## 1. Shivaji University , Kolhapur
### Question Bank For Mar 2022 ( Summer ) Examination

Subject Code : 81692 Subject Name : Probability Theory and applications
Common subject Code (if any) -------

Q. 1) Select most correct alternative

1. Which of the following is true in case of convergence in probability
   a) $\lim_{n\to\infty}[|X_n - C| \geq \epsilon] = 0$ b) $\lim_{n\to\infty}[|X_n - C| \leq \epsilon] = 0$
   c) $\lim_{n\to\infty}[|X_n| \geq \epsilon] = 0$ d) both A & C

2. Convergence in probability of sample mean to population mean is known as ________.
   a) Weak law of large number c) Both (a) and (b)
   b) Central limit theorem d) None of these

3. A sequence of random variable $\{X_n\}$ ,n=1,2,3,.. is said to be convergent to constant C strongly if
   a] $\lim_{n\to\infty}[|X_n = C|] = 1$ b] $\lim_{n\to\infty}[|X_n - C| \geq \epsilon] = 0$
   c] *both A & B* d] $\lim_{n\to\infty}[|X_n - C| \leq \epsilon] = 0$

4. If $X_i$ have only two values $i^{\alpha}$ and $i^{-\alpha}$ with equal probabilities. W.L.L.N. can be applied to the independent random variables $X_1, X_2$, …… if ____.

   a) $\alpha = ½$ b) $\alpha > 1/2$ c) $\alpha \geq ½$ d) $\alpha < ½$

5. If $X_n \xrightarrow{p} x$ Then……..
   a) $X_n^2 \longrightarrow x$ b) $X_n^2 \xrightarrow{2} x^2$
   c) $X_n^2 \longrightarrow x^2$ d) $X_n^2 \xrightarrow{p} x^2$

6. Which of the following is order Statistics?
   a) Range b) Mean

c) Median d) Mode

7. If $X_n \xrightarrow{p} a$ where a> 0 Then……

a) $X^2_n \xrightarrow{p} a^2$

b) $(X_n - a) \xrightarrow{p} 0$

c) $\frac{1}{X^n} \rightarrow \frac{1}{a}$

d) all are true

8. Using the theory of order statistics we can find distribution?

a) Sample Mean

b) Sample Median

c) Sample Range

d) Both b & c

9. Central limit theorem gives Convergence in distribution of a Sample mean it says that sample mean($X_n$) converges in distribution to ……..

a) Population Mean

b) Normal distribution

c) Both a & b

d) None of these

10. If $Xn \xrightarrow{p} X$ then ……..

a) $X_n + c \xrightarrow{p} X$

b) $X_n \xrightarrow{p} x^2 + C$

c) $X_n + c \xrightarrow{p} x + c$

d) $Xn^2 \xrightarrow{p} x$

11. Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a random sample (r.s.) drawn from U(0, 1) the p.d.f. of largest order statistics is

a) $\beta_1\ (1, n)$

b) $\beta_1\ (n, 1)$

c) $\beta_1\ (1,1)$

d) None of these

12. A sequence of random variables $\{X_n, n \geq 1\}$ is said to converge to X in distribution If

a) $\lim_{n\to\infty} F_n(x) = F(x)$

b) $\lim_{n\to\infty} F_n(x) = 1$

c) $\lim_{n\to\infty} F_n(x) = 0$

d) None of these

13. A random sample of size n is drawn from U(0, 1) then p.d.f. of smallest order static is

a) $\beta_1\ (1, n)$ b) $\beta_1\ (1,1)$

c) $\beta_1\ (n, n)$ d) None of these

14. If $X_1$, $X_2$, $X_3$ is random sample from U(0, 1) then the distribution of sample range is

a) $\beta_2\ (2,2)$ b) $\beta_1\ (2,2)$

c) $\beta_2\ (1, n)$ d) $\beta_1\ (1, n)$

15. If $X_n \xrightarrow{P} X$ then

a) $X_n^2 \xrightarrow{P} X$ b) $KX_n^2 \xrightarrow{P} KX$

c) $(X_n^2 - X) \xrightarrow{P} 0$ d) $X_n^2 \xrightarrow{P} X^2$

16. If $P\ (X_n=0)=1-\frac{1}{n}$, $P\ (X_n=1)=\frac{1}{n}$, n=1,2…then

a) $X_n \xrightarrow{2} 1$ b) $X_n \xrightarrow{2} 2$

c) $X_n \xrightarrow{2} 0$ d) None of these

17. If $X_1$, $X_2$, $X_3$ is random sample (r.s.) from the p.d.f. $f(X) = 2X$, if $0< X < 1$ then p.d.f. of third order statistic (Y). where $0 < Y < 1$ is

a) $24Y^5(1 – Y^2)$ b) $Y^3 (1 – Y^2)$

c) $24 (1-Y^2)$ d) None of These

18. If Yn follows B(n, P) then proportion of success to the number of trials $(\sum Y_n n)$ converges to ………………. In probability as $n\rightarrow\infty$

a) P b) Q

c) $\frac{P}{n}$ d) None of these

19. If $X_1$, $X_2$, .. are independent and identically distributed (i.i.d) random variables having mean μ and variance $\sigma^2 < \infty$ then weak law of large Numbers states

a) $\bar{X}_n \xrightarrow{P} \mu$ b) $\bar{X}_n \xrightarrow{2} \mu$

c) $\bar{X}_n \xrightarrow{L} \mu$ d) None of these

20. A random sample of size n drawn from pdf f(x) having cdf F(x) then the pdf of largest order statistics is__________.

a) Nf(x)[F(x)]

b) nf(x)

c) nF(x)

d) $nf(x)[F(x)]^{n-1}$

21. Let $X_1$, $X_2$, ………,$X_n$ be a random sample (r.s.) drawn from Exp(θ), the p.d.f. of smallest order statistics is

a) Exp(θ)

b) Exp(nθ)

c) $Exp(\frac{\theta}{n})$

d) $Exp(\frac{n}{\theta})$

22. A r.s. of size n is drawn from pdf f(x) having cdf F(x) then the pdf of smallest order statistics is

a) $n\, f(x)\, [1 – F(x)]^{n-1}$

b) $n\, [f(x)]^{n-1}$

c) $[f(x)]^{n-1}$

d) $n\, [f(x)]^{n-1}\, [1 – F(x)]$

23. The pdf of $i^{th}$ order statistic is

a) $[f(x)]^{n-1}$

b) ${}^{n}c_{i}\, f(x)\, [F(x)]^{i-1}[1-F(x)]^{n}$

c) $\frac{n!}{[(n-i)!\,)(i-1)!]}\, f(x)\, [F(x)]^{i-1}[1-F(x)]^{n-i}$

d) none of the above

24. If $X_1$ , $X_2$ , $X_3$ is a r.s. drawn from exp ( ø = 3 ) then probability distribution of smallest order statistic is exponential with parameter ø =

a) 5

b) 9
c) 8
d) None of these

25. Which of the following is order statistic?
a) Mean
b) Median
c) Mode
d) None of these

26. Using central limit theorem sequence of i.i.d. r.v. each having Poisson distribution with parameter ( 1 ) we have
$\sum_{x=0}^{n} \frac{e^{-n} n^x}{x!} \rightarrow$ k where k = ?
a) 0
b) 1
c) ½
d) None of these

27. If $X_n \xrightarrow{p} a$ , a>0 then
a) $1/X_n \xrightarrow{p} 1/a$
b) $X_n^2 \xrightarrow{p} a$
c) $\sqrt{Xn} \xrightarrow{p} a$
d) $aX_n \xrightarrow{p} a$

28. $\frac{Xn}{Yn} \xrightarrow{p} \frac{X}{Y}$ is possible only when
a) $p( Yn = 0 ) = p(y = 0) = 1$
b) $p( Yn = 0 ) = p(y = 0) = 0$
c) $p( Xn = 0 ) = p(X = 0) = 0$
d) none of these

29. if $Xn \xrightarrow{p}$ X then

a) $Xn \xrightarrow{D} X$

b) $Xn \xrightarrow{2}$ 2X

c) $Xn - X \xrightarrow{p} 0$

d) $none\ of\ these$

30. The Central Limit Theorem states that,

a) if n is large, and if the population is normal, then the sampling distribution of
the sample mean can be approximated closely by a normal curve.

b) if n is large, and if the population is normal, then the variance of the sample
mean must be small.

c) if n is large, then the sampling distribution of the sample mean can be approximated closely by a normal curve.

d) if n is large then the distribution of the sample can be approximated closely
by a normal curve.

31. If $X_n \sim U\left(-\frac{1}{n}, \frac{1}{n}\right), n = 1,2, \ldots$ then,

a) $V(X_n) \nrightarrow 0$

b) $X_n \nrightarrow 0$ in probability but $X_n \rightarrow 0$ weakly

c) $X_n \nrightarrow 0$ weakly

d) $X_n \rightarrow 0$ in probability.

32. $X_i, i = 1,2,3, \ldots$ are independently distributed with the following distributions

$$P(X_i = -1) = P(X_i = 0) = P(X_i = 1) = \frac{1}{3} \quad for\ i = 1,3,5, \ldots$$

$$P(X_i = -1) = P(X_i = 1) = \frac{1}{2} \qquad for\ i = 2,4,6, \ldots$$

Let $Y_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then

a) $Y_n \to 1$ almost surely
b) $Y_n \to 0$ almost surely
c) $Y_n \to 0$ in probability but not almost surely
d) $Y_n \to 1$ in probability but not almost surely

33. Let X1, X2, ... be a sequence of independent and identically distributed Chi-square random variables, each having 4 degrees of freedom. Define $S_n = \sum X_i^2$ $n = 1,2,, \dots$ . If $\frac{S_n}{n} \xrightarrow{p} \mu$, as $n \to \infty$ then μ is equal to:

(a) 8 (b) 16 (c) 24 (d) 32.

34. If {Xn} is a sequence of i. i. d. random variables with finite mean and variance, then {Xn} satisfies :

(a) Central limit theorem but not necessarily weak law of large numbers
(b) Weak law of large numbers but not necessarily central limit theorem
(c) Both central limit theorem and weak law of large numbers
(d) Neither central limit theorem nor weak law of large numbers

35. Let $X_1, X_{2,\dots}$ be a sequence of i.i.d. N(0,1) random variables. Then, as n→ ∞, $\frac{1}{n}\sum X_i^2$ converges in probability to

(a) 0 (b) 0.5 (c) 1 (d) 2

36. Which of the following may not hold true for sequences of random variables?

a) $X_n \xrightarrow{p} X \Longrightarrow X_n \xrightarrow{d} X$
b) $X_n \xrightarrow{d} X \Longrightarrow X_n \xrightarrow{p} X$
c) $X_n \xrightarrow{a.s} X \Longrightarrow X_n \xrightarrow{d} X$
d) $X_n \xrightarrow{a.s.} X \Longrightarrow X_n \xrightarrow{p} X$

37. Let $X_1, X_2, \dots, X_n$ are iid exponential random variables with mean 1. What is the distribution of $X_{(n)} = \max(X_1, X_2, \dots, X_n)$ .

a) Exponential with mean 1
b) Exponential with mean n
c) Exponential with mean 1/n
d) None of the above.

38. Let { $X_n$, n ∈ N } be a sequence of random variables with p.m.f. given by,

$P[X_n = -2] = \frac{1}{3n}$ , $P[X_n = 0] = 1 - \frac{2}{3n}$ , $P[X_n = 2] = \frac{1}{3n}$

Then sequence converges,

a) In distribution, probability and quadratic mean to X=2
b) In distribution, probability and quadratic mean to X=0
c) Only in probability to X=0
d) Only in distribution to X=1

39. Let $X_1$, $X_2$,……. be i.i.d. random variables with mean 0 and variance 2.

Consider the following statements

a) $\frac{X_1+X_2\pm\cdots Xn}{n}$ converges to 0 in probability as n→∞
b) $\frac{X_1^2+X_2^2+\cdots X_n^2}{n}$ converges to 2 in probability as n→∞
c) Both a) and b)
d) Neither a) nor b)

40. Let $X_1$, $X_2$,…….$X_n$ be a random sample of size n from U(0, 1) then E[ $X_{(n)}$] is equals to

a) $\frac{n}{n+1}$
b) $\frac{1}{n+1}$
c) $\frac{1}{(n+1)^2}$
d) None of the above.

Q. 2) attempt any two questions

1) State and prove weak law of of large Numbers ( W.L.L.N) for i.i.d. random variables.

2) Define a) Hazard rate

b) Hazard function

c) Survival function

d) IFR distribution

e) DFR distribution

3) Define orders Statistics for a r. s. of size b drawn from a continuous distribution f(x). For a r. s. of size n from U(a, b). Obtain the distribution of i) Minimum ii) Maximum , Order Statistics.

4) State and prove central Limit Theorem (C.L.T.) using m.g.f.

5) Define order Statistics for a r. s. of size n drawn from a continuous distribution f(X). State & Prove the p.d.f. of Yi= $X_{(i)}$ , $i^{th}$ order Statistics. Hence find the p.d.f. of smallest and largest order statistics.

6) State and prove the joint p.d.f. of $i^{th}$ and $j^{th}$ order Statistics [ Yi & Yj]

7) State and Prove Chebychev's inequality for i) Discrete distribution and

ii) Continuous distributions

8) Define joint p.d.f of Yi & Yj . Let Y1 < Y2 < Y3 be an order Statistics of size 3 drawn from the uniform distribution (0,1). Find the distribution of sample range.

9) If T is a life time of a component having exponential distribution with parameter Ө , then find the

i) Distribution function of f(t)

ii) Survival function R(t)

iii) Hazard rate h(t)

iv) Verify E(T) = $\int_0^\infty R(t)\, dt$

10) Show that hazard rate of a series system of components having independent life time is summation of hazard rates of the components. Also life time of series system of independent components with independent IFR life times is IFR.

11) let $X_{(1)}$ be the 1st order statistics of random sample of size n drawn from the distribution having p.d.f.

If Z= n($X_{(1)}$ - then investigate the limiting distribution of Z.

12) Explain the term convergence in i) Probability ii) Distribution function iii) Quadratic mean let $X_1,X_2,\ldots,X_n$ be a sequence of i.i.d random variable with p.d.f. U(0,). Show that Ynas n , where $Y_n$ = max ($X_1,X_2,\ldots,X_n$)

13) a) Let {$X_n$ , n be a sequence of random variable with common distribution show that WLLN exist.

b) Let $X_1,X_2,\ldots,X_n$ be a random sample drawn from poisson distribution with parameter (1) then using CLT show that as n

14) Explain the following terms

i) Reliability of a component of system

ii) Reliability of a system

iii) Reliability of a series system

iv) Reliability of a parallel system

v) Reliability of 2 out of 3 system

15) Let $Y_1<Y_2<Y_3<Y_4<Y_5$ denote the order statistics of random sample of size 5 from exp(1) show that $Z_1=Y_2$ and $Z_2= Y_4-Y_2$ are independently distributed.

16) A continuous random variable X is show that E(X)= 7 $E(X^2)$= 53

i) what is the least value of P [ 3 < X < n ]

ii) what is the greatest value of P [ ]

iii) what is the value of k that guarities P [ ]

17) Obtain the minimal path and cut representation of the following coherent system

i) The 2 out of 3 system

ii) The series system of n independent components

iii) The parallel system of n independent components

18) Let { $X_{(n)}$ , n ≥ 1} be a sequence of i.i.d. r.v., each $X_n$ has continuous distribution with density

$$f(x)_{xy} = \begin{cases} e^{-(Xn-\alpha)} & , \ Xn \geq \alpha \\ 0 & ; O.W. \end{cases}$$

Let $Y_n$ = min ($X_1, X_2, \ldots, X_n$ . Show that $Y_n \xrightarrow{P} \alpha$ . as n → ∞.

19) Let $X_1, X_2, \ldots, X_n$ be i.i.d. r.v. having U(0,1). Find the distribution of r = $X_{(n)} - X_{(1)}$ , where $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be order Statistics. Have find E(r) & Var(r).

20) If {$X_n$, n ≥ 1) be a sequence of r. v. defined by, P[$X_n$ = 0 ] = 1 – 1/n and P[$X_n$ = 1] = 1/n

i) Test for convergence in quadratic mean to zero.

ii) Test for convergence in probability to zero.

Q. 3) attempt any four questions

1) Obtain distribution function of $i^{th}$ order statistic

2) Show that WLLN holds good for a sequence of i.i.d random variables $X_1$, $X_2$, ….$X_n$ having Poisson Distribution with parameter m.

3) Define

a) Convergence in probability

b) convergence in quadratic mean

4) Let $X_1$, $X_2$, ….$X_n$ be a random sample from exponential distribution with mean $\frac{1}{\theta}$. Find the distribution of smallest and largest order statistic.

5) State and prove WLLN for i.i.d random variables with finite variance.

6) Let $X_{(n)}$ be $n^{th}$ order statistic of random sample $X_1$, $X_2$, ….$X_n$ from U(0, $\theta$), $\theta > 0$ with distribution function Fn. Examine the Weak convergence of Fn to any distribution function F.

7) Obtain Minimal path vector and Minimal path set for series system. Also give its block diagram

8) Define Hazard rate, Hazard function and survival function

9) Define reliability of a system and obtain it for

a) Series system of two components

b) parallel system of two components

10) If $X \sim U(-\sqrt{3}\,,\sqrt{3})$ distribution then obtain upper bound for

$P[|X-\mu| > \left(\frac{3}{2}\right)\sigma]$

11) Explain convergence in quadratic mean

12) Explain convergence in $r^{th}$ mean

13) Let $X_{(1)}, X_{(2)}, X_{(3)}$ be the order statistic of random sample of size 3 from the U (0,1). Find the distribution of sample median

14) Obtain distribution of $i^{th}$ order statistic

15) Show that a series system with two components is a coherent system.

16) Define path vector and minimal path vector.

17) State and prove WLLN.

18) Explain convergence in probability of sequence of random variables to

a) constant b) variable

19) A random variable has mean 12 and variance 9 find the bounds on

$P[6 \leq X \leq 18]$

20) If X follows U (0,10) find the bound for $P[|X-5| > 4]$

21) Draw a reliability block diagram and give a structure function for 2 out of 4 system.

22) A system consists of seven identical components connected in parallel. What must be the reliability of each component if the overall reliability of the system is 0.9.

23) For a series system of two components having reliability 0.5 each. Find the reliability of the system.

24) Derive the reliability function for 2 out of 3 good system.

25) Derive the inter relation between survival function distribution function and hazard rate.

26) Let $\{X_k, k \geq 1\}$ be a sequence of independent random variable with

$$P(X_k = \pm 2^k) = \frac{1}{2^{2k+1}}$$

$$P(X_k = 0) = 1 - \frac{1}{2^{2k}}$$

Check whether WLLN holds or not.

27) Let $X_{(1)}$ be smallest order statistic corresponding to r. s. of size n drawn from the distribution having p.d.f. $f(x) = e^{-(x-\theta)}, x \geq 0, \theta > 0$ .Then show that $X_{(1)} \to \theta$ in probability as n → ∞.

28) Let $X_{(1)}, X_{(2)}, X_{(3)}, X_{(4)}$ be order statistic corresponding to random sample of size 4 drawn from the distribution having pdf

$$f(x) = e^{-x} \quad x > o$$

Find $P(X_{(4)} \geq 3)$

29) Define reliability of a system and obtain it for 3 out of 4 system.

30) State different forms of Chebyshev's inequality.