi(.)$ is a constant function.
B) $\varphi(.)$ is only a quadratic function.
C) $\varphi(.)$ is any continuous function.
D) $\varphi(.)$ is only a linear function.

29. If $X_1, X_2, \ldots, X_n$ is a random sample of size n from continuous Uniform Distribution over $(0, \theta)$, then a consistent estimator of θ/e ,where e is universal constant, is …...

A) Sample mean.
B) Sample geometric mean.
C) Sample harmonic mean.
D) Sample mean square.

30. If $X_1, X_2, \ldots, X_n$ is a random sample of size n from Geometric Distribution with parameter θ then a consistent estimator of θ is …...

A) Sample mean.
B) 1+ Sample mean.
C) 1-Sample mean.
D) (1+ Sample mean)$^{-1}$.

31. …… is a consistent estimator for θ based on a random sample $X_1, X_2, \ldots, X_n$ of size n from the following distribution.

$$f(x;\theta) = \begin{cases} \theta\, x^{\theta-1}; 0 < x < 1,\ \theta > 0. \\ 0; \qquad\qquad otherwise \end{cases}$$

A) sample mean/(1-sample mean).
B) 1/(1-sample mean).
C) Sample mean
D) 1/sample mean.

32. If $X_1, X_2, \ldots, X_n$ is a random sample of size n from Normal Distribution with mean μ and variance μ^2, then unbiased and consistent estimators of μ and μ^2 are……

A) sample mean and sample median respectively.
B) sample median and sample variance respectively.
C) sample mean and sample mode respectively.
D) sample mean and sample mean square respectively.

33. If $X_1, X_2, \ldots, X_n$ is a random sample of size n from Exponential Distribution with parameter θ, then ……is a consistent estimator of θ.

A) sample mean
B) sample variance
C) 1/ sample mean
D) sample mean square

34. If $X_1, X_2, \ldots, X_n$ is a random sample of size n from continuous Uniform Distribution over $(\theta, \theta + 1)$, then consistent and unbiased estimator of θ is …...

A) Sample mean.
B) Sample mean + 0.5.
C) Sample mean - 0.5.
D) Sample mean square.

35. If $X_1, X_2, \ldots, X_n$ is a random sample of size n from Exponential Distribution with location parameter θ, then ……is a consistent estimator of θ.

A) sample variance
B) sample mean - 1
C) 1/ sample mean
D) sample mean + 1

36. For Normal distribution, sample mean is -----------than sample median for large samples.

A) More efficient
B) Less efficient
C) Either (i) or (ii)
D) None of the above

37. If T_1 and T_2 are two unbiased estimators of the parameter θ then T_1 is more efficient than T_2 if -----------

A) $V(T_1) = V(T_2)$
B) $V(T_1)>V(T_2)$
C) $V(T_1) < V(T_2)$
D) None of the above

38. Mean square Error of an estimator T for the parameter θ is -----------

A) $E(T - \theta)^2$
B) $E(T^2) - \theta^2$
C) $E(T - 2\theta)^2$
D) $E(T^2) + \theta^2$

39. Mean squared error of an estimator T_n of the parameter $\tau\ (\theta)$ is ----------

A) bias + Var_θ (T_n)
B) [bias + Var_θ (T_n)]^2
C) (bias)^2 + [Var_θ (T_n)]^2
D) (bias)^2 + Var_θ(T_n)

40. Relative efficiency of an estimator with respect to the most efficient estimator always lies between……

A) 0 and 1
B) -1 and 1
C) -1 and 0
D) 0.5 to 1

41. If T_1 is a MVUE for parameter θ and T_2 is any other unbiased estimator for parameter θ with relative efficiency 'e', then the correlation coefficient between T_1 and T_2 is …….

A) e + 0.1
B) e - 1
C) $e^{0.5}$
D) $e^{1/3}$

42. If T_1 and T_2 are two unbiased estimators of parameter θ with relative efficiencies 'e_1' and 'e_2' respectively, then the correlation coefficient between T_1 and T_2 lies between …….

A) $\sqrt{e_1 e_2}\ \pm \sqrt{(1+e_1)(1-e_2)}$
B) $\sqrt{e_1 e_2}\ \pm \sqrt{(1-e_1)(1-e_2)}$
C) $\sqrt{e_1 e_2}\ \pm \sqrt{(1-e_1)(1+e_2)}$
D) $\sqrt{e_1 e_2}\ \pm \sqrt{(1+e_1)(1+e_2)}$

43. If T is the most efficient estimator of parameter θ and T_1 is less efficient estimator with relative efficiency 'e', then which one of the following is true?

A) $Var(T- T_1) = (1-e)Var(T)/e$
B) $Var(T- T_1) > (1-e)Var(T)/e$
C) $Var(T- T_1) < (1-e)Var(T)/e$
D) $Var(T- T_1) \neq (1-e)Var(T)/e$

44. If $s^2 = \frac{\sum(X_i-\bar{X})^2}{n}$ and $S^2 = \frac{\sum(X_i-\bar{X})^2}{n-1}$ are sample variance and sample mean square based on a random sample $X_1, X_2, \dots, X_n$ of size n from Normal Distribution with parameters μ and σ^2, then relative efficiency of S^2 with respect to s^2 is…….

A) $(1+\frac{1}{n})^{-2}$
B) $(1+\frac{1}{n})^{2}$
C) $(1-\frac{1}{n})^{-2}$
D) $(1-\frac{1}{n})^{2}$

45. If T_1 and T_2 are two unbiased estimators of parameter θ having same variance and relative efficiencies 'e', then the correlation coefficient between T_1 and T_2 is…….

A) 2e + 1
B) e - 1
C) 2e – 1
D) $e^{1/2}$

46. The sufficient estimator of the parameter λ of Poisson distribution based on a sample X_1, X_2, X_3 is given by -----------

A) $X_1 + 2X_2 + X_3$
B) $X_1 + X_2 + X_3$
C) $X_1 + X_2 + 2X_3$
D) $2X_1 + X_2 + X_3$

47. If X_1, X_2, -----, X_n is a random sample of size 'n' from U(-θ, θ) then with usual notations, sufficient statistic for parameter θ is -----------

A) $\bar{X}$
B) $(X_{(1)}, X_{(n)})$
C) $X_{(n)}$
D) None of the above

48. If X_1, X_2, X_3,……..,X_n is random sample of size 'n' from N $(0, \sigma^2)$, then sufficient statistic for σ^2 is -----------

A) $\sum_{i=1}^{n} X_i$
B) $\sum_{i=1}^{n} X_i^2$
C) $(\sum_{i=1}^{n} X_i)^2$
D) None of the above

49. With usual notations, a statistic T is said to be sufficient for parameter θ if -----------

A) $I_T(\theta) = nI(\theta)$
B) $f(x_1, x_2, \ldots, x_n \mid T = t) = h(x_1, x_2, \ldots x_n)$
C) $f(x_1, x_2, \ldots, x_n) = h(x_1, x_2, \ldots x_n)\, g(T, \theta)$
D) All of the above

50. If X_1, X_2, ……, X_n is a random sample of size 'n' from a Geometric Distribution with parameter 'p' then a sufficient statistic for p is --------

A) ΣX_i
B) Sample mode
C) Sample Maximum
D) Sample minimum

51. If X is a random sample of size one from a Normal Distribution population with mean 0 and variance σ^2 then sufficient statistic for σ^2 is -----------

A) X^3
B) X
C) X^2
D) None of the above

52. If the conditional distribution of $X_1, X_2, \ldots, X_n$ given $T = t$, does not depend on parameter θ, for any value of $T = t$, then the statistics T based on $X_1, X_2, \ldots, X_n$ is called......

A) Unbiased
B) Consistent
C) Sufficient
D) Efficient

53. With usual notations, if $f(x_1, x_2, \ldots, x_n; \theta) = g(\hat{\theta}, \theta)\, h(x_2, \ldots, x_n)$, then $\hat{\theta}$ is ……for θ.

A) Unbiased
B) Consistent
C) Sufficient
D) Efficient

54. If $X_1, X_2, \ldots, X_n$ is a random sample of size n from Bernoulli Distribution with parameter θ, then ……is a sufficient statistic for θ.

A) sample mean
B) sample minimum
C) sample median
D) sample maximum

55. If $X_1, X_2, \ldots, X_n$ is a random sample of size n from Geometric Distribution with parameter θ, then ……is a sufficient statistic for θ.

A) sample mode
B) sum of all sample observations
C) sample median
D) sample maximum

56. If $X_1, X_2, \dots, X_n$ is a random sample of size n from Exponential Distribution with location parameter θ, then ……is a sufficient statistic for θ.

A) sample mean
B) sample variance
C) sample minimum
D) sample maximum

57. If $X_1, X_2, \dots, X_n$ is a random sample of size n from Normal Distribution with parameters μ and σ^2, then ……are jointly sufficient for μ and σ^2.

A) sample mean and sample median
B) sample mean and sample minimum
C) sample minimum and sample maximum
D) sample mean and sample variance

58. If $X_1, X_2, \dots, X_n$ is a random sample of size n from continuous Uniform Distribution over (α, β), then ……are jointly sufficient for α and β.

A) sample mean and sample median
B) sample mean and sample minimum
C) sample minimum and sample maximum
D) sample mean and sample variance

59. If $X_1, X_2, \dots, X_n$ is a random sample of size n from Poisson distribution with parameter θ, then information contained in sample about parameter θ is………

A) $\frac{1}{\theta}$
B) $\frac{n}{\theta}$
C) $\frac{n}{\bar{X}}$
D) $\frac{\theta}{n}$

60. If $X_1, X_2, \dots, X_n$ is a random sample of size n from Bernoulli Distribution with parameter θ then information contained in sample about parameter θ is………

A) $\frac{1}{\theta(1-\theta)}$
B) $\frac{n}{\theta(1-\theta)}$
C) $\frac{n}{\bar{X}(1-\bar{X})}$
D) $\frac{\theta(1-\theta)}{n}$

61. If information contained in sample is same as information contained in the statistic based on that sample, then that statistic is called as ………

A) consistent
B) unbiased
C) efficient
D) sufficient

62. If $X_1, X_2, \dots, X_n$ is a random sample of size n from U(0, θ) distribution then the information contained in sample about parameter θ is………

A) nθ
B) θ
C) 0
D) 1

63. Information about parameter θ contained in a random sample $X_1, X_2, \dots, X_n$ of size n with the distribution $f(x, \theta)$, $\theta \in \Theta$ and likelihood function L is ………

A) $E\left[-\frac{\partial log f}{\partial \theta}\right]$
B) $E\left[-\frac{\partial log L}{\partial \theta}\right]$
C) $E\left[\frac{\partial^2 log f}{\partial \theta^2}\right]$
D) $E\left[-\frac{\partial^2 log L}{\partial \theta^2}\right]$

64. If nI_θ and $I_\theta(T)$ are the information functions about parameter θ with respect to sample and sufficient statistic T then ……….

A) $nI_\theta = I_\theta(T)$
B) $nI_\theta \leq I_\theta(T)$
C) $nI_\theta \geq I_\theta(T)$
D) $nI_\theta = I_\theta(T) = 0$

65. If $X_1, X_2, \dots, X_n$ is a random sample of size n from Geometric distribution with parameter θ, then information contained in sample about parameter θ is……….

A) $\frac{1}{\theta(1-\theta)}$
B) $\frac{n}{\theta(1-\theta)}$
C) $\frac{n}{\theta^2(1-\theta)}$
D) $\frac{1}{\theta^2(1-\theta)}$

66. The Cramer-Rao inequality provides ………. for the variance of an unbiased estimator T of θ.

A) Upper bound
B) lower bound
C) both A) and B)
D) none of the above

67. With usual notations, a statistic T is said to be a minimum variance bound unbiased estimator of $\Phi(\theta)$ if ……….

A) $V(T) < \frac{\Phi'(\theta)}{nI_\theta}$
B) $V(T) > \frac{\Phi'(\theta)}{nI_\theta}$
C) $V(T) = \frac{[\Phi'(\theta)]^2}{nI_\theta}$
D) $V(T) > \frac{\Phi'(\theta)}{I_\theta}$

68. The amount of information contained in a random sample $X_1, X_2, \dots, X_n$ of size n about the parameter θ for Exponential Distribution with parameter θ is……

A) $n\theta^2$
B) n/θ^2
C) $n\theta$
D) n/θ

69. The amount of information contained in a random sample $X_1, X_2, \dots, X_n$ of size n about the parameters μ and σ^2 for Normal Distribution with parameters μ and σ^2 are respectively ……

A) $n\sigma^2$ and $n\sigma^4$
B) n/σ^2 and $n\sigma^4$
C) $n\sigma^2$ and n/σ^4
D) n/σ^2 and $n/(2\sigma^4)$

70. With usual notations, if $Var(T) \geq \frac{(\varphi'(\theta))^2}{nVar(\frac{\partial}{\partial\theta}log f(x,\theta))}$ where T is an unbiased estimator of $\varphi(\theta)$ then above inequality is called as ……

A) Cauchy Schwarz Inequality
B) Bool's Inequality
C) Chebyshev's Inequality
D) Cramer Rao Inequality

71. If T is MVBUE of θ then which of the following is true?

A) T is consistent for θ
B) T is sufficient for θ
C) T is efficient for θ
D) all of the above

72. If $X_1, X_2, \dots, X_n$ is a random sample of size n from Exponential distribution with parameter θ then …….is MVBUE of 1/θ.

A) Sample mean
B) sample variance
C) sample median
D) sample mode

73. If $X_1, X_2, \dots, X_n$ is a random sample of size n from Geometric distribution with parameter θ, then sample mean is MVBUE of ………

A) θ
B) 1-θ
C) $\frac{1-\theta}{\theta}$
D) $\frac{\theta}{1-\theta}$

74. An estimator T is called as minimum variance bound unbiased estimator for parameter $\varphi(\theta)$ if……

A) it is unbiased for $\varphi(\theta)$ and it has smallest variance
B) it is unbiased for $\varphi(\theta)$ and it attains Cramer – Rao lower bound
C) it is unbiased for $\varphi(\theta)$ and it has maximum variance
D) it is biased for $\varphi(\theta)$ and it has smallest variance

75. If an estimator T is sufficient and unbiased for parameter $\varphi(\theta)$, then it is also called as……

A) efficient estimator
B) consistent estimator
C) minimum variance bound unbiased estimator
D) moment estimator

76. If $X_1, X_2, \dots, X_n$ is a random sample of size n from Binomial Distribution with parameters m and θ, then sample mean is MVBUE for ……

A) $m\theta$
B) θ
C) m
D) m/θ

77. If $X_1, X_2, \dots, X_n$ is a random sample of size n from Poisson Distribution with parameter θ, then sample mean is MVBUE for ……

A) $n\theta$
B) θ
C) n/θ
D) $1/\theta$

78. Maximum Likelihood estimator of the parameter θ based upon a random sample of size n from Geometric distribution is ……

A) sample total
B) sample mean
C) 1 + sample mean
D) $(1 + \text{sample mean})^{-1}$

79. ….... is Maximum Likelihood estimator of the parameter θ when a random sample of size n is drawn from Binomial distribution with parameters m and θ, where m is known.

A) sample total
B) sample mean
C) sample mean $/m$
D) $(m + \text{sample mean})^{-1}$

80. MLE of the parameter θ based upon a random sample of size n from rectangular distribution over $(0, \theta)$ is ……

A) sample median
B) sample maximum
C) sample mean
D) $(1 + \text{sample mean})^{-1}$

81. Moment estimator of the parameter θ based upon a random sample of size n from the distribution having p.d.f.

$$f(x,\theta) = \begin{cases} e^{-(x-\theta)}, & x > \theta, \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

is ……

A) sample mean
B) sample mean - 1
C) 1 + sample mean
D) 1 - sample mean

82. …… are moment estimators of the parameters μ and σ^2 respectively when a random sample of size n is drawn from $N(\mu, \sigma^2)$ distribution.

A) sample mean and sample median
B) sample mean and sample variance
C) sample mean and sample mode
D) sample mean and sample total

83. …… are moment estimators of the parameters α and β respectively when a random sample of size n is drawn from rectangular distribution over (α, β).

A) sample mean $-\sqrt{3 \times \text{sample variance}}$ and sample mean $+\sqrt{3 \times \text{sample variance}}$
B) sample mean $-$ 3 $\times$ sample variance and sample mean $+$ 3 $\times$ sample variance
C) sample mean $-$ sample variance and sample mean $+$ sample variance
D) sample mean $-$ sample variance^2 and sample mean $+$ sample variance^2

Q2) Long answer questions (8 marks each)

1. State Cramer Rao Inequality. Let $X_1, X_2, \dots, X_n$ be a random sample of size n taken from $N(\theta, \theta)$ distribution. Obtain unbiased and efficient estimator of θ.

2. Explain the method of maximum likelihood for estimating the parameter. Obtain moment estimators of parameters α and β of Gamma distribution based on a sample of size n drawn from it.

3. Define the following terms:
   1. Likelihood function and unbiased Estimator.
   2. Minimum variance unbiased Estimator.
   3. Consistent Estimator.
   4. Sufficient estimator through Neyman factorization criterion.

4. Explain the method of moments for estimation. If $X_1, X_2, X_3, \dots, X_n$ is a random sample of size n from a population having Gamma distribution with parameters α and β, estimate α, β by the method of maximum likelihood.

5. State and prove Cramer Rao inequality. State the condition of equality sign in Cramer Rao inequality.

6. Explain the concept of unbiasedness and consistency of an estimator. If $X_1, X_2, X_3, \dots, X_n$ is a random sample of size n from a population with p.d.f.

$$f(x, \theta) = \begin{cases} 1 & \theta < x < \theta + 1 \\ 0 & \text{otherwise} \end{cases}$$

then show that the sample mean is an unbiased and consistent estimator of $\theta + 1/2$.

7. Define the following terms:

   i) Parameter space
   ii) Estimator and estimate
   iii) Point estimation
   iv) Statistic
   v) Unbiased estimator.

8. Describe the concept of sufficiency. State and prove any two properties of sufficient statistic.

9. Define sufficient statistic. If $X_1, X_2, X_3, \dots, X_n$ is a random sample from $U(0, \theta)$ distribution, then
   (i) Find sufficient statistic for θ.
   (ii) Show that $T_1 = 2\bar{X}$ and $T_2 = \left(\frac{n+1}{n}\right) X_{(n)}$ are unbiased estimators of θ.
   (iii) Find relative efficiency of T_2 with respect to T_1.

10. Define Fisher information function and state Cramer Rao inequality. Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from $N(0, \sigma^2)$ distribution. Find the lower bound for the variance of an unbiased estimator of σ^2.

11. Describe the method of maximum likelihood of estimation. If $X_1, X_2, X_3, \dots, X_n$ is a random sample from rectangular distribution over (a, b) then find moment estimators of a and b.

12. Define consistent estimator and Neyman's Factorization criteria for sufficiency. Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Discuss the unbiasedness and consistency of the following estimators for the parameter μ.

    i) $T_1 = \frac{2\sum X_i}{n(n+1)}$ ii) $T_2 = \frac{n\bar{X}}{n+1}$

13. If $I(\theta)$ and $I_n(\theta)$ (θ) are Fisher's information about parameter θ contained in single observation and in n-observations respectively, then show that

i) $I_n(\theta) = nI(\theta)$ ii) $I_n(\theta) = E\left(-\frac{\partial^2}{\partial\theta^2} logL\left(\theta|\underline{x}\right)\right)$

where $L\left(\theta|\underline{x}\right)$ is a likelihood function of θ.

14. Define minimum variance unbiased estimator. Find the moment estimator and MLE of θ based on a sample of size n from the following distribution,

$$f(x,\theta) = \begin{cases} \frac{1}{2} e^{-|x-\theta|}, & -\infty < (x,\theta) < \infty \\ 0 & \text{otherwise} \end{cases}$$

15. Define the following terms:
    i) Standard Error of a statistic
    ii) Mean Square Error
    iii) Likelihood function
    iv) Fisher information function
    v) Relative efficiency.

16. Define consistent and sufficient estimator. If $X_1, X_2, X_3, \dots, X_n$ is a random sample from $U(0,\theta)$ distribution, then discuss unbiasedness and consistency of $T = \left(\frac{n+1}{n}\right) X_{(n)}$ where $X_{(n)} = \max(X_1, X_2, \dots, X_n)$ for θ.

17. State the procedure of finding MLE of parameter θ. Find the moment estimator and maximum likelihood estimator of θ based upon a random sample of size n taken from the following distribution,

$$f(x,\theta) = \begin{cases} \frac{e^{-x^2/(2\theta^2)}}{\theta\sqrt{2\pi}}, & -\infty < x < \infty, \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

18. Define sufficient estimator. If $X_1, X_2, X_3, \dots, X_n$ is a random sample of size n from $U(0, \theta)$ distribution then examine consistency of the estimators for estimating θ.

i) $T_1 = \max(X_1, X_2, \dots, X_n)$,

ii) $T_2 = \min(X_1, X_2, \dots, X_n) + \max(X_1, X_2, \dots, X_n)$,

iii) $T_3 = (n+1)\min(X_1, X_2, \dots, X_n)$

iv) $T_4 = 2\bar{X}$

19. Explain the method of maximum likelihood estimation. Find the moment estimators of parameters α and β based on a random sample of size n from the following distribution,

$$f(x, \alpha, \beta) = \begin{cases} \dfrac{1}{\Gamma\alpha\, \beta^{\alpha}} e^{-x/\beta} x^{\alpha-1}, & x > 0, \alpha > 0, \beta > 0 \\ 0 & \text{otherwise} \end{cases}$$

20. State and prove Cramer-Rao Inequality and define MVBUE.

Q3) Short answer questions (4 marks each)

1. Show that sample mean is unbiased estimator of population mean whereas sample variance is biased estimator of population variance.

2. Explain the iterative produce to derive maximum likelihood estimator of location parameter μ of Cauchy distribution.

3. Show that the sample mean is unbiased and consistent estimator of parameter p of $B(1, p)$ distribution based on a random sample of size n.

4. Obtain sufficient estimator of the parameter θ of the population with p.d.f.
$$f(x, \theta) = \begin{cases} \theta\, x^{\theta-1} & 0 < x < 1, \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$
when a random sample of size n is taken from it.

5. Let X_1, X_2, X_3 be observations from Poisson distribution with parameter θ and $T = 0.4X_1 + 0.2X_2 + 0.4X_3$. Obtain the relative efficiency of T with respect to $\bar{X}$, the sample mean.

6. Distinguish between estimator and estimate.

7. If a random sample of size n is drawn from $N(\mu, \sigma^2)$ distribution, then obtain the amount of information contained in the sample about μ.

8. Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from a distribution with p.d.f.

$$f(x, \theta) = \begin{cases} 1 & \theta - \frac{1}{2} < x < \theta + \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Obtain sufficient statistic for θ.

9. If $X_1, X_2, X_3, \dots, X_n$ is a random sample of size n drawn from $N(\mu, \sigma^2)$ distribution, then obtain the maximum likelihood estimators of μ and σ^2.

10. Show that the mean of a random sample of size n from Exponential distribution with mean θ is minimum variance bound unbiased estimator of the parameter θ.

11. Prove that two distinct unbiased estimators of parameter θ give rise to infinitely many unbiased estimators of θ.

12. Let X_1, X_2 and X_3 be a random sample of size 3 from a distribution with mean μ and variance σ^2. Let $T_1 = X_1 + X_2 - X_3$, $T_2 = 2X_1 - 4X_2 + 3X_3$ and $T_3 = (X_1 + X_2 + X_3)/3$. Examine whether T_1, T_2 and T_3 are unbiased for μ. Which is the most efficient estimator?

13. Define Pitman-Koopman form of Exponential family. Show that Binomial distribution with parameters n and p is a member of Exponential family.

14. If $X_1, X_2, X_3, \dots, X_n$ is a random sample of size n from Geometric distribution with parameter p, then find minimal sufficient statistic for p .

15. If $X_1, X_2, X_3, \dots, X_n$ is a random sample of size n from a distribution with mean μ and variance σ^2 then discuss unbiasedness and consistency of

(i) $T_1 = \frac{X_1 + X_3}{2}$ (ii) $T_2 = X_n$.

16. Define UMVUE. Show that UMVUE is unique when it exists.

17. If $X_1, X_2, X_3, \dots, X_n$ is a random sample from Exponential distribution with mean θ then obtain maximum likelihood estimator of θ.

18. Define Fisher's information. If $X_1, X_2, X_3, \dots, X_n$ is a random sample of size n from Bernoulli distribution with parameter θ, then obtain information contained in sample about parameter θ.

19. Define minimum variance unbiased estimator. If a random sample of size n is drawn from Poisson population with parameter θ, then show that there exist infinitely many unbiased estimators of θ.

20. Define Pitman-Koopman form of Exponential family. If the statistic T is sufficient for θ, then show that any monotonic function $\emptyset(T)$ will also be sufficient for θ.

21. Define sufficient statistic. If $X_1, X_2, X_3, \dots, X_n$ is a random sample drawn from $N(\mu, \sigma^2)$ distribution, where σ^2 is known, then obtain sufficient statistic for μ.

22. Obtain unbiased estimator for the parameter θ based on a random sample of size n drawn from $B(m, \theta)$ distribution, where m is known.

23. Obtain sufficient statistic for parameter θ based upon a random sample of size n from the following distribution,

$$f(x, \theta) = \begin{cases} e^{-(x-\theta)}, & x > \theta,\ \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

24. State Cramer-Rao Inequality. Find MVBUE of parameter θ based upon a sample of size n drawn from Exponential distribution with mean θ.

25. Show that sample mean is a sufficient statistic for the parameter p of a Geometric distribution.

26. Define: i) Relative efficiency, ii) Likelihood function.

27. If $X_1, X_2, X_3, \dots, X_n$ is a random sample of size n drawn from any distribution then show that sample variance is not an unbiased estimator of population variance. Determine unbiased estimator of population variance.

28. Show that a Geometric distribution with parameter p is a member of Exponential family. Hence or otherwise determine minimal sufficient statistic for p.

29. If $X_1, X_2, X_3, \dots, X_n$ is a random sample from $N(\mu, \sigma^2)$ distribution, then find sufficient statistic for vector parameter $\theta = (\mu, \sigma^2)$.

30. State Neyman's factorization criteria of sufficiency. Show that if T is sufficient for θ then g(T) is also sufficient for θ provided that g(.) is any one-to-one function.

31. If $X_1, X_2, X_3, \dots, X_n$ is a random sample of size n from Poisson distribution with parameter θ, then find MVBUE of θ.

32. State Neyman's Factorization criteria of sufficiency and using it obtain sufficient statistic for θ based on a random sample of size n from $B(m, \theta)$ distribution where m is known.