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DEPARTMENT OF STATISTICS

Class:-B.Sc-I

Paper-II (Elementary Probability Theory)

Question Bank

……………………………………………………………………………………………

Q. 1) Choose a correct alternative for each of following.

1) If A and B are two events, the probability of occurrence of both A and B is given by…….

   (A) P(AUB) (B) P(A ∩ B) (C) P(A) + P(B) (D) P(A) - P(B)

2) A ticket is drawn from 25 tickets numbered 1 to 25. Define an event as: the number drawn is odd number. The number of elements in this event is……….

   (A) 11 (B) 12 (C) 13 (D) 25

3) If A and B are events such that P(A) = 0.6, P(B) = 0.5 and P(A ∩ B) = 0.3 then $P(\bar{A} \cup B)$ is …….

   (A) 0.4 (B) 0.8 (C) 0.2 (D) 0.3

4) If P(A) = 0.7, P(B) = 0.8, then most appropriate lower and upper possible values of P(A ∩ B) are ……

   (A) (0.1, 0.2) (B) (0.5, 0.7) (C) (0, 0.7) (D) (0, 0.8)

5) Which of the following is the power set corresponding to sample space Ω ={1,2} ?

   (A) {{}, Ω} (B) {{1},{2}}

   (C) {{1}, {2}, {1, 2}} (D){{},{1},{2},{1,2}}

6) If A and B are any two events, then $P(A \cup B)$ is equal to……….

   (A) P(A) + P(B) (B) P(A) + P(B) - P(A ∩ B)

   (C) P(A) + P(B) - $P(A \cup B)$ (D) P(A) + P(B) + P(A ∩ B)

7) If A and B are two events defined on the sample space Ω of a random experiment, then occurrence of A but not B is given by

   (A) $\bar{A} \cap B$ (B) $A \cup \bar{B}$ (C) $A \cup B$ (D) $A \cap \bar{B}$

8) An event consisting of these elements which are not in A is called………

   (A) Simple event (B) Compound event

   (C) Complementary event (D) Primary event

9) If a sample space Ω has n elements then its power set will have……..

   (A) n/2 elements (B) 2^n elements (C) n elements (D) 2n elements

10) An event containing only one element is called………

   (A) Sure Event (B) Impossible Event

   (C) Compound Event (D) Elementary Event

11) The probability of getting 5 Sundays in the month of May is…….

(A) 2/7 (B) 3/7 (C) 1/7 (D) 6/7

12) A and B are two events, then probability of at least one of them will occur is given by……

(A) Multiplication law of probability (B) addition law of probability

(C) Conditional probability (D) none of these

13) Power set of a sample space having 3 sample points contains…... Subsets of the sample space.

(A) 3 (B) 6 (C) 8 (D) 10

14) In a group of 10 men, 6 are graduates. A group of 3 men are selected at random. The probability group consist of all graduates is…….

(A) 1/6 (B) 0.1 (C) 0.2 (D) none of these

15) If the letters of the word SUN are arranged at random, the probability that the letter U gets the middle position is……

(A) 2/3 (B) 1/6 (C) 1/3 (D) 5/6

16) If a coin tossed 3 times the sample space has ….. points.

(A) 2 (B) 3 (C) 8 (D) 16

17) If A and B are any two events then the probability that at least one of them will occur is denoted by………

(A) P(A) or P(B) (B) P(A) + P(B)

(C) $P(A \cup B)$ (D) $P(\bar{A}) \cup P(\bar{B})$

19) If A and B are any two events then the following statement is true.

(A) $P(A \cap B) \leq P(A)$ (B) $P(A \cap B) \leq P(B)$

(C) $P(A \cup B) \leq P(A) + P(B)$ (D) All of these

20) The probability of getting the sum greater than 9 in a throw of a pair of fair dice is………

(A) 1/36 (B) 2/9 (C) 1/6 (D) 5/9

21) The probability of getting 5 Sundays in a month of February of a leap year is…….

(A) 2/7 (B) 1/7 (C) 5/29 (D) 5/52

22) The sample space corresponding to the experiment "Three seeds are planted and total number of seeds germinated are recorded after a week" is ………

(A) {0, 3} (B) {0, 1, 2, 3} (C) {1, 2, 3} (D) [0, 3]

23) Which of the following pairs is a pair of mutually exclusive events in drawing a single card from a pack of 52 cards?

(A) a heart and a queen (B) an even number and a spade

(C) a club and a red card (D) an ace and an odd number

24) The probability of drawing one white ball randomly from a bag containing 6 red, 8 black, 10 yellow and 1 green ball is…………

(A) 1/25 (B) 0 (C) 1 (D) 14/25

25) If A and B are two events such that $A < B$, then…….

(A) P(A) = P(B) (B) $P(A) \geq P(B)$ (C) $P(A) \leq P(B)$ (D) None of these

26) If A^C is the complementary event of A, then $P(A^C)$ is……….

(A) 1 (B) 0 (C) P(A) (D) 1- P(A)

27) A set is called doubleton, if its cardinality is ……..

(A) 2 (B) ≥ 2 (C) ≤ 2 (D) ∞

28) A box has 4 red and 4 blue balls. Five balls are selected at random, then probability of getting at least 1 blue ball is given by…..

(A) 1 (B) 1/2 (C) 0 (D) None of these

29) I: A and B are mutually exclusive

II: $P(A \cap B) = 0$

III: A and B are independent events. Then

(A) I⇒II (B) I⇒III (C) III⇒I (D) All of the above

30) Let A and B be two events such that P(A) = 0.2 and P(B) = 0.8. Which of the following statement is always true?

(A) $P(A \cup B) = 1$ (B) $P(A \cap B) = 0.16$

(C) $P(A \cap B) \leq 0.2$ (D) None of the

31) If P(A) = 1/3, P(B) = 3/4, $P(A \cup B) = 11/12$ then P(B|A) =………..

(A) 1/6 (B) 4/9 (C) 1/ 2 (D) none of these

32) Let A and B be two events defined on Ω and P(B) > 0 then P(A/B)=P(A)/P(B)

(A) $B < A$ (B) $A \cap B = \phi$ (C) $A < B$ (D) none of these

33) If $B < A$ then $P(B|A)$ is ……

(A) 0 (B) 1 (C) P(A)/P(B) (D) P(B)/P(A)

34) For any event A, $P(\Omega/A)$ is……..

(A) One (B) Zero (C) P(A) (D) P(1/A)

35) If P(A) = 1/3, P(B) = 1/ 4, P(A|B) = 1/6 then P(B|A)=……..

(A) 1/4 (B) 1/8 (C)3/4 (D) none of these

36) If an event B has occurred and it is known that P(B) =1 then P(A|B) is equal to……

(A) One (B) Zero (C) P(B) (D) P(A)

37) If A and B are mutually exclusive events then P(A|B) = ……….

(A) P(A) (B) P(B) (C) One (D) Zero

38) If P(A) = 0.3, P(B) = 0.4 and A and B are mutually exclusive events then $P(A^C/B^C)$=……..

(A) 0.3 (B) 0.4 (C) 0.12 (D) 0.5

39) For an event A, P(A/A) is………

(A) One (B) Zero (C) P(A) (D) Not determined

40) If $A < B$ then $P(A|B)$ is……

(A) 0 (B) 1 (C) P(A)/P(B) (D)P(B)/P(A)

41) Let A and B be two events such that P(A) = 0.4, P(B) = k and P(AUB) = 0.6. If A and B are exclusive events then the value of k is…………

(A) 0.3 (B) 0.6 (C) 0 (D) 0.2

42) If A and B are exclusive events and P(A) = 0.3, P(B) = 0.4 then $P(A\cap B)$ =……….

(A) 0.3 (B) 0.4 (C) 0.7 (D) 0

43) A card is drawn from a pack of cards. If it is a picture card, the probability that it is a king is…..

(A) 1/3 (B) 1/4 (C) 1/12 (D) None of these

44) If A is an event defined on the sample space Ω and P(A) > 0 then following statement is false.

(A) $P(B/A) \geq 0$ (B) $P(\Omega/A) = 1$

(C) $P(B \cup C/A) = P(B/A) + P(C/A)$ (D) None of these

45) For an event A, $P(A/\bar{A})$ is………

(A) P(A) (B) $P(\bar{A})$ (C) 0 (D) 1

46) If $A < B$ then $P(B|A)$ is ………

(A) 0 (B) P(A) (C) P(B) (D) 1

47) If A and B are exclusive events and P(A) = 0.3, P(B) = 0.4 then $P(\bar{B})$=

(A) 0.7 (B) 0.5 (C) 0.6 (D) 0

48) If A and B are exclusive events and P(A) = 0.2, P(B) = 0.5 then $P(\bar{A}\cup\bar{B})$ =……..

(A) 0.2 (B) 0.3 (C) 0.4 (D) 0.7

49) If P(A) = 0.5, P(B) = 0.6 and P(B/A) = 0.9 then $P(A\cap B)$ =………

(A) 0.3 (B) 0.4 (C) 0.45 (D) 0.54

50) The probability that A card drawn from a pack of cards is a red card given that it is a king is ………

(A) 1/4 (B) 1/2 (C) 1/13 (D) 1/12

51) A function which generates probabilities is ……

(A) Mean (B) Variance

(C) Probability generating function (D) cann't be defined

52) The p.g.f of a r.v. X is P(s) then the p.g.f of X+3 is…….

(A) $s^3P(s)$ (B) $P(s) + s^3$ (C) P(s+3) (D) P(3s)

53) Let X be a r.v. then V(aX) is……

(A) aV(X) (B) V(X) (C) $a^2V(X)$ (D) 0

54) The distribution function of a discrete random variable is…..

(A) Non increasing (B) Non decreasing (C) Not constant (D) Not exponentail

55) If X & Y are independent random variable then P(X/Y) =…..

(A) P(X) (B) P(Y) (C) P(X, Y) (D) None of these

56) The expectation of a number of a throw of a single die is….

(A) 3 (B) 7/2 (C) 1/6 (D) 140

57) If X is discrete random variable with mean E(X) then $E[X-E(X)]^2$ is……

(A) Mean (B) Variance (C) S.D (D) Raw moment

58) A random variable is a ……defined on sample space.

(A) Probability (B) Function (C) Constant (D) Variable

59) If X & Y are two independent random variables then the Probability generating function of X+Y is…..
(A) $P_x(s).P_y(s)$ (B) $P_x(s) + P_y(s)$
(C) $P_x(s) - P_y(s)$ (D) None of these

60) The value of joint distribution function F(x,y) lies within the limits …..
(A) -1& 1 (B) -1 & 0 (C) -∞ & 0 (D) 0 & 1

61) A random variable X is said to be discrete if the sample space of X has …. Sample points
(A) Finite (B) Countably infinite
(C) Finite or countably infinite (D) Uncountably infinite

62) If X is a discrete r.v. , the expected value of s^x , for $|S| \leq 1$ is known as…….
(A) Probability distribution function (B) Characteristic function
(C) Probability generating function (D) Moment generating function

63) The p.g.f. of discrete r.v. X is $0.5 + 0.3S + 0.2S^3$. Then E(X) is…….
(A) 0.9 (B) 1 (C) 1.5 (D) 0.5

64) If X takes value 1, 2 with P(X=1) = 0.2 and E(X) = 2.2 then P(X=2) is ……..
(A) 0.5 (B) 0.1 (C) 0 (D) 1

65) For the following distribution

X :	0	1	2
P(x) :	k	5k	4k

The value of k is ……
(A) 1 (B) ½ (C) ¼ (D) 1/10

66) If P(x) is p. m. f. of a discrete r. v. X, then Σ p(x) is equal to ……..
(A) One (B) Zero (C) Infinity (D) None of these

67) If r. v. X takes values -1, 0, 1 with probabilities 0.3, 0.4, 0.3 respectively, then |X| takes values with probabilities
(A) (0.5, 0.5) (B) (0.4, 0.6) (C) (0.6, 0.4) (D) None of these

68) From the distribution function we can find
(A) Mean (B) Median (C) Mode (D) None of these

Q. 2) Long answer questions.

1) Define i) An event
ii) Sample Space
iii) Power set of a sample space
iv) Mutually exclusive events
v) Compliment of an event

2) With usual notation show that,
i) P(A∪B) = P(A)+ P(B) – P(A∩B)
ii) $P(A^C \cap B)$ = P(B) – P(A∩B)

3) Define probability measure and prove that P(∅)=0 and $P(A^C)$ = 1- P(A)

4) For two events show that, P(A∩B) ≤ P(A) ≤ P(A∪B) ≤ [P(A)+ P(B)]

5) Given P(A) = 3/4, P(B) = 5/8 then show that,

i) P(A∪B) ≥ ¾
ii) 5/8 ≥ P(A∩B)
iii) 3/8 ≤ P($A \cap B$) ≤ 5/8
iv) 1/8 ≤ P($A \cap B^C$) ≤ 3/8

6) Give an axiomatic definition of probability. Show that conditional probability satisfies the axioms of probability.
7) State and prove Baye's theorem.
8) State and prove multiplication law of probability.
9) State partition of sample space and Baye's theorem. A man is equally likely to choose one of the three routes C_1, C_2 and C_3 from his house to railway station. The probability of missing the train by the routes C_1, C_2 and C_3 are 2/5, 3/10, 1/10. He sets out on a day and misses the train. What is probability that the route C_2 was selected?
10) A fair coin tossed twice and the events are defined as follows
A: Head on the first toss
B: Head on the Second toss
C: Same face on the both tosses
Discuss the pair wise and mutual independence of events A, B and C.
11) Define cumulative distribution function. State & Prove properties of distribution function.
12) Explain the following terms giving suitable illustrations.
    i) Random variable
    ii) Discrete random variable
    iii) Probability mass function of discrete random variable
    iv) Distribution function of discrete random variable
13) Define probability generating function (p.g.f.) of a random variable X. Then find mean and variance from p.g.f..
14) Explain Pearson's coefficients of skewness and kurtosis.
15) If a random variable X has the p.g.f. $P_x(s) = \left(\frac{ps}{1-qs}\right)^n$ where p+q = 1 and |s| < 1, find the mean & variance of X.

Q. 3) Short answer questions.

1) Define i) a-priori definition of probability
    ii) Axiomatic definition of probability
2) If A and B are independent events with P(A) =1/2 and P(B) = 1/4 then obtain P(A∪B) and P(A/B).
3) If A and B are independent events then show that A^C and B^C are also independent events.
4) If A, B and C are mutually independent events then show that A∪B and C are also independent events.
5) If A and B are exclusive events then show that

i) P (A/B) = 0 ii) $P(A/B^C) = \frac{P(A)}{1-P(B)}$

6) Define i) mutually exclusive events ii) Independent events. Give an illustration of each.
7) If B⊂A, prove that P ($A \cap B^C$) = P (A) - P (B). Hence deduce that P (B) ≤ P (A).
8) Define i) Union of two events
   ii) Intersection of two events
   iii) Impossible event
   iv) Certain event
   v) Complementary event
9) An urn contains 6 blue, 5 white, and 7 red balls. A person draws 4 balls from the box what is probability that among the balls drawn i) two are red and two are blue ii) two are blue and two are white.
10) If a fair coin and a die are tossed together. Find i) Sample space ii) P(head on a coin and an even number) iii) P (number > 4).
11) If A and B are independent events then show that A and B^C are also independent events.
12) If Ω= (ω_1, ω_2, ω_3, ω_4), A= (ω_1, ω_2), B = (ω_2, ω_3), and C = (ω_1, ω_3) then discuss about pair wise independence and mutual independence of three events A, B and C.
13) If A and B are exclusive events then show that

    i) P (B/A) = 0 ii) P (A/ A∪B) = $\frac{P(A)}{P(A)+P(B)}$
14) If A and B are any two events then prove that P (A^C/B) = 1- P (A/B).
15) If P(A) =1/4 , P(A/B) = 1/3, P(B/A) = 1/2 then find P(A/B^C).
16) If A and B are independent and P(A) = 1/4 and P(B) = 1/3, Find i) P(A∪B) ii) P($A^C \cap B^C$).
17) Let P be the probability function on sample space Ω= (ω_1, ω_2, ω_3). Find P (ω_1), P (ω_2) if P (ω_1) = 2 P (ω_2) and P (ω_3) = 1/3.
18) If A and B are independent events then show that A^C and B are also independent events.
19) Derive the relation between distribution function and probability mass function.
20) Construct a discrete random variable on a sample space of tossing of three fair coins.
21) Define the following terms
    i) Probability mass function.
    ii) Median
    iii) Mode
22) Let $(X = x) = \frac{x+1}{10}$, for $x = 0,1,2,3$. Verify whether P(X) is probability mass function. If it is so, find the distribution function of X. Also evaluate P(0 < X < 3) and P(X ≤ 2).
23) Define mean & variance of a random variable and prove that
    $V(X) = E(X^2) - [E(X)]^2$

24) Define mean & variance of a random variable and find the effect of change of origin and scale on them.

25) Define probability generating function (p.g.f.) of a random variable X. What is the effect of change of origin and scale on p.g.f..

26) If a and b are constants, prove that

i) $E(a) = a$

ii) $E(aX+b) = aE(X) + b$

iii) $V(aX+b) = a^2V(X)$

27) The probability distribution of X is as follows:

X	0	1	2	3	4
P(X=x)	k	3k	5k	2k	k

Find i) k ii) E(X) iii) Var(X) iv) $P(X \geq 2)$ v) Mode of X

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