Question Bank For Mar 2022 (Summer) Examination

Subject Code :_81693 Subject Name : Statistics

Paper XIV

(Statistical Inference II)

Q. 1. Choose the most correct alternative (1 mark each)

1. Power curve is a curve obtained by plotting $\theta_1$verses….
   a) $probablityofTypeIerror$
   b) $probabilityofTypeIIerror$
   c) Probability of rejecting the null hypothesis at θ_1
   d) Probability of accepting the null hypothesis at θ

2. Let α denote the size of a test, then which of the following tests exists for testing simple null hypothesis against simple alternative hypothesis?
   a) UMP test of size α b) MP test of size α
   c) UMP test of size (1-α) d) Both a and b

3. If Λ denotes the likelihood ratio test statistic, then under certain regularity conditions which of the following is the asymptotic distribution of -2logΛ?
   a) Chi square distribution b) Normal distribution
   c) Gamma distribution d) t-distribution

4. The critical region of a likelihood ratio test criterion is always….
   a) Left tailed b) Right tailed
   c) Two tailed d) either (b) or (c)

5. Which one of the following statements is a false statement?
   I) A UMP test always exists.
   II) Null hypothesis is simple if it specifies the underlying distribution completely.
   III) Power of a test is the probability of rejecting Null Hypothesis when it is true.

   a)Statement I and II b) Statement II and III
   c) Statement III d) Statements I and III

6. The LR-test for testing $H_0: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$ based on sample from normal population leads to….
   a) One tailed t- test b) Two tailed t-test
   c) Two tailed F- test d) One tailed F- test.

7. Which one of the following non- parametric tests is applicable for a randomness of sample?
   a) Median test b) Sign test c) K-S test d) Run test.

8. The most preferred confidence interval for a parameter should be an interval ….
   a) with shortest width and largest confidence coefficient
   b) with largest width and largest confidence coefficient
   c) based on sufficient statistics
   d) both (a) and (b)

9. In SPRT, decision about the null hypothesis is taken after…..

a) fixed number of observations b) each successive observation

c) at least three observations d) only one observation

10. If α = P(Type I error) and β = P(Type II error), then in SPRT lower and upper cut off points (A and B) are given by….

a) $B = \frac{\alpha}{1-\beta}$ and $A = \frac{1-\alpha}{\beta}$ b) $B = \frac{\alpha}{1-\beta}$ and $A = \frac{\beta}{1-\alpha}$

c) $B = \frac{\beta}{1-\alpha}$ and $A = \frac{1-\alpha}{\beta}$ d) $B = \frac{\beta}{1-\alpha}$ and $A = \frac{1-\beta}{\alpha}$.

11. In SPRT of strength (α, β)= (0.02, 0.03) the stopping bounds (A, B) are given by….

a) $\left(\frac{97}{3}, \frac{2}{98}\right)$ b) $\left(\frac{97}{2}, \frac{3}{98}\right)$ c) $\left(\frac{98}{3}, \frac{2}{98}\right)$ d) $\left(\frac{98}{2}, \frac{3}{97}\right)$

12. Which of the following statements about SPRT are true?

I) Sample size (n) is fixed

II) P(Type I error) =α and P(Type II error)= β are fixed.

III) P(Type II error)= β is minimized for fixed α.

a) Only statement (I) is true. b) Only statement (II) is true.

c) Only statement (III) is true. d) All three statements are true.

13. The likelihood ratio test statistic for testing H_0: $\sigma^2 = \sigma_0^2$ against H_1: $\sigma^2 \neq \sigma_0^2$ based on a sample of size n from normal population N (μ, σ^2) leads to….

a) χ^2_{n-1} distribution. b) χ^2_{n-2} distribution.

c) t_{n-1} distribution. d) t_{2n-1} distribution.

14. Which of the following is most appropriate test for testing simple H_0 against simple H_1?

a) MP level α test b) MP level (1-α) test

c) UMP level (1-α) test d) Likelihood Ratio level (1-α) test

15. If a hypothesis is rejected at the level of significance 0.025, then it ….

a) must be rejected at any level b) must be rejected at the 0.01 level

c) must be rejected at the 0.05 level d) must not be rejected at any other level

16. A sample of one observation, say X is taken from the distribution $f(x) = \theta e^{-x\theta}$, x>0 for testing H_0: θ = 1 against H_1: θ=2. The hypothesis H_0 is rejected if X≤0.5, then the power of a test is….

a) $1-e^{0.5}$ b) $1-e^{-1}$ c) 1-e d) e

17. If random variable X has $N(\mu,\sigma^2)$-distribution then which of the following is a simple null hypothesis?

a) |μ|=0 b) μ=10 c) $\sigma^2=16$ d) μ=10, $\sigma^2=16$

18. A sample of size 144 from $N(\mu,\sigma^2)$ gives the sample mean $\overline{X}$=10 and sample variance s^2=36 then 95% confidence interval for μ is….(given $Z_{0.025}$=1.96)

a) (9.02, 10.98) b) (9.02, 9.98) c) (10.02, 10.98) d) (9.20, 10.98)

19. Which of the following statement is false?

a) Probability of rejecting H_0 when H_1 is true is known as type II error.

b) Neyman Pearson test leads to a most powerful test.

c) Probability of rejecting H_0 when H_0 is true is known as type I error.

d) All the above are true

20. The critical region of two sample Run test is….

a) Two tailed b) Right tailed c) Left tailed d)Either (a) or (b) or (c)

21. For exponential distribution with parameter θ, which of the following hypothesis is simple?

a)H: $\theta < 4$ b) H: θ=|2| c) H: $= \theta > 4$ d) None of these

22. Which of the following statement/s is/are true?

(I) NP-Lemma provides MP-test.

(II)Non-parametric tests are often less powerful.

(III)Size of test is desired to be less than or equal to power of the test.

a) Statement I b) Statement II c) Statement I and III d) All of them

23. Which of the following non-parametric test is applicable for paired samples?

a) Run test b) K-S test c) Sign test d) Median test

24. If we are interested in determining an upper bound for the average nicotine content of certain brand of cigarettes then this is a problem of….

a) Point estimation b) Interval estimation

c)Testing of hypothesis d) None of them

25. The LR test for testing H_0: $\sigma=\sigma_0$ against H_1: $\sigma\neq\sigma_0$ based on random sample of size n taken from $N(\mu,\sigma^2)$ where μ, is known leads to:

a) χ^2-test with n d. f. b) χ^2-test with n-1 d. f.

c) F-test d) Normal test

26. Which of the following non-parametric test is equivalent of the χ^2-test of independence of two attributes?

a) Median test b) Run test c) Mann-Whitney test d) K-S test

27. Given that $P(4.4 \leq \mu \leq 15.7)= 0.90$, Which of the following is correct?

a) The width of confidence interval is 11.3.

b) 4.4 and 15.7 are 90% confidence limits of μ.

c) Probability that μ does not lie in the interval (4.4, 15.7) is 0.1 .

d) All (a) to (c) are true

28. If $\beta(\theta)$ is the probability of type II error of a test for testing H_0: $\theta=\theta_0$ against H_1 and $\theta<\theta_0$ then $1 - \beta(\theta)$ gives the….

a) Power function b) Power of the test at θ

c) Both (a) and (b) d) Neither (a) nor (b)

29 Among the following statements, false statement/s is/are….

I) SPRT is a sequential test

II) For large samples median test leads to chi-square test

III) Median test is used for paired data only.

a) II and III b) I and II c) I, II, III d) III

30. K-S test for single sample is referred to as….

a) Test of randomness b) A test of goodness of fit

c) Both (a) and (b) d) Neither (a) nor (b)

31. Following is the arrangement of male (M) and female (F) in a queue

MMFMFFMFFMFFFMMMFFFM

Total numbers of runs in this queue are….

a) 09 b) 01 c) 20 d) 11

32. Some statements are given below:

I) In SPRT, the size of sample is random

II) Randomness of sample can be tested by using run test,

III) UMP tests always exist.

Among the above false statement is….

a) III b) II c) I d) IV

33. If $X_1, X_2, \ldots, X_n$ is a random sample of size n from exponential distribution with parameter θ then interval estimate of θ is obtained by using….

a) Normal distribution b) t-distribution

c) Chi-square distribution d) F-distribution

34. $T(X, \theta)$ which is a function of random sample $X = (X_1, X_2, \ldots, Xn)$ and parameter θ. The distribution of $T(X, \theta)$ is independent of θ and is used to find C. I. of θ is called as….

a) statistic b) likelihood function c) pivot d) sample space

35. The power of a statistical test for testing null hypothesis H_0 against alternative hypothesis H_1 is the probability of…

a) Reject H_0 when it is true b) Reject H_1 when it is true

c) Reject H_1 when it is false d) Reject H_0 when it is false

36. Which one of the following tests will be used only for two independent samples?

a) Mann Whitney Test b) K-S – Test

c) Sign – Test d) t – Test

37. If a statistical test T for testing simple null hypothesis against simple alternative is at least as powerful as any other test then it is known as….

a) UMP – test b) MP – test c) LR – test d) None of them

38. Which of the following Non-parametric test utilizes the empirical distribution function?

a) Median test b) Wilcoxon's signed rank test
c) Wald-Wolfwitz run test d) Kolmogorov -Smirnov test

39. If $X_1, X_2, \ldots, X_n$ is a random sample of size *n* from N(μ, σ_0^2), where σ_0 is known but μ is unknown then, with usual notations, what is(are) pivotal quantity(quantities) to find C. I. for μ?

a) $\frac{\sqrt{n}(\bar{X}-\mu)}{\sigma_0}$ b) $\frac{\sqrt{n}}{S}\bar{X}$
c) Both a) and b) d) None of the above

40. If $X_1, X_2, \ldots, X_n$ is a random sample of size n from N(μ, σ^2), where μ is known, then what is(are) pivotal quantity(quantities) to find C. I. for σ^2?

a) $\sum_{i=1}^{n}\left(\frac{X_i-\bar{X}}{\sigma}\right)^2$ b) $\sum_{i=1}^{n}(X_i-\mu_0)^2$
c) Both a) and b) are true d) None of the above is true

41. A random sample of size n individuals is selected from a population to study some population characteristic. If X individuals are possessing this characteristic in this sample of size n, then with usual notations, what is (1–α) level confidence interval for population proportion P of this characteristic for large n?

a) $\left(\frac{X}{n}-\frac{Z_{\alpha/2}}{\sqrt{n}}\sqrt{\frac{X}{n}\left(1-\frac{X}{n}\right)},\ \frac{X}{n}+\frac{Z_{\alpha/2}}{\sqrt{n}}\sqrt{\frac{X}{n}\left(1-\frac{X}{n}\right)}\right)$

b) $\left(\frac{X}{n}-\frac{t_{(n-1,\ \alpha/2)}}{\sqrt{n}}\sqrt{\frac{X}{n}\left(1-\frac{X}{n}\right)},\ \frac{X}{n}+\frac{t_{(n-1,\ \alpha/2)}}{\sqrt{n}}\sqrt{\frac{X}{n}\left(1-\frac{X}{n}\right)}\right)$

c) Both a) and b).

d) None of the above.

42. If (L(X), U(X)), where L(X) and U(X) are real valued functions of X, L(X) < U(X) < ∞, is confidence interval for θ based on random sample X then what is length of this confidence interval?

a) U(X) b) (U(X) + L(X))/2 c) U(X) – L(X) d) (U(X) – L(X))/2

43. If $X_1, X_2, \ldots, X_n$ is a random sample of size n from N(μ, σ_0^2), where μ is unknown and σ_0 is known.Then with usual notations, what is (are) (1–α) level confidence interval(s) for μ?

a) $\left(\bar{X}-\frac{\sigma_0}{\sqrt{n}}Z_{\alpha/2},\ \bar{X}+\frac{\sigma_0}{\sqrt{n}}Z_{\alpha/2}\right)$ b) $\left(\bar{X}-\frac{S}{\sqrt{n}}t_{(n-1,\frac{\alpha}{2})},\ \bar{X}+\frac{S}{\sqrt{n}}t_{(n-1,\frac{\alpha}{2})}\right)$
c) Both a) and b) are true. d) None of the above is true

Q.2. Long answer questions (8 marks each)

1. Define power of test. State and prove Neyman-Pearson Lemma

2. Define Most Powerful Test, Uniformly Most Powerful Test

   If $X \geq 2$ is the critical region for testing $H_0 : \theta=2$ against $H_1 : \theta=1$ based on the sample from exponential distribution with parameter θ, then obtain α, β and power of the test.

3. Obtain $100(1-\alpha)\%$ confidence interval for difference between two population means based on two independent large samples of size n_1 and n_2.

4. Define UMP test of size α. Obtain UMP test of size α for testing $H_0 : \theta=\theta_0$ against $H_1 : \theta>\theta_0$ when a sample of size n is drawn from exponential population with parameter θ.

5. Define MP and UMP test. Assuming X has $N(\mu, 4)$ distribution, obtain UMP test of level 0.05 to test $H_0 : \mu=7$ against $H_1 : \mu<7$.

6. Use N-P Lemma to obtain MP critical region to test $H_0 : \mu=\mu_0$ against $H_1 : \mu=\mu_1$ $(\mu_1>\mu_0)$ based on sample of size n from $N(\mu, \sigma^2)$ when σ^2 is known. Obtain power of the test.

7. Derive SPRT of strength (α, β) to test $H_0 : \theta=\theta_0$ against $H_1 : \theta=\theta_1$ $(\theta_1>\theta_0)$ based on sequence of observations from $B(n, \theta)$ population.

8. Define SPRT. Derive SPRT of strength (0.05, 0.02) to test $H_0 : \theta=2$ against $H_1 : \theta=3$ based on sequence of i. i. d. observations from exponential population with mean θ.

9. Describe the procedure of SPRT. Derive SPRT of strength (α, β) to test $H_0 : \mu=\mu_0$ against $H_1 : \mu=\mu_1$ $(\mu_1>\mu_0)$ based on sequence of observations from $N(\mu, 1)$ distribution.

10. Explain the procedure of likelihood ratio test. Derive LRT for testing $H_0 : \sigma^2=\sigma^2_0$ Vs $H_1 : \sigma^2\neq\sigma^2_0$ based on sample of size n from $N(\mu, \sigma^2)$ population.

11. Derive LR test for testing $H_0 : \mu=\mu_0$ against $H_1 : \mu\neq\mu_0$ based on sample of size n drawn from $N(\mu, \sigma^2)$ distribution considering cases i) σ^2 is unknown and ii) σ^2 is known.

12. Obtain $100(1-\alpha)\%$ confidence interval for difference between two population proportions based on two independent large samples.

13. Obtain $100(1-\alpha)\%$ confidence interval for difference between means based on two independent small samples of size n_1 and n_2 from $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ populations.

14. Describe the procedure of Run test for randomness and two samples K-S test.

15. Describe the procedure of Median test and Mann-Whitney U test.

16. Explain Run test and Mann-Whitney U test for two samples

17. Explain procedure for sign test and signed rank test.

Q.3. Short answer questions (4 marks each)

1. Describe procedure to obtain interval estimator of population median using order statistics

2. If X has p. d. f. $f(x) = \frac{2x}{\theta^2}$; $0 \leq x < \theta$. Obtain Type I error and power of test for testing $H_0 : \theta=4$ against $H_1 : \theta=5$ if C. R. {x/x>4}

3. Obtain UMP test for testing $H_0 : p=1/2$ against $H_1 : p>1/2$ based on sample of size n from B(1, p) considering level of significance 0.1

4. Obtain 100(1-α)% confidence interval for population median based on large sample.

5. Obtain 90% confidence interval for population proportion based on large sample of size n

6. Define the terms; size of test, power function, pivotal quantity and critical region

7. Obtain UMP test for testing $H_0 : \theta=2$ against $H_1 : \theta=1$ based on sample of size n from B(15, θ)

8. Obtain 95% C. I. for mean μ of $N(\mu, \sigma^2)$ population based on sample of size 100 when σ^2 is unknown.

9. Define the terms; confidence coefficient, MP critical region, UMP test and p-value.

10. Define pivotal quantity and power of the test. Differentiate between parametric and non-parametric tests

11. Define; Simple and composite hypothesis, Critical value, Confidence interval and Level of significance

12. Suppose 'X' has Bernoulli distribution with probability of success θ. It is proposed to test $H_0 : \theta=0.5$ against $H_1 : \theta=0.3$ based on sample of size 5. The C. R. is $\Sigma Xi > 3$. Find probabilities of Type I and Type II errors. Also find power of test.

13. Obtain 100(1-α)% confidence interval for difference between means based on samples from two independent normal populations

14. Obtain likelihood ratio test for testing $H_0 : \mu=\mu_0$ against $H_1 : \mu\neq\mu_0$ when a sample is drawn from N(μ, 625) population.

15. Obtain UMP test for testing $H_0 : \lambda=2$ against $H_1 : \lambda>3$ based on sample of size n from P(λ) population. Use level α=0.02.

16. Derive SPRT of strength (0.05, 0.02) for testing $H_0 : \lambda=2$ against $H_1 : \lambda=3$ when observations are drawn sequentially from P(λ) population.

17. Obtain SPRT of strength (α, β) for testing $H_0 : \lambda=\lambda_0$ against $H_1 : \lambda=\lambda_1$ when observations are drawn from P(λ) population.

18. Obtain SPRT of strength (α, β) for testing $H_0 : P=P_0$ against $H_1 : P=P_1$ when observations are drawn from B(n, P) population.

19. Derive SPRT of strength (α, β) for testing $H_0 : \theta=2.5$ against $H_1 : \theta=3.5$ in case of observations drawn from exponential distribution with parameter θ.

20. Derive MP test for testing $H_0 : \lambda=2$ against $H_1 : \lambda=1$ when sample of n observations is drawn from $P(\lambda)$ distribution.
21. Derive SPRT of strength (α, β) for testing $H_0 : \theta=2$ against $H_1 : \theta=3$ in case of observations drawn from exponential distribution with mean θ.
22. Write procedure of sign test for single sample.
23. Describe the procedure of Kolmogrov -Smirnov test for two independent samples.
24. Describe the procedure of one sample Wilcoxon's signed rank test.
25. Explain the procedure of single sample Kolmogrov -Smirnov test.
26. Explain procedure for Mann-whitney U test.
27. Explain the test for randomness.
28. Explain non-parametric test procedure for testing goodness of fit for one sample.
29. Explain median test for two independent samples
30. Explain advantages of non-parametric methods over parametric methods.
31. Explain likelihood ratio test and sequential probability ratio test procedures.
32. Derive UMP test of size α for testing $H_0 : \theta=\theta_0$ against $H_1 : \theta<\theta_0$ based on r. s. of size n from exponential distribution with parameter θ.
33. Obtain $100(1-\alpha)\%$ confidence interval for mean of exponential distribution with mean θ.
34. Explain in brief general procedure of determining confidence interval.
35. Explain in brief concept of p-value.
36. Describe likelihood ratio test and state its properties.